1 **EVALUATION OF THE STIFFNESS MATRIX OF AN INDETERMINATE TRUSS USING MINIMIZATION TECHNIQUES**

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ABSTRACT

For an existing structure the evaluation of the Stiffness matrix may be hampered by certain physical limitations such as material deterioration resulting from prolonged use in a corrosive environment.The following is a method that allows the determination of the member stiffness of an indeterminate truss through a Minimization technique of an Error Function. Thus exact sectional and material properties needed for thorough structural analysis do not have to be known a priori.

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ACKNOWLEDGMENT

The author wishes to express his gratitude to Mr. Mohammed H. Arafa* for the invaluable assistance he rendered in the preparation of this paper.

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INTRODUCTION

Analysis of existing old structures is usually difficult to perform due to the fact that material properties change over time. This limitation is frequently encountered in industrial buildings housing a corrosive environment. Therefore standard structural analysis methods become inapplicable if accurate results are sought; this is because presently known methods of analysis hinge upon the availability of the member properties and the geometry of the structure. The following method overcomes such a difficulty through the application of known forces at the nodes and the subsequent measurement of the associated displacements. Helou(1) presented the method but the presentation was limited to determinate trusses. The following is an extension of the same principles, albeit in more general terms, in order to make the method equally applicable to indeterminate systems.

In structural mechanics the force , displacement equilibrium equation is written in the following form

$$\{F\} = [K] \{X\} \quad (1)$$

In which
{F} is the force vector applied at the nodes,
[K] is a global stiffness matrix,
{X} is the associated displacement vector at the nodes.
For an exact solution of equation 1 , the following statement is true

$$\{F\}-[K] \{X\} = 0.0 \quad (2)$$

And when equation 1 is not exact an error vector E may be introduced as follows.

$$\{E\} =\{F\} - [K] \{X\} \quad (3)$$

A typical element in the error vector of equation 3 is of the form

$$E_i = F_i - \sum_{i=1}^{n} K_{ij} X_i \quad (4)$$

where n is the number of degrees of freedom.

The problem is now reduced to minimizing to zero the error vector of equation 4.For this to be achieved an error function has to be constructed. This is done by squaring both sides of equation 4,i.e. by forming the inner product of the right hand side of equation 4 with itself and carrying out the summation over the number of loading conditions. The necessity of using more than one loading vector will be made clear later on in the text. The error function takes the form.

$$EF=\sum_{j=1}^{m}\sum_{i=1}^{n}[F_i^j-\sum_{l=1}^{n}K_{il}X_l^j]^2 \tag{5}$$

In which m is the number of loading cases .

The solution proceeds by taking the first derivative of the error function, EF, with respect to each unknown element of the stiffness matrix and setting it equal to zero , i.e.

$$\frac{\partial EF}{\partial K_i}=0.0 \tag{6}$$

This operation will result in a set of linear simultaneous equations equal in number to the elements of the structure.

$$\sum_{j=1}^{m}[J^j]^T(\{F\}^j-[K]\{X\}^j)=0.0 \tag{7}$$

in which [J] is a jacobean matrix defined as follows

$$[J] = \begin{bmatrix} \frac{\partial EF_1}{\partial k_1} & \cdots & \frac{\partial EF_1}{\partial k_m} \\ \vdots & \cdots & \vdots \\ \frac{\partial EF_n}{\partial k_1} & \cdots & \frac{\partial EF_n}{\partial k_m} \end{bmatrix} \tag{8}$$

which will yield to

$$\sum_{j=1}^{m} [J^j]^T \{F\}^j - \sum_{j=1}^{m} [J^j]^T [K] \{X\}^j = \sum_{j=1}^{m} [J^j]^T [J] \{k\} \tag{9}$$

where {k} is the vector of element stiffnesses in local coordinates furthermore with $[J_j]^T[J]$ invertible the solution for {k} is written formally as

$$\{k\} = (\sum_{j=1}^{m} [J^j]^T [J]\ [K])^{-1} . \sum_{j=1}^{m} [J^j]^T \{F\}^j \tag{10}$$

From the solution it remains to be is shown that {J} {K} = {K} {X} This will be done in the course of the illustrative example.

ILLUSTRATIVE EXAMPLE

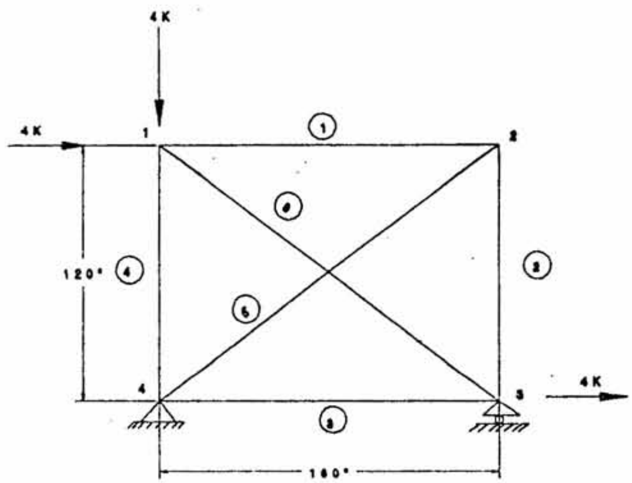

Figure 1-a
LOAD CASE NO.1

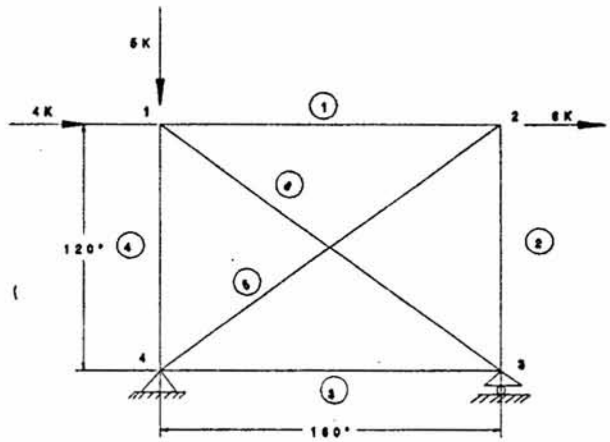

Figure 1-b
LOAD CASE NO. 2

For the indeterminate truss shown in Figure 1 a and b. All elements have an area of 4 in^2 and modulus of elasticity = 30000 ksi .The global reduced stiffness matrix obtained by standard structural analysis methods is

$$\{K\}=\begin{bmatrix} k1+.64k6 & . & . & . & SYM \\ -.48k6 & k4+.36k6 & . & . & . \\ -k1 & 0 & k1+.64k5 & . & . \\ 0 & 0 & .48k5 & k2+.36k5 & . \\ -.64k6 & .48k6 & 0 & 0 & k3+.64k6 \end{bmatrix}$$

To assure the existence of a solution two loading cases are used. The following are the loading cases together with the associated displacements used in the present numerical experiments.

$$\{F^1\}=\begin{Bmatrix} 4 \\ -4 \\ 0 \\ 0 \\ 4 \end{Bmatrix} \qquad \{X^1\}=\begin{Bmatrix} .01017 \\ -.002694 \\ .007154 \\ -.001694 \\ .007654 \end{Bmatrix}$$

and

$$\{F^2\}=\begin{Bmatrix} 4 \\ -5 \\ 6 \\ 0 \\ 0 \end{Bmatrix} \qquad \{X^2\}=\begin{Bmatrix} .01663 \\ -.001604 \\ .01733 \\ -.004104 \\ .006037 \end{Bmatrix}$$

The error vector is written as

$$\begin{Bmatrix} E1 \\ E2 \\ E3 \\ E4 \\ E5 \end{Bmatrix} = \begin{Bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \end{Bmatrix} - \begin{bmatrix} k1+.64k6 & . & . & . & SYM \\ -.48k6 & k4+.36k6 & . & . & . \\ -k1 & 0 & k1+.64k5 & . & . \\ 0 & 0 & .48k5 & k2+.36k5 & . \\ -.64k6 & .48k6 & 0 & 0 & k3+.64k6 \end{bmatrix} \begin{Bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \end{Bmatrix}$$

The Jacobean Matrix is

$$[J] = \begin{bmatrix} X1-X3 & 0 & 0 & 0 & 0 & .64X1-.48X2-.64X5 \\ 0 & 0 & 0 & X2 & 0 & -.48X1+.36X2+.48X5 \\ -X1+X3 & 0 & 0 & 0 & .64X3+.48X4 & 0 \\ 0 & X4 & 0 & 0 & .48X3+.36X4 & 0 \\ 0 & 0 & X5 & 0 & 0 & -.64X1+.48X2+.64X5 \end{bmatrix}$$

Upon performing the operation described in equation 10 the unknown elements stiffnesses are the readily obtained. They are the same as would be obtained by evaluating EA/L for each element.

$$\{K\} = \begin{bmatrix} 749.36 \\ 1000.63 \\ 749.87 \\ 1000.44 \\ 600.28 \\ 599.48 \end{bmatrix} kip/in$$

CONCLUDING REMARK

From the previous presentation and example it is apparent that the proposed method requires a complete set of data i.e. A displacement reading must be available at every degree of freedom of the structure.This is a shortcoming that perhaps can be avoided through further research in this area.

REFERENCES

1- Helou,A.H., Retrieval of System Properties of Existing Structures, International Association for Bridge and Structural Engineering, Colloquium 1993 , Copenhagen, Denmark .

2- Matzen,V.C. And McNiven, H.D., Investigation of the Inelastic Characteristics of a Single Story Steel Structure Using System Identification and Shaking Table Experiments, Report No.EERC 76-20 Earthquake Engineering Research Center, University of California, Berkeley, August 1974.

3- Shield, P.C.,Elementary Linear Algebra,Worth Publisher INC.

4- Touqan,A.R.,Najah91, An Engineering Analysis Program for Framed Structures,November 1990.

3- White,R.N.,Gergely,P.,Sexsmith,R.C.,Structural Engineering, Combined Edition, John Wiley and Sons.