An-Najah National University

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RENEGING BEHAVIOR ON DRIVE-THRU QUEUES: A CASE STUDY-BASED APPLIED MATHEMATICAL INVESTIGATION

By

Ayman Wasef Mohammad Dbeis

Supervisor

Prof. Khaled Al-Sahili

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This Thesis was Defended Successfully on 02/03/2023 and approved by

Prof. Khaled Al-Sahili
Supervisor — Signature

Dr. Wasel Ghanem
External Examiner — Signature

Prof. Sameer Abu-Eisheh
Internal Examiner — Signature

Dedication

To those who saw me through my eyes.

Acknowledgement

First and foremost, I am deeply indebted to Prof. Khaled Al-Sahili for being an outstanding supervisor. His constant support and invaluable suggestions made this work successful, as he has had an immense impact on my academic evolution. For all of that, I offer my heartfelt thanks.

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Declaration

I, the undersigned, declare that I submitted the thesis entitled:

RENEGING BEHAVIOR ON DRIVE-THRU QUEUES: A CASE STUDY-BASED APPLIED MATHEMATICAL INVESTIGATION

I declare that the work provided in this thesis, unless otherwise referenced, is the researcher's own work, and has not been submitted elsewhere for any other degree or qualification.

Student's Name: Ayman Wasef Mohammad Dbeis

Signature:

Date: 02/03/2023

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Abstract

Drive-thru service was never exclusive for fast food restaurants only, but it is still more common for them. During the COVID-19 pandemic, drive-thru services extended to the health sector as well, to maintain the physical distancing. Palestine barely includes fully-active integrated drive-thru services; however, an expansion for full infrastructure drive-thru services is expected in the near future.

Whenever there is a need to obtain the service by queuing and waiting, queued customers show prominently their desire to be served as fast as possible. Impatient customers' behaviors appear clearly in two main acts: reneging and balking. Reneging, as the main scope of this study, is one of ignored behaviors in queuing analysis, which makes analyzing and managing queues not reflecting the reality of queuing. This is due to the limited knowledge about such an event and factors leading to its occurrence.

This research aims to reach a deeper understanding of the queuing theory and the queue's characteristics, to make it more reflective and interpretive to real life queuing problems. This is done by analyzing the behavior of impatient reneging customers, and identifying how their decision is affected by the queue's characteristics.

Through non-participant observations, the required data were manually collected using surveillance camera recordings for Rio Café's drive-thru service in Al-Bireh City. Given the purpose of this study, the data were mathematically analyzed to determine the probability distribution functions of the reneging behavior and the factors affecting waiting customers' decision. In addition, linear and non-linear regression models were developed to predict the reneging rate and the relative frequency of reneging customers.

The study showed that the exponential distribution is the probability distribution function of the time intervals between successive reneging events of at least one reneging customer per time interval. However, the reneging rate does not follow the Poisson distribution.

It was concluded that the long waiting time is not the only factor causing impatient customers to renege on the queue, but other important factors affect this behavior. These factors are integrated and jointly distributed, leading to the customer's decision to leave the queue.

Keywords: Reneging Behavior, Impatient Customers, Retention, Queuing Theory, Joint Probability Distribution, Spatial-Dynamic Modelling.

Chapter One
Introduction

1.1 Background

When hearing “a drive-thru service”, where customers are served by driving through it without the need to leave their vehicles, the first thing that comes to the reader’s mind is fast-food restaurants. Actually, it started in the United States with drive-in restaurants, where customers are served by a staff while they are parked, which was soon first replaced by drive-through in 1931 (1).

Drive-thru service was never exclusive for fast-food restaurants only, but it is still more common for them. Popular Mechanics Magazine (2) published that the Grand National Bank in St. Louis in the USA provided a drive-up service in 1930, so customers can drive up to a window that was built close to the curb, to make deposits through it without leaving their cars. Since then, the drive-thru service has spread to other countries. In the UK, drive-thru services have been there when Westminster Bank was the first to adopt a drive-thru banking service in 1959, as many banks followed after (3). In 2010, Tesco launched the first drive-thru supermarket in the UK (4).

During the COVID-19 pandemic, drive-thru services extended to the health sector as well. Hussain et al. (5) addressed the use of drive-thru services in many pharmacies around the world, especially in Arab Gulf countries during the pandemic, which was originally started in the USA in 1990. Furthermore, drive-thru testing stations were widely used across the world and in Middle Eastern countries, which proved their efficiency in detecting and isolating infected cases (6).

Nowadays, there are many drive-thru services around the world such as ATMs, pharmacies, car wash stations, grocery stores, and more service providers are racing to adopt this service as an added-value for their businesses. This kind of service suits the pattern of our fast life. In fact, it seems it suits more the human nature in being comfortable, while getting things done. With such a minimal physical effort, customers in drive-thru services are being served without the need to leave their own vehicles.

Despite the world-wide spread of such a service, Palestine barely includes fully-active integrated drive-thru services. Even some food franchise restaurants officially claim they have a drive-thru lane, these are not actively or regularly functioning. On the other hand, some pharmacies are newly having a drive-thru service. To be more exact, it is not a full neither an integrated drive-thru service, while it tends to be more as a picking up service for pre-orders. Even though, an expansion for full infrastructure drive-thru services is expected in the near future in Palestine, especially that this service is actively introduced for many types of services in the Middle East and regional countries.

Nevertheless, most drive-thru services are not sufficiently taking queuing consequences into consideration. Waiting times and queues' lengths are important factors in determining customers' satisfaction. In addition, customers' behaviors and decisions to join or leave the queue in response to waiting times and queues' length, affect the queue's characteristics itself, and may negatively affect other customers' behaviors or decisions under particular circumstances. For instance, long queues and waiting times make customers dissatisfied with the service, and may prompt them to leave the line or not return again. This will cost the business its sales and may cost it its reputation. Therefore, it is paramount to understand the queuing issues and customers' behaviors responding to the queue's characteristics.

The basic form of the queuing theory aims to keep the queue in equilibrium; described by May (7) as a steady-state queue, by ensuring that the arrival rate of customers is less than the served customers' rate. Otherwise, the queuing system will collapse, and queuing performance measures, such as the waiting time and the queue length, will go to infinity.

1.2 Problem Definition

Whenever there is a need to obtain the service by queuing and waiting, queued customers show prominently their desire to be served as fast as possible, as human nature hates waiting, especially if it is for unknown times. In the meantime, impatient customers' behaviors appear clearly in two main acts: reneging as leaving the joined queue without getting the service; while balking, as a special case of reneging, is not joining at all (8).

The author argues that some customers' behaviors toward certain queues' characteristics that are greater or lower than their acceptable preferences might cause more problems than other behaviors in some cases under particular circumstances. For instance, in drive-thru services that have no room in their waiting lines for customers' turn back, a disturbance would occur when an impatient customer decides to leave the queue. This disruption would, somehow, affect the decision of other waiting customers to wait more time, by renegers that make the line seem longer for those contemplating to join and may prompt some to balk (9). This case would be more clarified if a group of (n) impatient customers decide to give up the service simultaneously. In such a case, especially if the departing customers are obviously related together from other customers' point of view, a lobby would be created affecting not only other customer's acceptable waiting time, but also the service time and the decision of choosing the service itself again.

In other words, reneging or any customers' behavior toward the queue after entering the system, may be more important to study in case of drive-thru services than other behaviors before entering the system, such as balking.

Reneging, as the main scope of this study, is one of ignored behaviors in queuing analysis, knowing that it is a daily event in any random queue, which makes analyzing and managing queues not reflecting the reality of queuing. This is due to the limited knowledge about such an event and factors leading to its occurrence. A worthy addition to the queuing theory will be added by analyzing this behavior, especially that customer(s) reneging on a queue is, somehow, their expressed dis-satisfaction with the provided service and related waiting time, as a result of inadequate service time or other factors.

In fact, longer waiting times mean higher consumptions in goods customers are waiting to have, as Ülkü et al. (10) showed, and so in energy of the facility they are waiting in or vehicles they are driving thru. In other words, time is money, and there is a cost of waiting. This cost is not necessarily directly related to money, but it could be converted in terms of it. For example, customers will not come back again to a service known that has a long queue or a slow service rate. Komashie et al. (11) showed that there is a strong negative relationship between waiting time and customer satisfaction.

Furthermore, by knowing the reneging probability of occurrence and the probability distribution function of each of the factors leading to it, such a behavior will be more understandable and predictable. Therefore, service providers and decision makers can estimate expected economic loss when reneging occurs. This means that understanding this behavior is the first step in treating and decreasing causative factors, which through that, less customers will leave the line and more profits will be earned. Furthermore, addressing reneging affecting factors should keep the queue under steady state (arrival rate is less than the service rate), and increase the customers' satisfaction with the service experience.

1.3 Objectives

This research aims to reach a deeper understanding of the queuing theory and the queue's characteristics, to make it more reflective and interpretive to real life queuing problems. This is done by highlighting the behavior of queuing customers, especially those who are not patient enough to wait more time in the line until they are served, and how their decision (either staying in the line or leaving it) is affected by the queue's characteristics.

Explaining the reneging behavior helps the service provider to exactly know the causes, and possibilities of its occurrence. This prompts the provider to address the queue's characteristics that are leading to such an event, to reduce the number of reneging customers as much as possible, especially if the size of the losses that could incur or the revenues that could be earned are identified.

Furthermore, by addressing the causative queue's characteristics, the quality of the service system will be improved, and a higher customer satisfaction will be gained, especially when it comes to reducing waiting and/or service times. In addition, by considering the spatial aspect in analyzing the queuing system and such impatient customers' behavior, a service provider with multiple drive-thru sites can better manage the system from a macroscopic perspective.

Given the purpose of this research, the specific objectives are:

1. To mathematically determine the probability distribution function of reneging behavior;
2. To define the factors affecting the reneging customer's decision in leaving the line; and to mathematically determine the probability distribution functions of reneging's affecting factors;
3. To explore incorporating the reneging behavior into the queuing theory's main equations;
4. To theoretically develop the basis of a spatial-dynamic logical "app" for collecting, analyzing, and showing related data based on queuing theory equations and reneging behavior.

1.4 Case Study

It was challenging to select a case of a drive-thru service in Palestine that has a reasonable demand, where reneging events could be well observed and analyzed, especially if such a service is not widely spread at the local level. The selected site was Rio Café's drive-thru service at Al-Huda Gas Station, in Al-Balou area in Al-Bireh City, Palestine as one of the highest demand sites among all Rio Café's drive-thru services, with an average daily demand of more than 350 customers.

In fact, the selected site fulfills an important condition of visibility of the waiting line, so that the entire queue length is clearly visible for the observer through the images captured from surveillance camera recordings (Appendix A, Figure A-I).

The selected case study is a single-channel drive-thru service, with both the arrival rate and the service time randomly distributed (M), and following the "First-In-First Out" (FIFO) system. The number of queued customers frequently exceeds the maximum designated queue length (three-customers as the service provider painted, as shown in (Appendix A, Figure A-I); however, the place provides ample room for additional vehicles to join the queue beyond the painted spots. As such, the selected case is theoretically considered as infinite queue length (∞), see (Appendix A, Figure A-II). Therefore, the stochastic approach should be followed in analyzing such a queue denoted as [M/M/1 (∞, FIFO)], as May (7) presented.

1.5 Literature Review

Numerous literature address the queuing theory and its applications, which have been used in various fields. The research ranges from the discussion of the theory, its mathematics, applications with the operational aspects of a system, drive-thru services, and impatient customers' behaviors toward the queue. The focus in this review is on the operational aspects and impatient customers' behaviors.

1.5.1 Mathematics of Queuing Theory

May (7) presented a comprehensive discussion of the queuing theory from a macroscopic and microscopic perspectives. The author also presented the various systems of deterministic or stochastic service time and arrival rate, as well as single and multi-channel systems, and the associated formulas and constraints for each. However, May (7) focused on the case of (M/M/n), when arrival and service rates follow Markov chain (M/M), with (n) number of channels.

Other literature solved mathematically the problem of (M/G/1), when the arrival rate is Markovian (M), while the service rate is generally distributed (G), for a single server. This is known as Pollaczek–Khinchine (P-K) formula in continuous time. Haigh (12) showed related equations for mean queue size and expected waiting time for such a case. Furthermore, Chan et al. (13) solved the P-K formula in discrete time, using Little's law ($L = \lambda W$), where L is the length of the queue, λ is the arrival rate, and W is the mean time spent at the queue (both waiting and being served).

1.5.2 Application of Queuing Theory

Sathiyabala and Vidhya (14) indicated that the application of queuing theory has increased in various fields of banking sector, healthcare, traffic control, computer parallel system, and distributed systems. The paper analyzed the instances of using the queuing theory in various applications and the benefits acquired.

In the banking system, an application of queuing theory was proposed to determine the optimum service level for ATMs in Ghana (15). The research found, through administered questionnaires, that 52% of 106 responds felt that one to five-minute interval is a reasonable time to wait in a queue, while on average, it was eight minutes as an acceptable interval of time. The bank's ATM system consisted of two ATM machines, which serve as a multiple-channel system (M/M/s). It was found that adding an additional ATM would achieve less cost with less waiting time, with an optimum number of 3 ATMs, which would decrease the waiting time from 32 minutes to 48 seconds with a reasonable cost. It was also found that the optimal service rate should be 0.55 customer per minute (CPM) instead of the current of 0.51 CPM, to keep the customers' waiting time less than the acceptable average (8 minutes) while maintaining the capacity (2 ATMs). In addition, multiple queues (two lines), unlike expected, generated a longer waiting time. Therefore, based on the First-In-First-Out (FIFO) system, the paper recommended adopting a single queue (one line), which would perform better in such cases.

Kyritsis and Deriaz (16) built a web application called "QueueForMe" using machine learning for predicting waiting time, allowing anyone to create his/her own virtual queue and allowing clients to join. The dataset used to train the machine included the time a client joins the queue, waiting time, service time, and the total time in the system for total of 52,444 reported clients in 3 banks in Ogun State in Nigeria, over a period of 4 weeks. A neural network model was used, for its continuous learning capabilities, in setting up the machine learning experiment to finally have a fully trained neural network with a mean absolute error of only 3.35 minutes in predicting clients' waiting time.

In the web application, a creator can log in and create a queue that clients can join to have information about, such as their position in and the estimated waiting time predicted by machine learning, with a simulation of customers joining where it is possible to select the distribution of arrivals, distribution of service time, and the number of channels. In addition, the updated future version would allow the creator to define a set of queue parameters where joining clients can respond to them. Thus, the neural network can learn more about queues' patterns and extract more valuable information to use it later with new features and predefined parameters' responses of all previous queuers.

Furthermore, the queuing theory has been applied widely in medical sector. A queuing theory modeling was applied recently in Australia to determine how many ICU beds will be required, and to predict when the system fails during the COVID-19 pandemic (17). The authors used Little's theorem to determine the number of patients in the system (L), with an arrival rate equals to the proportion of confirmed COVID-19 patients require an ICU admission per day (λ), and a mean of 10 days a patient remains in the system (W). The modeled uninterrupted exponential growth scenario showed that the public health system in Australia would fail when the number of needed ICU beds exceeds capacity, where it was approximately 2300 beds.

With an ICU admission rate of 5%, the number of required ICU beds was 10% of the confirmed cases, which means the system would break the moment the number of confirmed COVID-19 cases reaches 23,000. Furthermore, if the ICU admission rate is 10%, the system would break in day 26, while if the ICU admission rate is reduced to 2.5%, the number of beds would be sufficient for more than a month.

Dbeis and Al-Sahili (18) investigated the performance of the queues on ATMs in Rafidea Street in Nablus City during the COVID-19 lockdown, and optimized the service with maintaining physical distancing and other required health measures, when the Palestinian government panned vehicular travel and imposed paying the public sectors' employees on specified paydays via ATMs only. Under the paydays schedule, the analysis showed long queues up to 39 persons with an average waiting time reaching two hours. The researchers discussed four alternatives to manage the queues, where the optimum one considerably reduced average waiting time to only 4.3 minutes, and average number waiting to be served to a maximum of 3 persons during the peak hour. This was attained by rearranging paydays' categories based on demand size with an extra payday. It also included adding two mobile ATMs according to the spatial distribution of demand points and ATMs in the coverage built area based on an impedance walking distance of 10-minute of walking at a speed of 1.2 m/s; a value that is normally used for design purposes according to AASHTO (19).

In Nigeria, Kembe et al. (20) applied the queuing theory for a multichannel queuing model in the Riverside Specialist Clinic in Makurdi City. Data were collected for four weeks through observations, and the queue characteristics were analyzed. Results were used in TORA software to find the optimal server level of 12 doctors against the present of 10 doctors, where the optimal number achieved the minimum total cost (expected service cost and expected patients waiting cost in the system). Although expanding the service facilities and increasing the server level will increase the service cost; however, once the service quality improves, the cost of patients waiting time in lines decreases and so does the total cost.

Komashie et al. (11) used the queuing theory to build an integrated model of patients and staff satisfaction, to optimize the satisfaction level for both staff and patients in healthcare sector, through the concept of Effective Satisfaction Level (ESL). Data were collected using a questionnaire with a five-point Likert scale, by interviewing 68 doctors and nurses in two accident and emergency departments in London. As expected, patients' satisfaction increased as waiting time fell down. They used the Ratio of Patients Waiting Time (Δ_p), which is the ratio of difference between expected waiting time and actual waiting time to expected waiting time.

In the same manner, for measuring staff's satisfaction rate, they used Service Time Ratio (Δ_s) as the ratio of the difference between actual service time and ideal service time to ideal service time. The analysis of the staff's questioners showed that the majority were dissatisfied when they spent less time with patients than the expected ideal service time they thought. The best-fit analysis on empirical data resulted in the following double hyperbolic tangent equation for staff satisfaction level (U):

$$U(\Delta_s) = 0.25\, tanh(1.72\Delta_s) + 0.76\Delta_s\, tanh(-4.43\Delta_s) + 0.95 \qquad (1.1)$$

They defined Effective Satisfaction Level (ESL) as the maximum level of satisfaction on the optimum Total Satisfaction Curve (TSC) at a given value of ideal service time expressed in terms of Δ_p. The technique used to build TSCs was the weighted sum of the satisfaction level for both patients and staff at a given value of ideal service time. The experimental results, for an ideal service time of 2 hours, showed that when expected waiting time was 0.5 hours, the patients' satisfaction level reached its optimum, while the staff's satisfaction level inclined to almost zero. On the other hand, the ESL reached its optimum for both patients and staff when expected waiting time was 2 hours same as the ideal service time. The presented argument showed that the synergy between these two satisfaction levels is the key to sustainable improvement in healthcare quality.

1.5.3 Drive-Thru Services

Abu Farha et al. (21) investigated the awareness, perception, and barriers of drive-thru pharmacy services among pharmacists in Jordan. Using a questionnaire developed with a Likert scale of five degrees, 226 pharmacists were interviewed to assess their perception towards such a service. Most of pharmacists reported that drive-thru pharmacy service made them feel more like a fast-food worker than a pharmacist, and this could negatively affect the image of pharmacy profession (74.6%). However, the majority (88%) agreed that such a service would serve sick patients, elderly, disabled people, or women with child in the car. While, 75.5% pharmacists reported it would reduce parking problems. The study highlighted the need to better introduce the concept of drive-thru pharmacy service among pharmacists in Jordan, since it was still new in the country and the Middle East.

Hussain et al. (5) stated that drive-thru pharmacy service was one of many measures to ensure safety and protection during COVID-19, where patients received their pre-ordered and pre-packed medicine, without the need to exit their vehicles, with little human interaction. They addressed the use of such a service in many countries around the world, especially in the Arab Gulf countries such as UAE and Qatar.

Greene and Kannan (22) developed a trip generation model predicting morning peak hour volumes for the land uses 936 (Coffee/Donut shops without drive-thru window) and 937 (Coffee/Donut shops with drive-thru window); published and coded by the Institute of Transportation Engineers' (ITE) Trip Generation Manual, 8th Edition. The manual contained only three variables determining trip generation volumes: gross floor area, number of seats, and peak hour of adjacent street traffic. While, the study investigated if other site-specific factors had a statistical relationship affecting the trip generation volume, such as Average Daily Traffic (ADT), geographic, and demographic variables. Using video recordings and manual traffic counts, they collected data at 13 different coffee/donut shops around Western New York, during the morning peak hour period. The resulted equations were as shown in Equations (1.2) and (1.3).

$$Parking\ Lot\ Trips = 76.509 - 7.231D + 0.003P + 0.007ph_{am} - 43.927DT \quad (1.2)$$

$$Drive - Thru\ Trips = -113.414 - 21.215NL + 0.007P + 2.108Age + 0.037PH_{am} + 0.002ADT + 68.607DT \quad (1.3)$$

Where:

D	:	Distance from interstate highway
P	:	Population within 3/4-mile radius
ph_{am}	:	Morning peak hour
DT	:	Drive-thru presence (0 if a drive-thru is not present, and 1 if it is)
NL	:	Number of lanes
Age	:	Median age within 3/4-mile radius
ADT	:	Average daily traffic

To determine the expected number of queued vehicles at shops with a drive-thru window, they used the Drive-Thru Trip Generation Model to determine the arrival rate. In order to estimate the average queue length required for the drive-thru lane at a confidence level of 95%, the following formula was used:

$$\sum_{n=0}^{n=\alpha} P(n) = \sum_{n=0}^{n=\alpha} \rho^n(1-\rho) \geq 0.95 \quad (1.4)$$

Where:

$P(n)$: Probability of exactly 'n' vehicles in a drive-thru lane

ρ : Traffic intensity $= \frac{\lambda}{\mu}$

λ : Arrival rate

μ : Service rate

Once the value of Equation (1.4) is attained, the resulted number of vehicles can be multiplied by the vehicle's average length, to determine a corresponding queue length required for the drive thru lane.

Dougherty (23) proposed a block-queuing system to alleviate the problem of pollution in urban areas with minimum extra infrastructure, by dividing the queue into two sections: an active section at the front, where drivers are allowed to keep their engines running; and a passive section, where drivers are asked to turn off their engines until the active section clears.

In a drive-thru service system, Reilly and Berglin (24) presented a new model for determining the optimal block-size under block-queuing, with maximum number of vehicles that minimizes the expected amount of fuel consumption. Using the Pollaczek–Khintchine (P-K) formula in Equation (1.5) for average length of the waiting line (L_q), for a randomly distributed arrival rate (λ) and a generally distributed service rate (μ) in a single-server queuing system (M/G/1).

$$L_q = \frac{\rho^2 + \lambda^2\sigma^2}{2(1-\rho)} = \frac{(r+1)\rho^2}{2(1-\rho)}, \qquad for\, r \geq 0 \tag{1.5}$$

Where:

ρ : Traffic intensity

σ^2 : Service-time variance $= \frac{r}{\mu^2}$

r : Service process type (0 for M/D/1, 1 for M/M/1)

If r = 0, the service process is deterministic and the queuing system is M/D/1. If r = 1, the queuing system is M/M/1 and the service times follow a (negative) exponential distribution. By setting the first derivative of Dougherty's equation (23) for fuel consumption to zero, and substituting for L_q, they found the nominal size of block-size (b) as:

$$b_0 = \sqrt{\frac{f(r+1)\rho^2}{i(1-\rho)\left(\frac{1}{\mu} - \frac{l}{v}\right)}} \tag{1.6}$$

Where:

f	:	Fuel used to restart engine
r	:	Service process type (0 for M/D/1, 1 for M/M/1)
ρ	:	Traffic intensity
i	:	Idling engine's rate of fuel consumption for an
l	:	Average length of a vehicle
v	:	Speed of vehicles advancing in the queue

Also, by analyzing the enhanced model, they found that a necessary and sufficient condition for a block-queuing benefit is:

$$b_{LB} = \left\lceil \frac{\frac{f}{i}}{\frac{1}{\mu} - \frac{l}{v}} \right\rceil \leq \left\lfloor \frac{(r+1)\rho^2}{(1-\rho)} \right\rfloor = b_{UB} \tag{1.7}$$

Otherwise, if $b_{LB} > b_{UB}$, it is better to keep engines running. Through two block-queuing optimization examples, they concluded that block-queuing is most beneficial under heavy traffic conditions.

1.5.4 Behaviors of Impatient Customers

Wang et al. (8) surveyed queuing systems with impatient customers with various dimensions. They defined the main three impatient behaviors (balking, reneging, and retrial) and viewed mathematical equations, explaining some of them as they were proposed in literature. Balking, when an impatient customer decides not to joining the queue at all. They drew the attention to a fact that a customer takes his/her decision to join the queue or not in accordance with the queue length, since the waiting time is invisible to him/her. They reviewed the assumptions made by Haight (25), where it was assumed that customers had N value representing a threshold of queue length. A customer balks, if the observed length of the queue is greater than N; otherwise, the customer joins the queue. Unlike balking, Haight (26) assumed an impatient customer will renege on the queue after a certain waiting time, regardless the queue length when the customer arrives.

Even though, customers may decide to leave the queue, if it appears that the time consumed will exceed their threshold waiting time T. While, "Retrial" is a different phenomenon, when a customer who may balk at entering the system or renege after a waiting time, can join the orbit of customers to repeat the request after a random amount of time.

As a theoretical investigation, van Tits and van der Veeken (27) checked the general validity of queuing formulas for expected waiting time and queue length when the service times are not exponentially distributed and follows more general distributions, with varying balking mechanisms, using simulation models. The model used the technique of regenerative simulation, where the system starts empty every time when a customer arrives at an idle server. Based on the simulation, the theoretically calculated average time corresponding to each different number of cycles (10,000- 40,000 cycles) laid in the 95% confidence interval, using an arrival rate of 5 minutes per customer and the standard balking mechanism where the probability of an arrival joining the queue can be calculated using Equation (1.8).

$$P_r = 1/(1 + r) \tag{1.8}$$

Where:

P_r : probability of an arrival joining the queue

r : Queue length

The study investigated five different distributions the service time may follow (shifted negative exponential, gamma, log normal, uniform, and transformed beta distributions), performing two series of simulations with these distributions. It was found that the average waiting time and queue length are the same in all distributions. While the average queue length did not conflict with the theoretical formulas, the average waiting time differed greatly, meaning that its formula is only valid when the service time is exponentially distributed. Furthermore, it was found that the waiting time is affected by the value of service time's variance. By applying a linear regression, they developed a formula of average waiting time ($\bar{W}$) as a function of the variance of service time (ST), as shown in Equation (1.9).

$$\bar{W} = 1.08572 + 0.08049\, VAR(ST) \tag{1.9}$$

In addition, extra series of simulations was carried out to find out whether the previous linear relationship can be applied to other balking mechanisms. Following Haight (25), the study used three other different queue-joining probabilities (balking mechanisms), and by means of regression analysis, they estimated coefficients, using the formula below:

$$\bar{W} = 0.18559\left(E(ST)\right)^2 + 0.11957\sqrt{E(ST)}\, VAR(ST) \tag{1.10}$$

Where:

$E(ST)$: Expected value of service time

Although the previous empirical Equations (1.9) and (1.10) were derived from simulations with the Gamma distribution for service time, they stated that it is not generally valid for all various balking mechanisms.

Other papers proved and showed the mathematical form for the probability of reneging and balking formulas (28,29).

To see the effect of the capacity of the system on its effectiveness, Wara et al. (30) conducted a queuing model simulation of an M/M/1 system, with a retention of reneged customers and balking, as Kumar and Sharma (31) defined retention as employed strategies to convince reneged customers to stay in the line. The simulation was carried out to test variant queuing system sizes (starting with 200 and ending with 900 customers) against certain performances' measures for both system and queue, such as: expected size and expected waiting time. Obtained results showed that the greater the system's capacity, the shorter the queue and customers' waiting time. In other words, service providers can decrease their losses from impatient customers leaving the line by increasing the capacity, where reneged customers can wait for more time in a larger capacity system before giving up the service.

Liao (32) proposed a queuing model for an assumed fast-food restaurant, using reneging rate and balking index to derive the number of lost customers; and thus, the business loss. The probability of balking used was estimated using Equation (1.11).

$$P(\text{Balking}) = \frac{\theta(n-S)}{S} \tag{1.11}$$

Where:

n	:	Number of customers in the system when customers arrive at the peak hour
S	:	Number of servers in the system
θ	:	Balking index (the average intention of not joining the queue)

Concluded results showed that by decreasing the balking index, the balking customers decrease, which increases the queue length and makes customers have to wait more time. Therefore, the lower the balking index, the higher the reneging rate. Despite the assumption of many data and estimation of probabilities, this case showed that adding more servers will increase the cost, but maximize the net profit by decreasing the total number of lost customers due to the decreasing queue length, which requires a good management to make proper choices.

1.5.5 Summary

In summary, the queuing theory has been applied in various fields including analyzing, optimizing, and managing queues. However, its use for spatial distribution of services to manage service (demand) and arrival rates in order to improve the system's performance was found in only one literature recently published (18). Furthermore, impatient behaviors (balking and reneging) have been discussed extensively in literature, but without determining such behaviors' probability distribution functions. Moreover, the literature considered the waiting time and the queue length as the only factors affecting reneging and balking behaviors, respectively, without mentioning other important factors such as the customer's rank and the arrival time as critical factors related to the customer's decision in joining or leaving the queue.

Considering the above, this research attempts to fill in the gap, by determining the probability distribution function for reneging behavior, as a special case of balking; and the affecting factors as well. Through that, it would be possible to accurately analyze, manage, and optimize drive-thru queues and other services that have the same characteristics and factors affecting the impatient customers' decision either to stay in the queue or join it in the first place.

1.6 Thesis Structure

This thesis is mainly composed of five chapters. Chapter One introduces the subject along with its background, the objectives of the research, and the reviewed literature related to the queuing theory and impatient customers' behaviors toward the queue, especially the reneging behavior. Chapter Two describes the methodology used in the study, while Chapter Three shows how required data were collected. Chapter Four discusses the data analysis process to achieve the research objective's, including determining the probability distribution functions that reneging behavior and affecting factors are following. Finally, the summary, conclusions and recommendations of this thesis are shown in Chapter Five.

Chapter Two
Methodology

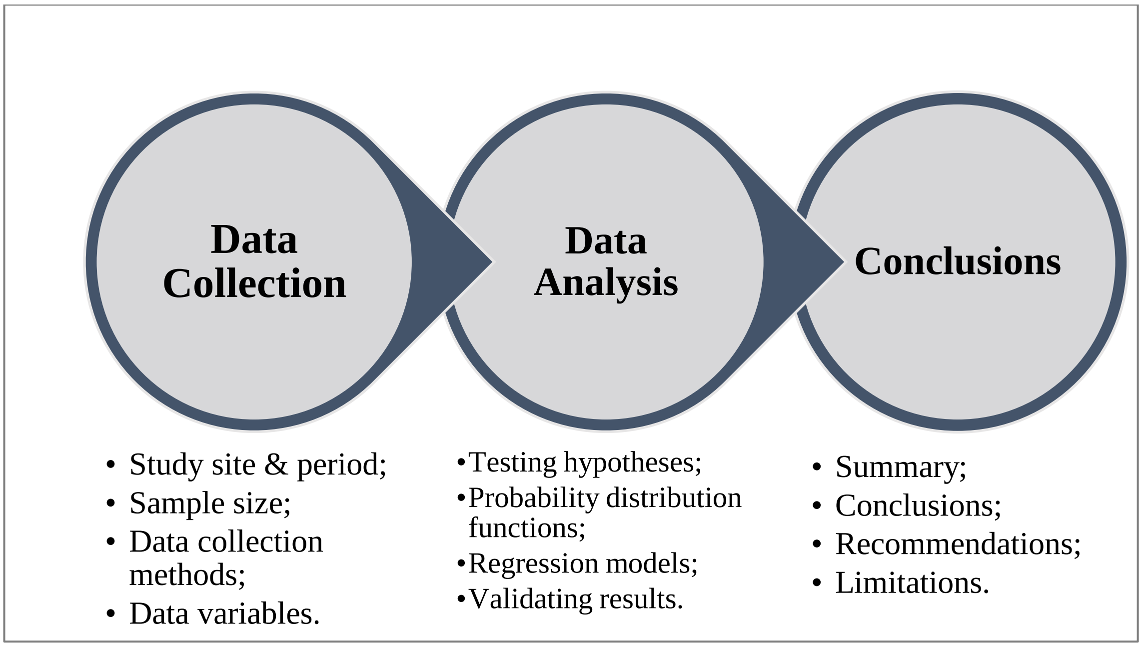
2.1 Introduction

Regarding the objectives of this research, the purpose is to reach a deeper understanding of impatient customers' reneging behavior toward the queue they are joining, as a necessary step to make queuing theory more reflective to real-life queuing problems in terms of waiting time and queue length. The developed models and identified probability distribution functions that reneging behavior and affecting factors follow, will assist service providers to manage their queues, and save waiting times for queued customers.

The procedure of achieving the pre-defined four objectives is divided into two main steps: Data collection, and data analysis, as shown in Figure 2.1. Each objective was passed through both steps to be achieved.

Figure 2.1

Main Steps of the Study



2.2 Data Collection

For the purpose of the study, the required data were collected from one drive-thru service embedded to Rio Café located at Al-Huda Gas Station in Al-Bireh City. All collected data are genuine, that date back to September 2021.

Through non-participant observations, the required data were manually collected, using surveillance camera recordings provided by the café's administration as softcopy-video data. This was done after determining the needed data variables and identifying the study time of day, as well as estimating the minimum sample size.

In general, the collected data focused on queuing active periods foreseen through field observations for multiple services susceptible to queuing, to detect as much as possible of reneging customers through observing the queue performance and impatient customers' behaviors after they joined it.

2.3 Data Analysis

Depending on the related data for the relevant objective, the collected data were mathematically analyzed after checking its adequacy. The process of analysis consists of four main sub-processes, as the following:

2.3.1 One-Way ANOVA Approach

To test if the means of each of the arrival rate and the service time are significantly different between workdays and between workhours, one-way Analysis of Variance (ANOVA) approach was used, based on the F distribution shown in Equation (2.1), at a confidence level of 100(1-α)%, where α is a chosen significant level.

$$F_0 = \frac{SS_B/g-1}{SS_W/n-g} = \frac{MS_B}{MS_W} \tag{2.1}$$

Where:

g	:	Number of groups	
n	:	Number of total observations	
n_j	:	Number of observations in group j	
SS_B	:	Sum of squares between groups	$= \sum_{j}^{g} n_j(\bar{x}_j - \bar{x})^2$
SS_W	:	Sum of squares within groups	$= \sum_{j}^{g}\sum_{i}^{n_j}(x_{ij} - \bar{x}_j)^2$
MS_B	:	Mean squares between groups	
MS_W	:	Mean squares within groups	

The hypotheses were stated as:

- Null hypothesis (H0): μ1 = μ2 = μ3 = μi (there is no significant difference);
- Alternative hypothesis (H1): μ1 ≠ μ2 ≠ μ3 ≠ μi (there is a significant difference).

If the right-tailed probability (P-value) of $f_0 \geq \alpha$, then there is no evidence to reject H_0, otherwise it is rejected. The exact P-value was computed using Excel software, using the function "F.DIST.RT" for f_0, with Degrees of Freedom (DF)1 = g – 1, and DF2 = n – g.

However, the one-way ANOVA approach was done using Data Analysis Tools in Microsoft Excel software.

2.3.2 Correlation between Random Variables

Pearson's correlation coefficient was used as a quantitative measure of the strength of the linear relationship between each pair of the selected random variables (factors) affecting the reneging behavior. The correlation coefficient is defined as the following:

$$P_{xy} = \frac{\sum_i^n y_i(x_i - \bar{x})}{\sqrt{\sum_i^n (y_i - \bar{y})^2 \sum_i^n (x_i - \bar{x})^2}} \tag{2.2}$$

Where:

P_{xy} : Pearson's correlation coefficient between *x* and *y* random variables;

n : Number of total observations.

The closer the value of P_{xy} is to 1, the stronger is the positive-linear relationship between the tested variables. On the other hand, the linear relationship is strongly negative between the two variables as much as the value of the coefficient is closer to -1. However, the closer the value of $|P_{xy}|$ is to zero, the weaker is the linear relationship.

2.3.3 Probability Distribution Functions of Continuous Random Variables

To test if a random variable is following a hypothesized probability distribution function, the Chi-square goodness-of-fit test was used, as Equation (2.3) shows.

$$\chi_0^2 = \sum_i^k \frac{(O_i - E_i)^2}{E_i} \tag{2.3}$$

Where:

O_i : Observed frequency in the ith class interval

E_i : Expected frequency in the ith class interval

k : Number of class intervals (bins)

The null and alternative hypotheses were stated as:

- H_0: the population follows the hypothesized distribution;
- H_1: the population does not the hypothesized distribution.

The Chi-square goodness-of-fit test statistic with a number of DF = k – p – 1, where p is the number of parameters of the hypothesized distribution estimated by the sample statistics; and it is denoted as $\chi^2_{\alpha,k-p-1}$ at a confidence level of 100(1-α)%.

If the P-value of $\chi^2_{0,k-p-1} \geq \alpha$, or $\chi^2_{\alpha,k-p-1} \geq \chi^2_0$, then it is unable to reject the H_0 that the distribution of the tested random variable is as it is hypothesized. Otherwise, the null hypothesis is rejected. The exact value of P-value was computed through Excel software using the function "CHISQ.DIST.RT".

2.3.4 Regression Models

Two types of regression models were developed in this research: simple linear regression, and multiple non-linear regression. The second type of regression was developed based on the form of a multiple linear regression shown in Equation (2.4), where each variable can represent an included non-linear function. However, both types were developed using Data Analysis Tools in Microsoft Excel software.

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon \quad (2.4)$$

Where:

β_0 : Intercept

β_k : Regression Coefficient of variable x_k

k : Number of regressor variables

ε : Random error assumed to be zero

2.3.4.1 Statistical Tests

To check the adequacy of a regression model, several statistical tests were conducted, as the following:

1. ANOVA Approach

ANOVA approach was used to test the overall significance of a regression model, based on the F-test statistic shown in Equation (2.1), at a confidence level of 100(1-α)%, as Equation (2.5) shows. It should be noted that the denominator in Equation (2.5) is the variance of error, where its square root is the regression standard error (se); the average distance between observed values and the regression line.

$$F_0 = \frac{SS_R/k}{SS_E/(n-k-1)} = \frac{MS_R}{MS_E} \tag{2.5}$$

Where:

k	:	Number of regressor variables	
n	:	Number of observations	
SS_R	:	Regression sum of squares	$= \sum_{i}^{n} (\hat{y}_i - \bar{y})^2$
SS_E	:	Error (residuals) sum of squares	$= \sum_{i}^{n} (y_i - \hat{y}_i)^2$
$\hat{y}_i$	:	Regression estimated value	
MS_R	:	Regression mean square	
MS_E	:	Error (residuals) mean square	

The hypotheses statements for ANOVA approach were as the following:

- H_0: $\beta_1 = \beta_2 = \cdots = \beta_k = 0$;
- H_1: $\beta_j \neq 0$, for at least one coefficient.

If the P-value of $f_0 \geq \alpha$, then there is no evidence to reject H_0, otherwise it is rejected. In other terms, if $f_0 > f_{\alpha,k,n-p}$; the H_0 should be rejected. The exact P-value was computed using Excel software, using the function "F.DIST.RT" for f_0, with DF1 = k, and DF2 = n – p. However, the ANOVA approach was done using Data Analysis Tools in Microsoft Excel software.

2. T-Test Statistic for Regression Slope

T-test, shown in Equation (2.6), was used to test the significance of each individual coefficient in the regression model. The hypothesis is that each coefficient equals a constant at a confidence level of 100(1-α)%.

$$T_0 = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)} \qquad (2.6)$$

Where:

$se(\hat{\beta}_0)$: Standard error of the intercept in simple linear regression $= \sqrt{MS_E\left(\frac{1}{n} + \frac{\bar{x}^2}{SS_x}\right)}$

$se(\hat{\beta}_1)$: Standard error of the estimated coefficient in simple linear regression $= \sqrt{\frac{MS_E}{SS_x}}$

SS_x : Sum of squares of the variable *x* $= \sum_i^n (x_i - \bar{x})^2$

$se(\hat{\beta}_j)$: Standard error of the estimated coefficient of regressor x_j in multiple linear regression $= \sqrt{\frac{\sigma^2}{\left(1 - P_{x_1x_2}{}^2\right)\sum_i^n (x_{j_i} - \bar{x}_j)^2}}$

σ : Regression standard error $= \sqrt{\frac{SS_E}{n-p}}$

$P_{x_1x_2}$: Pearson's correlation coefficient between x_1 and x_2 variables

The hypotheses to test if an individual regression coefficient β_j equals a value of β_{j0} is:

- H_0: $\beta_j = \beta_{j0}$;
- H_1: $\beta_j \neq \beta_{j0}$.

The H_0 is rejected if $|t_0| > t_{\alpha,n-p}$, or when two times the P-value of $|t_0| < \alpha$. Otherwise, there is no evidence to reject the null hypothesis. An important special case of the previous hypothesis that when H_0: $\beta_j = 0$. In this case, if the H_0 is not rejected, this indicates that the regressor x_j can be deleted from the model.

A 100(1 - α)% confidence interval on the regression coefficient β_j, j = 0,1, ..., k in the linear regression model is given by the following equation:

$$\hat{\beta}_j - t_{\alpha/2,n-p}se(\hat{\beta}_j) \leq \beta_j \leq \hat{\beta}_j + t_{\alpha/2,n-p}se(\hat{\beta}_j) \quad (2.7)$$

3. Coefficient of Determination (R^2)

To check the adequacy of regression models, R^2 was used as a ratio of sum of squares (Equation (2.8)). The value of R^2 falls between 0 and 1, representing the strength of the regression model in explaining the variability of the data.

$$R^2 = (Multiple\ R)^2 = \frac{SS_R}{SS_E + SS_R} \quad (2.8)$$

By adding new variables to the model, the value of R^2 increases. However, this does not mean that the new model with extra variables is much better than the original one. Therefore, especially for multiple regression models, the adjusted R^2 statistic was used to assess the fit of the model, as Equation (2.9) shows. The difference between the two statistics is that the value of the adjusted R^2 will only increase if the newly added variable reduces the residuals (error) mean square.

$$R^2{}_{adj} = 1 - \frac{SS_E/(n-p)}{(SS_E + SS_R)/(n-1)} \quad (2.9)$$

Chapter Three
Data Collection

3.1 Introduction

This study aims to define the factors affecting the reneging event, and determine the probability distribution for these factors and such an event, through observing the queue performance and impatient customers' behaviors after they joined it. The required data for this purpose was collected for Rio Café's drive-thru service located at Al-Huda Gas Station in Al-Bireh City. The data were collected manually through non-participant observations, using surveillance camera recordings provided by the café's administration as softcopy-video data.

Before collecting the data, the study time of the day was determined focusing on the active periods to detect as possible reneging customers. Then, the required sample size was estimated, and the data variables were determined and defined.

It should be mentioned that diverted customers before entering the queue (balking customers) were not taken into consideration, as reneging behavior is the main scope of this study.

3.2 Study Period

In order to clearly observe the performance of the queue and impatient customers' behaviors, the study was conducted in the summer during daily active-periods when high demands of customers are expected to arrive.

Based on field observations, it was expected to have at least two peak hours. The first one is in the early morning when commuters are traveling to work, and the second in the afternoon when commuters are traveling back home. Therefore, data were collected for the following hours: 7:00-10:15 and 14:00-18:00, in a total of 7.25 hours a day, for 17-fulltime workdays from September 1 to 20, 2021.

The daily study duration was divided into 15-minute time intervals; 29-time intervals a day, for a total of 493-time intervals for the whole study duration of 17 days. Therefore, data were collected in each interval separately.

3.3 Sample Size

The minimum sample size of workhours needed to be observed for this study was calculated using the following Equation (3.1), based on the estimation of the proportion of data item in the population (33).

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 (P)(1-P), \qquad for\ E > 0 \tag{3.1}$$

Where:

Z : Standard normal random variable

α : Significance level

E : Amount of tolerable error

P : Proportion of the data item

Since reneging events occurred in 113 different time intervals out of 493, the proportion of reneging was estimated as 23% of the workhours. With a tolerance of 5 minutes (0.08 hours), the minimum sample size is 107 workhours, for a two-tailed 95% confidence interval with a Z-score of 1.96. In this study, data were collected for 123.25 workhours (493-time intervals of 15-minutes) as a representative sample size for the study's population.

3.4 Data Collection Methods

The data were manually collected through non-participant observation using surveillance camera recordings, where the position of the camera fits very well with the clear visibility condition of the waiting line mentioned before.

It was expected from the outset that a large amount of data would be collected. The most important feature of using such recordings is that it enables the observer to review previously collected data, and collect data for different variables at separate times for the same time interval. Therefore, it will be possible to clearly observe the whole performance of the queue, its characteristics, and waiting customers' behaviors without losing an observation.

3.5 Data Variables

To achieve the objectives of this study, the data were collected on arriving customers, under two assigned categories of variables: queuing performance variables and reneging behavior variables. The data for the queuing performance variables were collected in each time interval of the study duration to examine the queue's characteristics and understand the queuing pattern that forms in the case study's drive-thru service. On the other hand, the reneging behavior variables, such as the reneger's rank and the time the reneging customer waited in the line, were selected as the expected factors affecting the reneging behavior. The associated data were collected only when a reneging event occurred, for each reneging customer in each related time interval.

The reneging customer's rank and the queue length were in terms of passenger cars. The observed types of vehicles, according to AASHTO (19), were: Passenger Car, Single Unit Truck, and Bus. Therefore, non-passenger car types vehicles were converted (in terms of their length) to an equivalent of two passenger cars in the relevant variables. See Appendix A, Figure A-III.

The selected variables are shown in Table 3.1 with their description and the related category. Descriptive statistics are shown for the numerical values of the variables, such as mean, standard deviation (SD), and sample size (Table 3.2).

Given the definition of Response Waiting Time (RWT) in Table 3.1, some impatient customers were detected weaving their vehicles to the service line in order to join the queue. However, the long queue length and their last rank prevented them from fully stopping. These impatient customers were considered as renegers, with a minimum RWT less than one second denoted as zero in Table 3.2.

In general, during the observation of 493-time intervals (each interval is 15 minutes), 164 reneging customers were detected in 113-time intervals. Furthermore, data for all variables were collected for all observations in each time interval, except for service time, which was collected randomly as described in detail in Chapter Four.

Table 3.1

Selected Data Variables and Related Category

Category	Variable	Description
Queuing Performance Variables	Day	The day of arrival
	Date	The date of arrival
	Arrival Time	The exact time interval, related to the arrival hour, a customer arrives at the drive-thru service's queue
	Arrival Rate (customers/15 minutes)	The total number of arriving customers per time interval
	Service Time (minutes)	The time taken to serve a customer from the vehicle's arrival to the service window until its departure after being served
Reneging Behavior Variables	Reneging Rate (Renegers/15 minutes)	The total number of reneging customers per time interval.
	RWT (minutes)	Response Waiting Time (RWT): The elapsed waiting time in the queue between the arrival and the departure of an impatient customer without getting the service
	Rank	The rank of the reneging customer in the queue in terms of passenger car the moment a reneging event occurs, as the customer at the service window has the rank 1
	Queue Length	The overall queue length at the moment a reneging event occurs, in terms of passenger cars, including the customer in service

Table 3.2

Descriptive Statistics for the Selected Variables

Variable	Average	SD	VAR	Min.	Max.	Total	Sample Size
Arrival Rate (customers/15 minutes)	5.5	2.46	6.05	0	14	2713	493
Service Time (minutes)	2.01	0.9	0.81	0.33	6.93	986.27	490
General Reneging Rate (Reneger/15 minutes)	0.33	0.69	0.48	0	4	164	493
Reneging Rate| When Reneging Occurs (Reneger/15 minutes)	1.45	0.67	0.45	1	4	164	113
Renegers' RWT| When Reneging Occurs (minutes)	0.58	0.94	0.88	0	4.23	95.52	
Renegers' Rank| When Reneging Occurs	3.11	0.93	0.86	2	5	510	164
Queue Length| When Reneging Occurs	3.23	0.94	0.88	2	6	529	

Chapter Four

Analysis

4.1 Introduction

In this chapter, the previously collected data were analyzed to achieve the objectives of this research, especially to determine the probability distribution function of reneging behavior and affecting factors, after defining them and explaining their roles. This started by analyzing the arrival rate and the service time; the two main variables forming and affecting the characteristics of the queue in the studied drive-thru service. This was done to examine the queuing pattern and performance, and to understand the reneging behavior of impatient customers.

The Chi-square goodness-of-fit test was mainly used to determine how likely the data variables follow the hypothesized distributions. Other statistical tests were also conducted such as F-test, to ensure the best estimation and to validate the generated regression models for the intended variables.

4.2 Distribution of Arrival Rate

It was concluded from the literature review, that most arrival rates follow a Poisson distribution. The total number of observed arrival customers was 2,713 who arrived during the study duration of 493-time intervals of 15 minutes (the main sample size), with an average arrival rate (λ) of 5.5 customers per 15-minute time interval (i.e 22 customers per hour), and a standard deviation of 2.46.

Before determining the distribution of the arrival rate, the collected sample size was checked using the Central Limit Theorem (CLT) method, to confirm that the collected sample size is representative to the arrival rate's population, where the minimum required sample size could be calculated using Equation (4.1).

$$n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2, \qquad for\ E > 0 \tag{4.1}$$

For a two-tailed 95% confidence interval, Z-score is 1.96, standard deviation is as calculated from the collected sample (5.5 customers/time interval), and a tolerable error of 5% of the calculated mean (0.275 customer/time interval) is used. As a result, the minimum sample size should be 308, while the collected sample was 493 records; indicating that the collected sample size is representative.

Therefore, the Chi-square goodness-of-fit test was used to check the assumed distribution that the arrival rate follows a Poisson distribution, at a significance level of 5%, as shown in Appendix B, Table B-I.

If X is the number of events in a Poisson process, then X is a Poisson random variable with the following Probability Mass Function (PMF) shown in Equation (4.2), where λ is a rate parameter; the mean number of events per unit length or unit time.

$$f(x) = \frac{e^{\lambda}\lambda^{x}}{x!}, \qquad for\ x \geq 0\ \ and\ \lambda > 0 \tag{4.2}$$

With 8 classes of customers' arrival rate (shown in Appendix B, Table B-I), and one parameter of the hypothesized Poisson distribution estimated by sample statistics (λ), the number of degrees of freedom (DF) is 6, which gives that the computed χ_0^2 of 6.083 is less than the critical $\chi_{0.05,6}^2$ of 12.592. Since the exact P-value for the computed $\chi_{0,6}^2$ is 0.414; which is much greater than α = 0.05, then it is unable to reject the null hypothesis; H_0 that the data of the arrival rate are following a Poisson distribution.

A comparison between observed frequencies of arrival rate and those expected from the hypothesized Poisson distribution is presented in Appendix B, Figure B-II.

4.3 Distribution of Service Time

The customers' service time data were randomly selected from 493-time intervals, so one service time observation per time interval, except for three-time intervals during which there were no arrivals or the vision was blocked. In total, a sample size of 490-service time observations was collected, with a mean of 2.01 minutes and a standard deviation of 0.9 minutes.

The sample size was also checked using Central Limit Theorem (CLT) method, as Equation (4.2) showed, for a 95% confidence interval. A tolerable of 5% of the calculated mean (0.1 minutes) is used, which confirms that the collected sample size is representative as long as it is greater than the minimum size of 312-time intervals.

The hypothesis of customers' service time distribution was tested against Exponential, Gamma and Log-Normal distributions. The P-value for Chi-square test was almost zero for both Exponential and Gamma (for shape parameters 2 and 3) distributions with a service rate of 0.5 customer per minute.

For the Log-Normal distribution, Appendix B, Table B-III shows the computations of Chi-square goodness-of-fit test for exponential distribution of service time, with a mean of 0.6 and a standard deviation of 0.46 for the Natural Logarithm of service time (X).

Assuming that Ln (X) is normally distributed, with mean θ and standard deviation ω for Ln (X), then X is Log-Normally distributed with the following Probability Density Function (PDF):

$$f(x) = \frac{1}{x\omega\sqrt{2\pi}} exp\left[-\frac{(ln(x) - \theta)^2}{2\omega^2}\right], \qquad for\ 0 < x < \infty \tag{4.3}$$

Results showed no strong evidence to reject the null hypothesis that the customers' service time follows a Log-Normal distribution, since the computed P-value is 0.634 for $\chi_0^2 = 8.873$ that exceeds 0.05 at a significance level of 5% and a number of DF = 11. Appendix B, Figure B-IV shows the actual data compared with theoretical customers' service time; that is, Log-Normally distributed.

Referring to Appendix B, Table B-III, it is noticed that last four classes (5.5 minutes or more), which represent high service times, contain one or zero observations, while the first class (0.5 minutes) contains only two observations. Therefore, an extra analysis was conducted to detect the data set outliers.

The method used in outlier(s) detection was the Interquartile Range (IQR) rule, using the first and the third quartiles. Since the service time's distribution was proven as a Log-Normal, the IQR scale used was 1.5, as Montgomery and Runger (34) recommended for the Normal distribution. Therefore, the rule was applied to the Natural Logarithm (Ln) of service time (X), which is normally distributed. See Appendix B, Table B-V for values of the first and third quartiles, and the IQR range used for Ln (X), where IQR is absolute difference between Q1 and Q3, UB is Upper Bound = Q3+1.5(IQR), and LB is Lower Bound = Q1-1.5(IQR).

Applying the IQR rule indicated that there are ten outliers in the sample. Eight of them because they are less than the LB and they correspond to service time values of less than 0.6 minutes, while the remaining two outliers because they are greater than the UB, and they correspond to service time values greater than 5.5 minutes. The variance of the sample is 0.82 $(\text{minutes})^2$, which is relatively high due to the high variation of provided goods and variation of corresponding service times. Therefore, these ten outliers (which represents only 2.0% of the sample size) were not excluded and were considered as natural outliers, representing the natural variation in the population of service time. Referring to Appendix B, Table B-III: Chi-Square Goodness-of-Fit Test for Log-Normal Distribution of Customers' Service Time, the probability of service time (X) exceeding 5.5 minutes is 0.008, which means that the expected frequency is about 4 cases. In other words, the integration of Log-Normal distribution function for a service time value equals to or less than 5.5 minutes is 0.992 (less than 1), which indicates that it is natural to have greater values than 5.5 minutes. This confirms that the detected outliers should be considered in the analysis, as the service time is log-normally distributed.

4.4 One-Way ANOVA Approach

Average values of the data collected on the arrival rate and the service time variables were categorized according to day and time of observation for each variable, as Tables B-VI and B-VII in Appendix B show. The one-way ANOVA approach, based on the F distribution, was used to test if the means of each of the two variables are significantly different during both workdays and workhours.

The obtained results show that there is no strong evidence to reject the null hypothesis that there is no statistical significant difference between the service time's means during both workdays and workhours, since the P-value is 0.46 and 0.23, respectively. On the other hand, the means of the arrival rate are not significantly different during workdays, with a P-value of 0.12; however, there is a significant difference between them during workhours as the P-value for the F distribution almost approaches zero. One-Way ANOVA Tables are presented in Appendix B, Tables B-VIII, B-IX, B-X, and B-XI, for the means of the arrival rate according to the study workdays, the means of the arrival rate according to the study workhours, the means of the service time according to the study workdays, and the means of the service time according to the study workhours, respectively.

It was important to use the ANOVA approach in testing the significant difference in means and obtain such results, which meets the expectation that the service time should not significantly vary during workdays or workhours, even with a noticeable variation in goods provided and related service times. This confirms the random selection for the service time observations, since it does not matter from which time interval a service time reading was randomly selected or at which moment during the time interval the selection was. While for the arrival rate, it was hypothesized that the means of arrival rates should vary at peak workhours not in workdays, which leads to determining these peak hours, as it will be described in Section 4.6. In general, all obtained results emphasize that the study duration is representative and not biased.

4.5 Correlation between Arrival Rate and Service Time

It is expected to have a strong negative correlation between the arrival rate and the service time, since it is hypothesized that these two main variables are dependent on each other; where the average service time should accordingly decrease responding to the increase in the arrival rate of customers. This hypothesis was formed based on personal field observations for many services susceptible to queuing and operated by humans. Referring to Tables B-VI and B-VII in Appendix B, the dependency level between the averages of these two variables was estimated by Pearson's correlation coefficient for two categories. One category was based on the averages of the arrival rate and the service time according to workdays (Figure 4.1), while the other one was for their averages according to workhours (Figure 4.2).

The results show that the correlation coefficient is negative in both categories, but its strength significantly varies among the two categories. The correlation is negatively stronger between the averages of the arrival rate and the service time when they are categorized according to workhours (-0.80), while the coefficient is only -0.21 when the averages are calculated based on workhours, as Tables B-XII and B-XIII in Appendix B show. The difference between the two coefficients is explained by the results obtained before from the one-way ANOVA approach; in which, there is no significant difference between the averages for each of the two variables between workdays. While between workhours, the averages of the service time are not significantly different, but the averages of the arrival rate are, since it is expected to detect such a difference according to the variation in the arrival rates between off-peak and peak hours.

Figure 4.1

Average Arrival Rate and Average Service Time, according to the Study Workdays

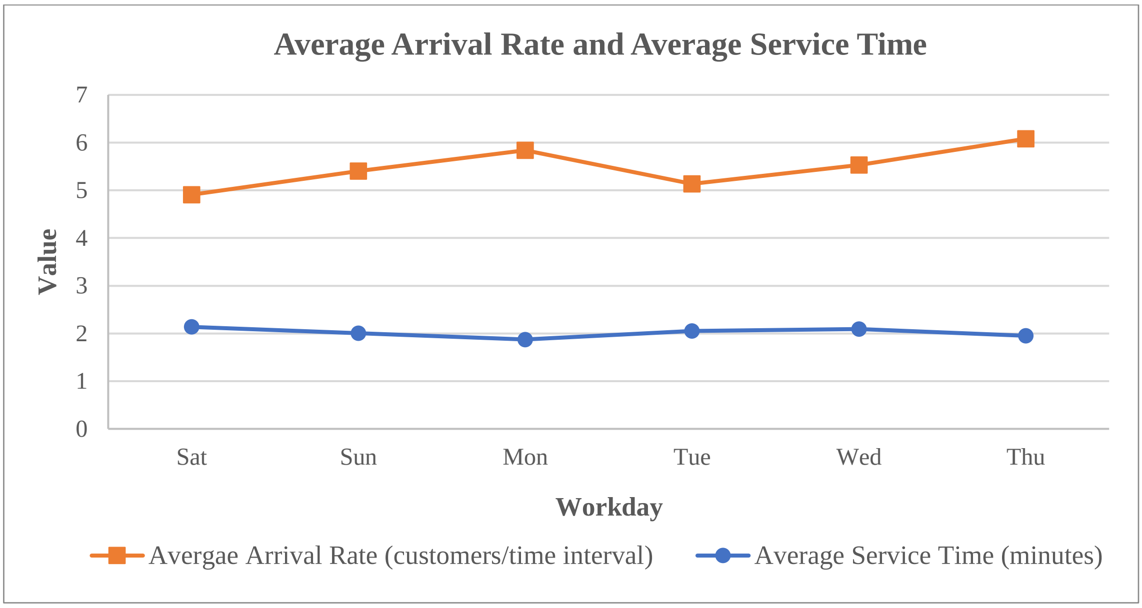
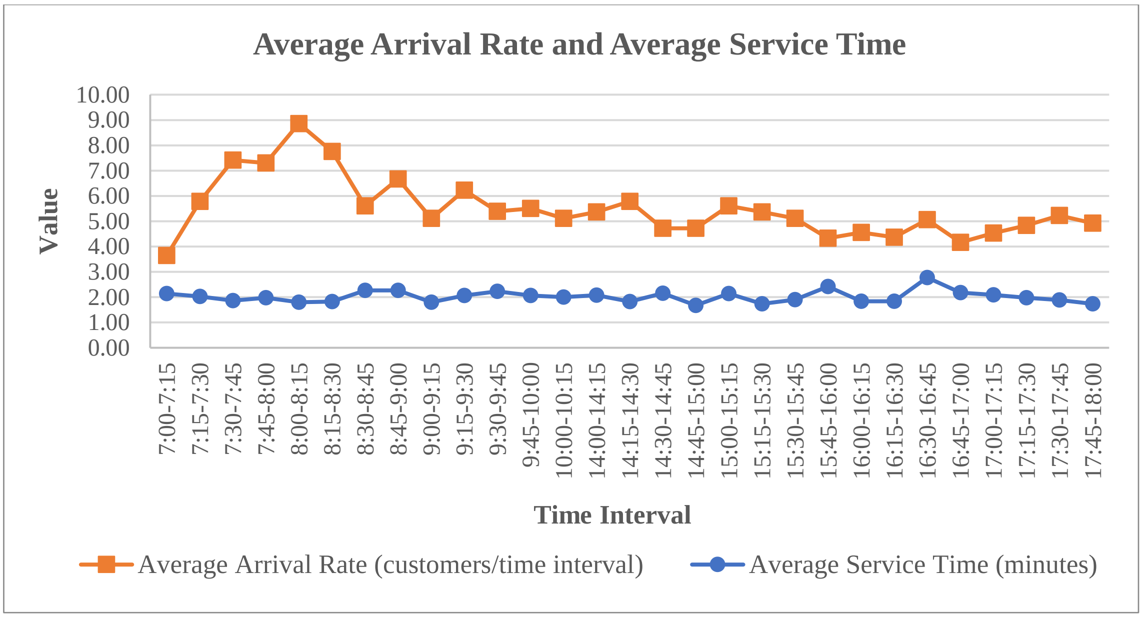


Figure 4.2

Average Arrival Rate and Average Service Time, according to the Study Workhours



Therefore, both coefficients confirm the hypothesis that the service time is negatively dependent on the arrival rate, but it is more clarified that there is a strong negative correlation between the averages of them based on workdays. This confirms that the service in Rio Café's drive-thru lane is performing normally in relationship between the arrival rate and the service time.

4.6 Peak Hours

The peak hours were determined, as Table 4.1 shows, based on the maximum Hourly Volume (HV) of the average arrivals for each workhour, which is the summation of the average arrival rates categorized based on workhours for every four consecutive time intervals in a day. Three peak hours were identified:

Table 4.1

Average Hourly Volume for the Study Workhours

Time Interval	Average Arrival Rate (customers/time interval)	Average Hourly Volume (customers)
7:00-7:15	3.64	24.14
7:15-7:30	5.78	29.36
7:30-7:45	7.42	31.33
7:45-8:00	7.31	29.53
8:00-8:15	8.86	28.89
8:15-8:30	7.75	25.14
8:30-8:45	5.61	23.61
8:45-9:00	6.67	23.39
9:00-9:15	5.11	22.22
9:15-9:30	6.22	22.22
9:30-9:45	5.39	-
9:45-10:00	5.50	-
10:00-10:15	5.11	-
14:00-14:15	5.36	20.58
14:15-14:30	5.78	20.83
14:30-14:45	4.72	20.42
14:45-15:00	4.72	20.81
15:00-15:15	5.61	20.42
15:15-15:30	5.36	19.36
15:30-15:45	5.11	18.36
15:45-16:00	4.33	18.31
16:00-16:15	4.56	18.14
16:15-16:30	4.36	18.11
16:30-16:45	5.06	18.58
16:45-17:00	4.17	18.75
17:00-17:15	4.53	
17:15-17:30	4.83	19.50
17:30-17:45	5.22	
17:45-18:00	4.92	
Average	5.48	22.26

1. A primary AM peak hour of 7:30 to 8:30 in the early morning, with an average HV of 31.33 customers;
2. A secondary PM peak hour of 14:15 to 15:15 in the mid-afternoon, with an average HV of 20.83 customers;
3. A sub-secondary peak hour in the evening from 17:00 to 18:00, with an average HV of 19.50 customers.

The first two peak hours are formed when commuters are traveling to work and traveling back home, respectively. The third peak hour is often formed when people go out to enjoy their time after the end of their workhours, and/or when commuters who work to such an hour are traveling back home. In addition, the last peak-hour is argued to be formed seasonally and may be at a different time according to the season and the facility. For this facility, Rio Café’s drive-thru service, and based on field observations, other sub peak-hours were formed after 18:00 in the summer season. However, this is beyond the daily study-duration.

4.7 Characteristics of Rio Café’s Queue

It was shown in sections 4.2 and 4.3 that the arrival rate and the service time follow random distributions: Poisson and Log-Normal, respectively, which considers both of arrival and service rates are following a Markovian chain. In other terms, the queuing system in Rio Café’s drive-thru service is denoted as M/M/1, (∞, FIFO) system, for a random arrival rate, a random service rate, a single-channel queue with an infinite length, and a “First-In-First- Out” service, respectively.

The queuing performance equations for such a system are shown in Appendix B, Table B-XIV, as May (7) presented, which were used to determine the characteristics of the queue in Rio Café’s drive-thru service, as Table 4.2 shows.

Based on the whole study period of 493-time intervals corresponding to 123.25 workhours in 17 workdays, Table 4.2 shows that the queue has a high intensity rate of 0.74, with a relatively high average waiting time of almost three times the average service time (2.01 minutes). Despite that, the average number of customers in the system (waiting and being served) is only 2.85 customers, which matches very well with the maximum designated queue length of three customers (including the customer in service) as the service provider painted (see Appendix A, Figure A-I). However, when queue is present, the service provider needs to increase the queue length to accommodate for five customers: four waiting customers, and one in service. In addition, it should be noted that the probability to have more than 3 customers in the system is 0.3, which means that about one third of the time there, is no room in the designated queue length for the queued customers.

Table 4.2

Characteristics of the Queue in Rio Café's Drive-Thru Service

Performance Measure	General Study Period		When Reneging was Detected	
	All Time Intervals	Morning PH Only	All Time Intervals	Morning PH Only
λ (customers/time interval)	5.5	7.8	7.45	9.58
μ (customers/time interval)	7.46	8.02	7.32	8.38
ρ	0.74	0.97	1.02	1.14
ρ (0)	0.26	0.03	N/A	N/A
ρ (1)	0.19	0.03	N/A	N/A
ρ (2)	0.14	0.03	N/A	N/A
ρ (3)	0.11	0.03	N/A	N/A
E (m) – customers	2.11	31.36	N/A	N/A
E (m/m>0) – customers	3.85	33.33	N/A	N/A
E (n) – customers	2.85	32.33	N/A	N/A
E (v) – minutes	7.80	62.40	N/A	N/A
E (w) – minutes	5.70	60.45	N/A	N/A

On the other hand, Table 4.2 shows that in the morning peak hour, the average number waiting to be served exceeds 30 customers, with an average waiting time for more than 60 minutes, at a high intensity rate of 0.97. Furthermore, there is a probability of 0.88 that arrival customers are queuing out of the designated line of three customers in the morning peak hour. In fact, field observations did not show such excessive records, except for the arrival rates. Since queuing performance equations depend on the averages of arrival and service rates, high variation in their data considerably affects them, which affects in turn the obtained results. See Table 3.2, which showed a high variance in the data of both the arrival rate and the service time.

When reneging behavior was detected, the average intensity rate was more than 1 in the morning peak hour and in all analyzed-time intervals. In such a case, when intensity equals to or greater than 1, May (7) recommends using the deterministic approach or microscopic simulation instead of the stochastic equations shown in Appendix B, Table B-XIV. As it will be described later in detail, the intensity rate is not necessary to exceed 1 for reneging behavior to occur, and it is not the only factor affecting this impatient customer's behavior.

Since the literature did not provide straight forward equations related to reneging behavior, and the stochastic approach is more generalizable for other services and systems, the following sections attempt to fill in the gap by mathematically determining the probability distribution function for the reneging behavior and the factors affecting it.

4.8 Reneging Behavior

In this study, reneging behavior was detected in 113-time intervals out of 493, in which 164 (out of 2713) customers reneged on the queue after they already joined it and waited for a certain time before giving up the service and leaving the line, with an average rate of reneging of 0.33 customers per time interval. Furthermore, the occurrence of this behavior was not regularly detected, as there were various number of time intervals with no detected reneging customers between successive time intervals in which the behavior was detected.

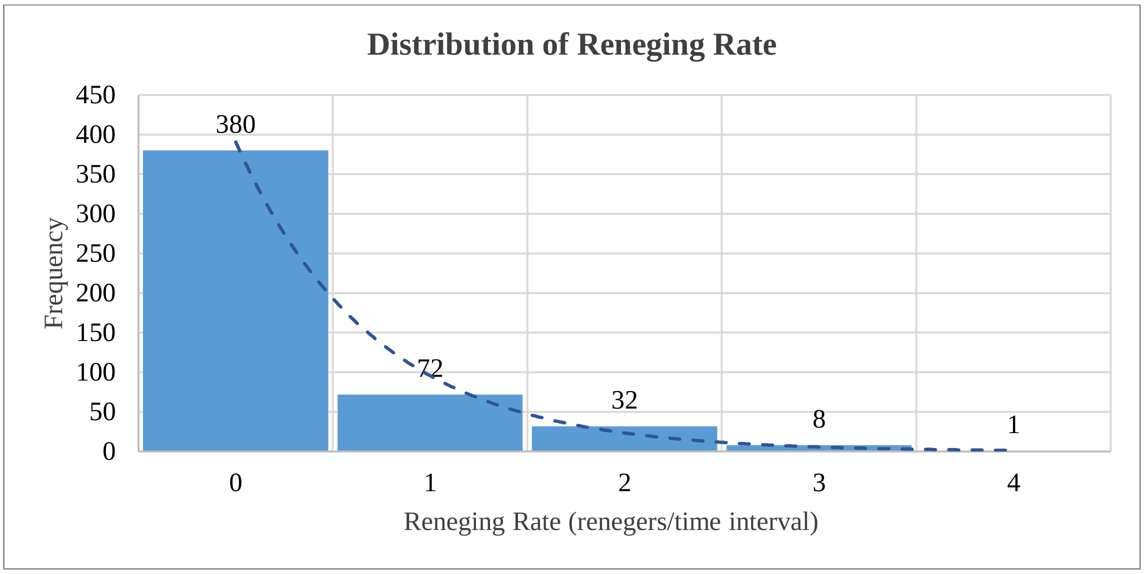
The following two sub-sections describe in details the distribution of reneging behavior, its rate of occurrence, and the time intervals between successive intervals in which it was detected.

4.8.1 Distribution of Reneging Rate

The total number of reneging customers varied from time interval to another, as Figure 4.3 shows. Its properties matched those assumed for a Poisson process, especially that the probability of simultaneous events in a subinterval of time tends to be zero is approaching zero and all events are independent to each other (34). Therefore, it was hypothesized that the reneging rate is following a Poisson distribution; however, this null hypothesis was rejected since the P-value of the conducted Chi-square goodness-of-fit test was almost zero.

Figure 4.3

Distribution of Reneging Rate



Despite that, the reneging rate still can be modeled as a Poisson process and matches its properties described by Montgomery and Runger (34). Therefore, it is argued that the reneging rate is either generally distributed or it is following a special Poisson distribution. However, a Poisson regression was modeled, where Y is the natural logarithm (Ln) of the reneging rate's relative frequency, while X is the reneging rate, as presented in Appendix C, Table C-I and Figure C-II. The equation obtained from the regression is shown in Equation (4.4).

$$Y = -1.4078x - 0.2326 \tag{4.4}$$

The equation shows the best estimation of the reneging rate's relative frequency for the study duration, with a value of $R^2 = 0.980$. Furthermore, the overall significance of the model was tested using ANOVA approach, where the significance F is 0.001 at a confidence level of 95%, indicating strong evidence to reject the null hypothesis that the model's slope equals to zero. The regression statistics, the ANOVA results, and the regression analysis's outputs are shown in Appendix C, Tables C-III, C-IV, and C-V, respectively.

Let R_{RF} be the reneging rate's relative frequency, then the Equation (4.4) can be solved for R_{RF} and expressed as follows:

$$R_{RF} = e^{-1.4078x - 0.2326} \tag{4.5}$$

The reason for building the above equation based on the relative frequency is that it generalizes the distribution for any period of time, not only for the study duration. Therefore, if the service provider or the decision maker is interested in the reneging rate for a chosen period of time, the total number of reneging customers can be counted or estimated for a period ranging from hours to days or months, depending on the period of interest.

A comparison between actual relative frequencies of reneging rate and expected ones obtained from Equation (4.5) is presented in Appendix C, Figure C-VI.

4.8.2 Distribution of Time Intervals between Successive Reneging Events

To determine the probability to detect a reneging customer within a certain time of starting the observation, the time between successive reneging events should be known. In fact, the collected data did not include the exact time between successive reneging events occurred in the same time interval. Therefore, it was assumed here that the reneging event is the occurrence of reneging behavior in different time intervals, regardless the number of reneging customers in each.

The hypothesized distribution of reneging events was tested against exponential distribution, assuming reneging events occur according to a Poisson process. By that, X is the number of time intervals until the occurrence of one reneging event, as it follows an exponential distribution with the PDF shown in Equating (4.6), with a mean number of events $\lambda > 0$ per time interval.

$$f(x) = \lambda e^{-\lambda x}, \qquad for\ 0 \leq x < \infty \tag{4.6}$$

As mentioned before, reneging events occurred in 113-time intervals out of 493 15-minute time intervals, which means that the mean rate of occurrence (λ) is 0.23 reneging event per time interval (regardless of the number of renegers in each interval). Table 4.3 shows the computations of Chi-square goodness-of-fit test for exponential distribution of number of time intervals between successive reneging events.

The results obtained show no strong evidence to reject the null hypothesis that the reneging event is exponentially distributed, based on the computed P-value of 0.669, which exceeds 0.05 for $\chi_0^2 = 2.367$ at a significance level of 5% and DF = 4. Figure 4.4 shows the observed and exponentially distributed expected frequencies for number of time intervals between successive reneging events.

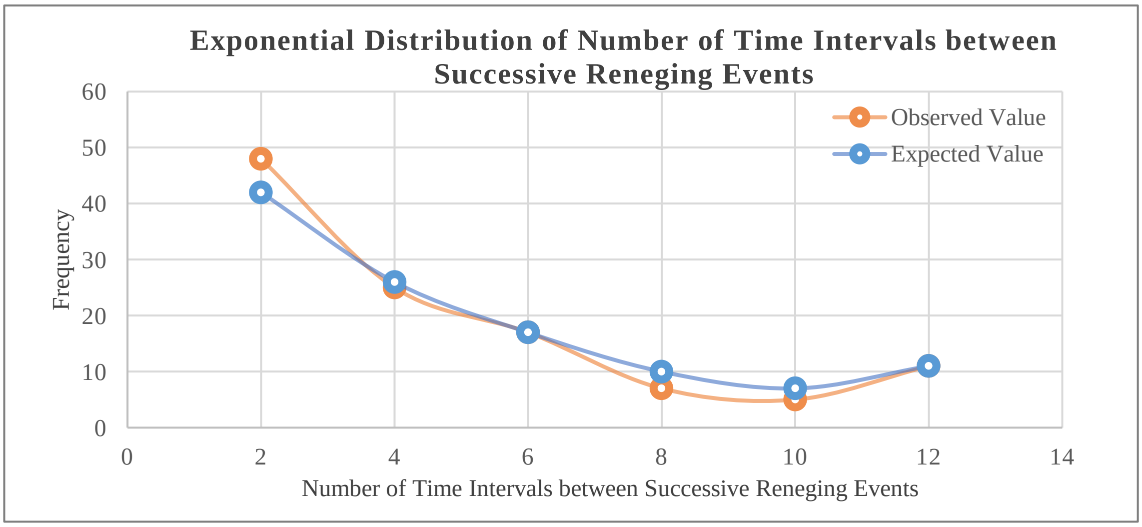
Table 4.3

Chi-Square Goodness-of-Fit Test for Exponential Distribution of Number of Time Intervals between Successive Reneging Events

No. of Time Intervals (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
2	48	0.369	0.369	42	0.857
4	25	0.601	0.232	26	0.038
6	17	0.748	0.147	17	0.000
8	7	0.841	0.093	10	0.900
10	5	0.900	0.059	7	0.571
x > 10	11	1.000	0.100	11	0.000
$\sum$	113	-	1.000	113	2.367
Right-Tailed Probability of $\chi_{0,4}^2$					0.669

Figure 4.4

Exponential Distribution of Number of Time Intervals between Successive Reneging Events



4.9 Factors Affecting Reneging Behavior

The reviewed literature addressed only one main factor affecting reneging behavior, which is RWT, as reneging occurs if RWT is equal to or greater than the impatient customer's threshold waiting time. In fact, this threshold varies among customers, depending on their personal preferences and the waiting time each of them tolerates. In such a case, the reneging issue is not generalizable and cannot be well understood, unless other factors accompany and affect the occurrence of such an event are determined.

When reneging occurred, the data were collected for all reneging events detected in the study duration, corresponding to each individual reneging customer, including the arrival time, the arrival rate, and the service time. Table 4.4 shows the average reneging rate and corresponding average value of several collected data variables, based on the daily study-workhours when reneging behavior was detected, where:

- R_R: Reneging rate (renegers/time interval);
- RWT: Response waiting time (minutes);
- $Rank$: Reneging customer's rank;
- L: Queue length (customers);
- λ: Arrival rate (customers/time interval);
- st: Service time (minutes);
- μ: Service Rate = $15/st$ (customers per time interval);
- ρ: Queue intensity = λ/μ.

It is expected that the main factors affecting reneging behavior are: queue intensity, RWT, and reneging customer's rank. The queue intensity is not directly evident to impatient customers, since the reneger is only affected by what is observed of the arrival and service rates after joining the queue. However, it is a reliable factor representing the degree to which the service's capacity is utilized. While reneging customer's rank is representing how far the customer from the service window is, RWT could be the time a customer tolerates to assess the speed of departure and decides whether to stay in the line or give up the service.

Since it was concluded the reneging rate's distribution is non-linear, there is no need to check its correlation with any factor, as Pearson's correlation coefficient is based on linearity assumption. However, this coefficient is still an indicator that could be adopted to confirm the factors affecting reneging behavior. Table C-VII in Appendix C shows the Pearson's correlation coefficient between previously shown variables in Table 4.4.

Table 4.4

Average Value of Selected Data Variables according to the Daily-Study Time Intervals, when Reneging Behavior was Detected

Time Interval	Average R_R	Average RWT	Average $Rank$	Average L	Average λ	Average st	Average ρ
7:00-7:15	1.0	0.02	3.00	3.00	7.00	1.10	0.51
7:15-7:30	1.0	0.22	3.00	3.00	6.25	2.09	0.88
7:30-7:45	1.7	0.36	3.20	3.20	10.00	1.40	1.00
7:45-8:00	1.8	0.22	3.11	3.11	9.00	2.01	1.29
8:00-8:15	1.6	0.40	3.64	3.82	10.29	1.71	1.18
8:15-8:30	1.7	1.24	3.53	3.80	9.22	1.86	1.15
8:30-8:45	1.3	0.34	3.20	3.20	6.25	2.63	1.12
8:45-9:00	1.4	1.15	3.10	3.40	7.57	2.19	1.08
9:00-9:15	1.0	0.66	3.00	3.00	6.33	1.89	0.79
9:15-9:30	1.8	0.75	3.83	4.00	9.75	2.08	1.34
9:30-9:45	1.8	0.16	3.56	3.56	7.60	1.83	0.91
9:45-10:00	1.8	0.17	3.14	3.14	8.25	3.30	1.61
10:00-10:15	1.0	2.70	2.00	2.00	7.50	1.76	0.88
14:00-14:15	1.7	0.27	3.00	3.00	9.33	2.94	1.73
14:15-14:30	1.4	0.17	2.43	2.43	6.20	2.09	0.83
14:30-14:45	1.0	2.45	2.00	2.00	3.00	0.67	0.13
14:45-15:00	1.1	0.18	2.63	2.75	5.00	2.11	0.71
15:00-15:15	2.0	0.62	3.00	3.10	6.80	2.45	1.21
15:15-15:30	1.0	0.87	3.33	3.33	6.00	1.91	0.71
15:30-15:45	1.0	2.32	4.00	4.00	5.00	0.53	0.18
15:45-16:00	1.8	0.38	3.14	3.29	6.25	1.99	0.84
16:00-16:15	1.5	0.81	2.83	3.33	7.75	1.85	1.00
16:15-16:30	1.0	0.30	2.75	2.75	6.25	1.85	0.78
16:30-16:45	1.0	0.79	3.00	3.00	6.25	2.08	0.84
16:45-17:00	1.0	0.20	3.00	3.50	7.00	1.53	0.71
17:00-17:15	2.0	1.00	2.75	2.75	6.00	2.26	0.87
17:15-17:30	1.5	0.31	3.00	3.00	7.50	2.19	1.12
17:30-17:45	1.6	0.27	2.75	2.88	7.00	1.92	0.85
17:45-18:00	1.0	2.22	3.00	4.00	7.00	3.17	1.48

The obtained results show a high correlation between some variables, such as: average rank & average L, average λ & average ρ, and average st & average ρ. These variables are dependent on each other, indicating a collinearity. Therefore, the variable with the highest correlation with R_R was selected from each pair and the other one was eliminated. The correlation between other variables is not high; less than 0.5, and could be accepted.

On the other hand, the correlation between average R_R and other variables is not high, and could be considered as moderate, indicating a poor linear correlation. However, the distribution of R_R is non-linear as mentioned before, and this correlation is just an indicator to confirm the factors.

Therefore, reneging customer's rank, RWT and ρ were considered as the main factors affecting reneging behavior with the highest correlation coefficients with average R_R. Furthermore, it is expected that the arrival time of customers affects their decision to stay in the queue or leave it, based on the importance of this time to them or how sensitive it is, especially at peak hours when arriving customers have important commitments, such as traveling to work. Therefore, it would be more important to study the probability of occurrence of a reneging event under these factors rather than the expected reneging rate.

4.10 Distribution of Factors Affecting Reneging

After identifying the factors affecting reneging behavior, it becomes necessary to determine their probability distribution functions, in order to mathematically understand how they affect this behavior and how far reneging customers interact with their ranges.

The following sub-sections discuss the conditional probability distribution functions of the three-selected main factors affecting reneging behavior, which are: queue intensity, RWT, and reneging customer's rank, when reneging behavior occurs.

4.10.1 Distribution of Queue Intensity

The queue intensity rate has an important role in explaining impatient customers' reneging behavior on a service line after waiting for a certain time to be served, since it represents the degree of utilization of the service's capacity.

When reneging behavior was detected, it was expected to have higher intensity rates in peak hours, as more customers arrive to the queue. In fact, the analysis shows that this hypothesis is partially true; however, higher intensity rates were detected in some off-peak hours, as Figure 4.5 shows, where the shaded area represents the three peak hours. It is known that off-peak hours have fewer arriving customers; therefore, it seems like customers tend to order products that relatively need more time to prepare in off-peak hours, during which reneging customers face higher intensity rates due to longer service times (less service rates) not higher arrival rates.

To determine the distribution of the queue intensity, it was assumed that renegers would expect that the number of customers that can be served during a fixed time interval (service rate) will be based on the service time they observed before they decide to leave the line. Other assumptions were made, as:

- Reneging customers fully observe the service time before they leave the line;
- The service time reneging customers observe is the same service time randomly selected per time interval;
- The randomly selected service time represents the average service time for the time interval it was selected from.

Therefore, for each time interval in which reneging behavior was detected, regardless of the number of renegers per each, the intensity rate was computed by dividing the arrival rate by the service rate for the same time interval of 15 minutes. This intensity will be called as the general queue intensity rate.

The goodness-of-fit for a log-normal distribution was tested for the general queue intensity using Equation (4.3), the moment a reneging event occurs, using the Chi-square test at a significance level of 5%, as Table 4.5 shows, assuming the natural logarithm of the queue intensity rate (X) is normally distributed, with a mean (θ) of -0.13 and a standard deviation (ω) of 0.56 for Ln (X).

The obtained result shows that the P-value of χ_0^2 = 3.366 with a DF of 5 is 0.643, which is much greater than the critical P-value of 0.05, indicating that there is a strong enough evidence that the data of general queue intensity rates fit well with a log-normal distribution. Figure 4.6 shows a comparison between observed and expected frequencies of general queue intensity rate.

Figure 4.5

Average Queue Intensity in the Daily-Study Time Intervals at Which Reneging Behavior was Detected

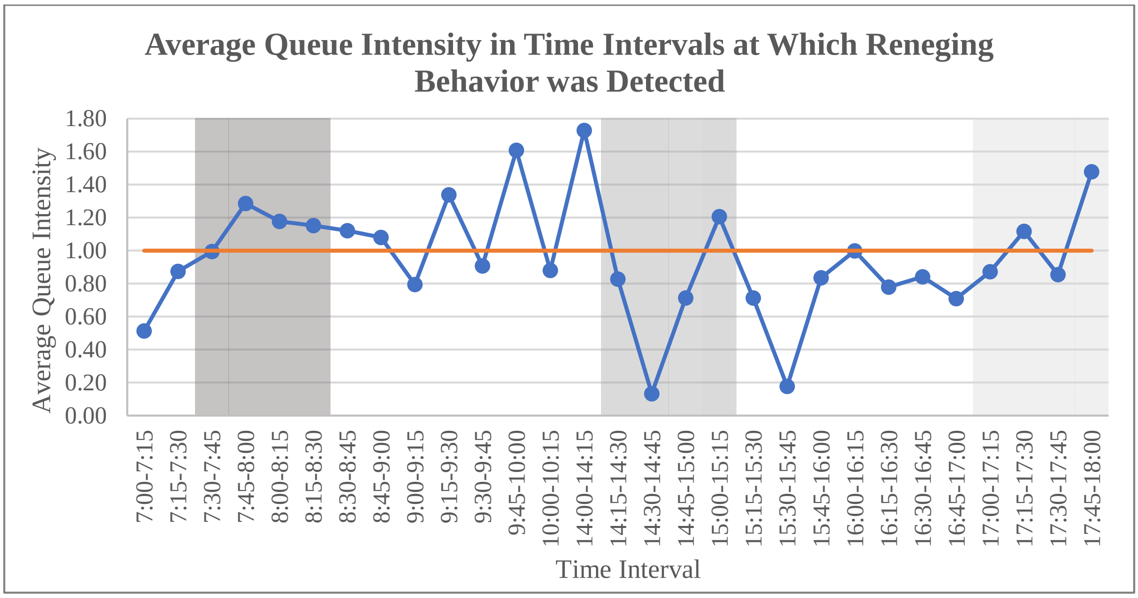


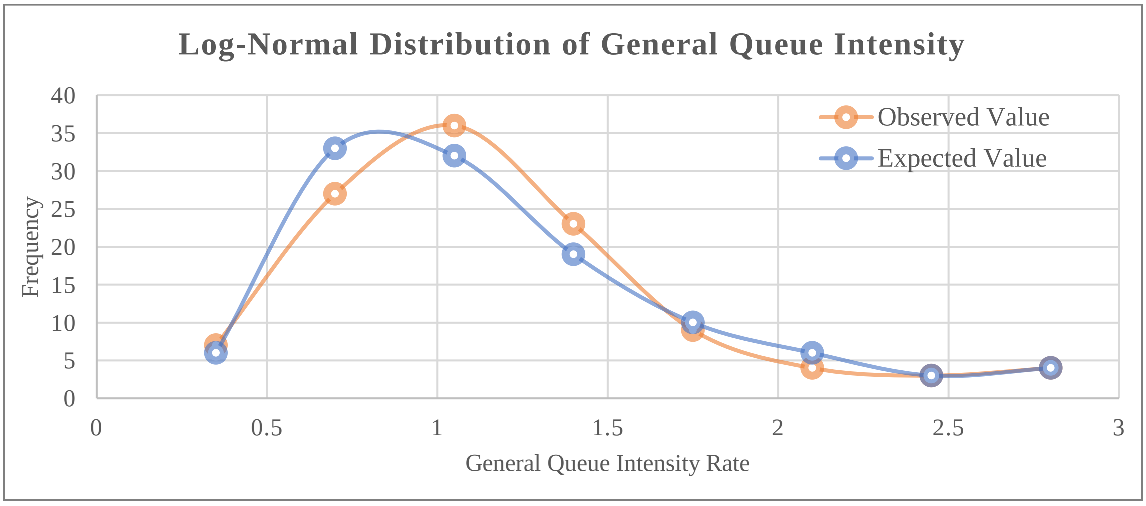
Table 4.5

Chi-Square Goodness-of-Fit Test for Lognormal Distribution of General Queue Intensity, when there is a Reneging Event

Queue Intensity (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
0.35	7	0.050	0.050	6	0.167
0.7	27	0.343	0.293	33	1.091
1.05	36	0.625	0.282	32	0.500
1.4	23	0.798	0.172	19	0.842
1.75	9	0.891	0.093	10	0.100
2.1	4	0.940	0.049	6	0.667
2.45	3	0.967	0.026	3	0.000
x > 2.45	4	1.000	0.033	4	0.000
$\sum$	113	-	1.000	113	3.366
Right-Tailed Probability of $\chi_{0,5}^2$					0.643

Figure 4.6

Log-Normal Distribution of General Queue Intensity, when there is a Reneging Event



However, reneging customers are not affected by the whole arrival rate, but only by what they observe of it during their RWT. Therefore, the general queue intensity should be replaced by the perceived queue intensity, which is the intensity perceived by the reneging customer the moment of leaving the queue, based on the reneger's rank and the service rate the reneging customer estimates for the Perceived Waiting Time (PWT), as it will be explained in Section 4.12. The estimated service rate (μ_E) can be calculated using Equation (4.7), with the same assumptions made in calculating the general queue intensity, and queued customers in front of the reneging customer arrive to the queue during a time period equals to the reneger's PWT.

$$\mu_E = \frac{PWT}{st}, \qquad for\ st > 0 \tag{4.7}$$

The perceived queue intensity (ρ_P) can be calculated by dividing the reneging customer's rank, as it is the number of observed arrived-customers by the reneger, by the estimated service rate (μ_E), as presented in Equation (4.8). In such linear queues, it is assumed that customers can perceive only their rank, where for other services the rank could be replaced by the queue length, if the customer can fully observe and perceive it.

$$\rho_P = \frac{Rank}{\mu_E}, \qquad for\ Rank\ \geq 2\ and\ \mu_E > 0 \tag{4.8}$$

The distribution of ρ_P was tested against log-normal distribution, using Chi-square goodness-of-fit test, where the P-value of $\chi^2_{0,4}$= 5.087 is only 0.279. The obtained results show a poor fit between the data of ρ_P and log-normal distribution; however, the P-value is greater than 0.05 and the null hypothesis that the data of ρ_P are lognormally distributed cannot be rejected except for stronger evidence.

Therefore, the Chi-square goodness-of-fit test was used again to test the hypothesized distribution of ρ_P against Maxwell–Boltzmann distribution. The obtained results show stronger evidence to reject that the perceived queue intensity is log-normally distributed, but it follows a Maxwell–Boltzmann distribution (Equation (4.9)), with a P-value of 0.602 for the $\chi^2_{0,5}$ of 3.641.

Table C-VIII in Appendix C shows the computations of Chi-square test of Maxwell–Boltzmann distribution at a significance level of 5%, and a b parameter of 0.89. The optimal value of b parameter was calculated using Solver analysis tool in Microsoft Excel software, to meet the mean of 1.42 and the variance of 0.45 (Equations (4.10) and (4.11), respectively). The actual data were compared with the theoretical perceived queue intensities that follows a Maxwell–Boltzmann distribution, as presented in Appendix C, Table C-IX.

$$f(x) = \sqrt{\frac{2}{\pi}} \frac{x^2 exp\left(-\frac{x^2}{2b^2}\right)}{b^3}, \qquad for\ x\ and\ b > 0 \tag{4.9}$$

$$\mu = E(X) = 2b\sqrt{\frac{2}{\pi}}, \qquad for\ b > 0 \tag{4.10}$$

$$\sigma^2 = V(X) = \frac{b^2(3\pi - 8)}{\pi}, \qquad for\ b > 0 \tag{4.11}$$

Referring to Figure 4.6 and Appendix C, Figure C-IX, it is obvious that the highest probability to renege on a queue is when the waiting customer faces a queue intensity approaching 1 or slightly higher. However, it is notable that the number of reneging customers increases as the queue intensity increases until it reaches its maximum value, then it starts decreasing as the intensity increases. This is explained as reneging customers who join the queue at higher intensity rates than its maximum value are really in dire need of the service, where the total number of these customers are less than the total of those who join the queue at less intensity rates. Therefore, with less reneging customers at higher intensities, the probability of reneging decreases.

4.10.2 Distribution of RWT/st Ratio

As RWT represents the time an impatient customer waits for the service before reneging and leaving the line, the approach of testing its distribution was against the reliability functions. To be more generalizable and applicable for other services, where service time varies depending on the type of the service, reneging customer's RWT was divided by the service time a reneger observes before leaving the line, with the same assumptions made in determining the distribution of the queue intensity. The theoretical distribution of obtained ratio (RWT/st) was compared with Weibull distribution (Equation (4.12)), which is often used to model the time until failure in many systems (34).

$$f(x) = \left(\frac{\beta}{\delta}\right)\left(\frac{x}{\delta}\right)^{\beta-1} exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \qquad for\ x, \delta\ and\ \beta > 0 \qquad (4.12)$$

Where:

δ : Scale parameter

β : Shape parameter

Table 4.6 shows the computations of Chi-square test at a significance level of 5%, using δ and β parameters as 0.22 and 0.56, respectively. Scale and shape parameters (δ and β) were calculated using Solver analysis tool in Microsoft Excel software, by changing their values in Equations (4.13) and (4.14), to find the optimal value of the mean ($\mu = 0.37$) and the variance ($\sigma^2 = 0.50$) of the random variable X (RWT/st ratio). Weibull PDF is shown in Equation (4.12).

When the random variable X follows a Weibull distribution, the equations for its mean and variance are:

$$\mu = E(X) = \delta\Gamma\left(1 + \frac{1}{\beta}\right), \qquad for\ \delta\ and\ \beta > 0 \qquad (4.13)$$

$$\sigma^2 = V(X) = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \mu^2, \qquad for\ \delta\ and\ \beta > 0 \qquad (4.14)$$

With eight classes and two parameters of the hypothesized distribution were estimated by sample statistics, the DF used was 5 to compute the P-value of χ_0^2 = 1.729, which is 0.885, as Table 4.6 shows.

Table 4.6

Chi-Square Goodness-of-Fit Test for Weibull Distribution of Renegers' RWT/st Ratio, when there is a Reneging Event

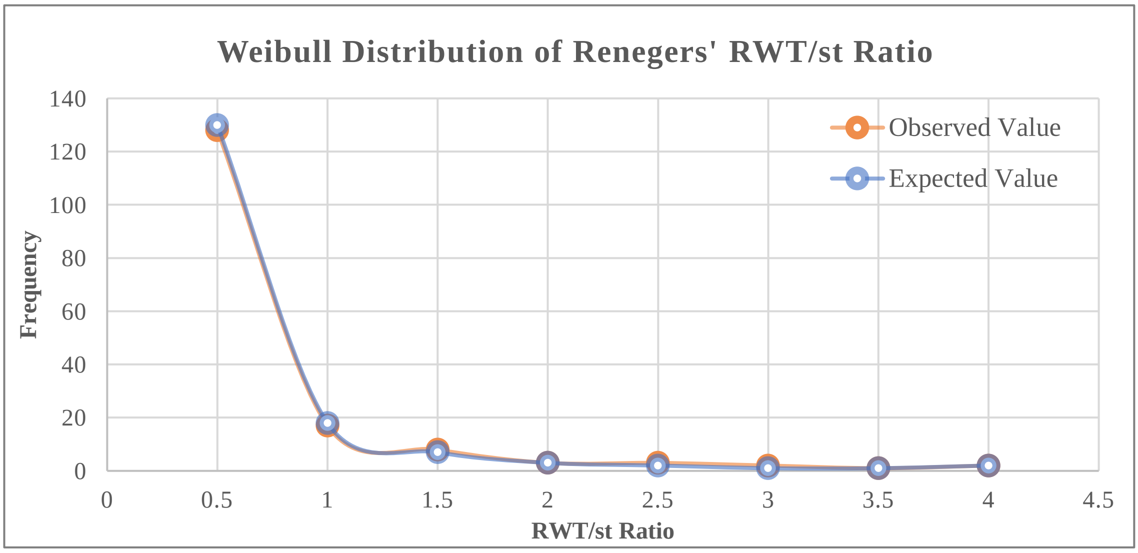
RWT/st (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
0.5	128	0.795	0.795	130	0.031
1	17	0.903	0.108	18	0.056
1.5	8	0.947	0.044	7	0.143
2	3	0.968	0.021	3	0.000
2.5	3	0.980	0.012	2	0.500
3	2	0.987	0.007	1	1.000
3.5	1	0.991	0.004	1	0.000
x > 3.5	2	1.000	0.009	2	0.000
∑	164	-	1.000	164	1.729
Right-Tailed Probability for $\chi_{0,5}^2$					0.885

The computed P-value shows a satisfactory agreement between actual and theoretical values for a Weibull distribution. In other terms, there is no strong evidence to reject the null hypothesis that RWT/st ratio has a Weibull distribution. Furthermore, the high match between actual and theoretically expected values is highly notable in Figure 4.7, which confirms the results obtained.

As shown in Figure 4.7, it is clear that the distribution of reneging customers' RWT/st ratio follows a dumping sinusoidal wave, where it is argued that it is a complex Weibull-trigonometric distribution.

Figure 4.7

Weibull Distribution of Renegers' RWT/st Ratio, when there is a Reneging Event



In addition, it is noted that as RWT/st ratio increases, the probability of reneging decreases, with a sharp decline from RWT/st ≤ 0.5 to RWT/st ≤ 1, and a smooth decline as RWT/st ratio is greater than 1. The majority of reneging customers who leave the queue at RWT/st ≤ 1 have less patience than others who stayed in the line at higher RWT/st ratios. With much fewer reneging customers who are willing to wait for a ratio of RWT/st greater than 1, the probability of reneging is decreasing, as who have this willingness to wait for more time needs the service more than others. However, it is expected that RWT/st is affected by other factors, especially the general queue intensity and the arrival time of customers, as it will be explained later.

4.10.3 Distribution of Reneging Customers' Rank

As mentioned before, the rank of customers represents the time distance between each of them and the window of the drive-thru service. Moreover, it represents the number of arrivals who are fully observed by the queued customer during the waiting time.

The hypothesized renegers' rank distribution was tested against several discrete and continuous distributions, and it was concluded that there is no fit for any one, except for the Sine distribution (Equation (4.15)) with a scale parameter (b) of 5 as the maximum detected reneger's rank. Goodness-of-fit test was conducted, as Table 4.7 shows, where obtained results indicate a reasonable fit, with a P-value of 0.348 for $\chi^2_{0,2}$ = 2.112, implying it is acceptable to consider that renegers' rank is following a Sine distribution.

$$f(x) = \frac{\pi}{2b} sin\left(\pi \frac{x}{b}\right), \qquad for\ x \in [0, b]\ and\ x \geq 2 \qquad (4.15)$$

Furthermore, it is claimed that the renegers' rank follows a complex Sine wave compound with another function. Figure 4.8 shows a comparison between observed and expected frequencies of reneging customers' rank, while it is noted that observed values has a semi-symmetrical curved shape about the most frequent rank.

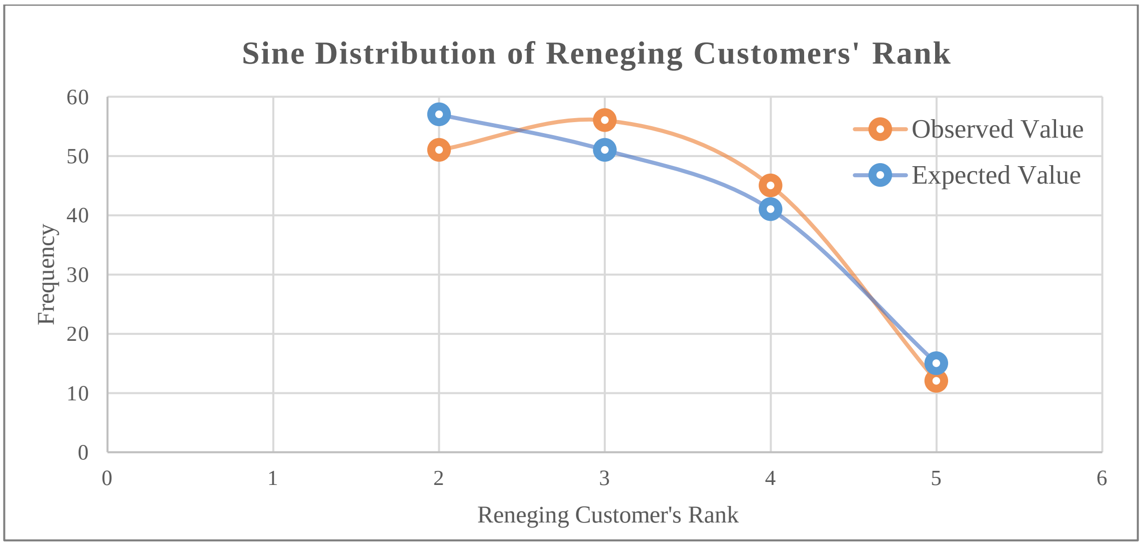
Table 4.7

Chi-Square Goodness-of-Fit Test for Sine Distribution of Renegers' Rank, when there is a Reneging Event

Rank (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi^2 = \sum \frac{(O_i - E_i)}{E_i}$
2	51	0.345	0.345	57	0.632
3	56	0.654	0.309	51	0.490
4	45	0.904	0.250	41	0.390
x > 4	12	1.000	0.096	15	0.600
$\sum$	164	-	1.000	164	2.112
Right-Tailed Probability for $\chi^2_{0,2}$					0.348

Figure 4.8

Sine Distribution of Renegers' Rank, when there is a Reneging Event



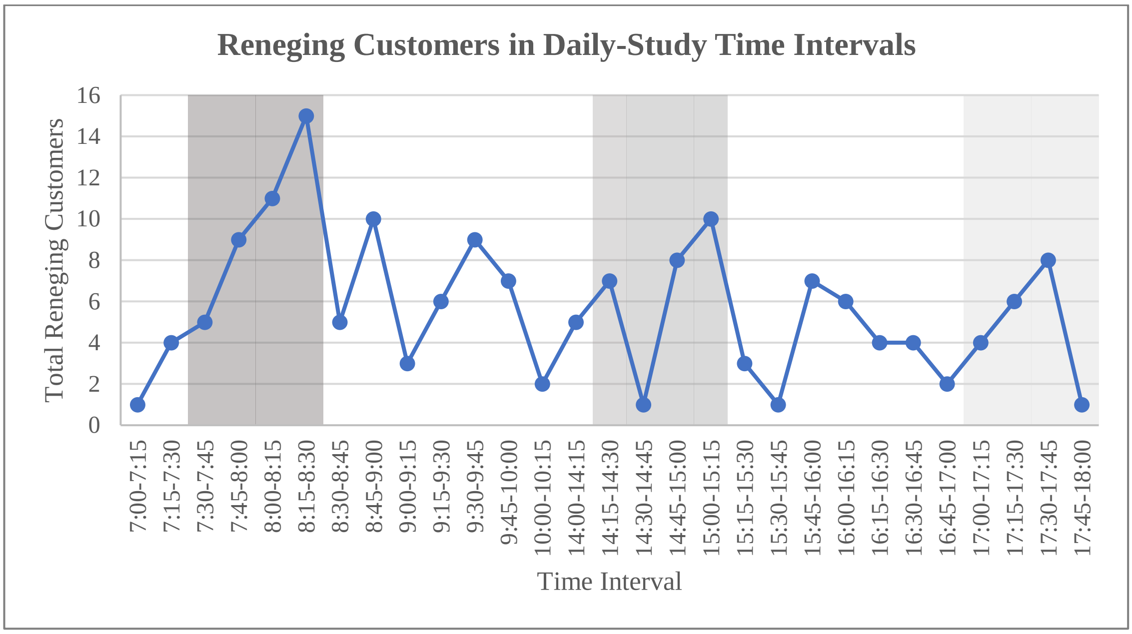
From the obtained results, it is clear that as the rank of customers increases, the probability of reneging decreases, since the number of customers who join long queues is usually few, which their length is already observed by customers before deciding to join it. In this regard, it is expected that the longer the queue is, the fewer reneging customers are detected, but the more customers are balking. Same as RWT/st ratio, it is also expected that reneging customers' rank is dependent on other factors, such as the arrival time of customers, as it will be explained later.

4.11 Sensitivity of Arrival Time

As previously explained, there was a significant difference between arrival rates' averages based on workhours or time intervals customers arrive at, due to the variation in arrivals between peak and off-peak hours. When more customers arrive to the queue at peak hours, it is expected to have more renegers as well. Figure 4.9 shows the total number of renegers in each of the daily-study time intervals, where the shaded area is the peak hours.

Figure 4.9

Total Number of Reneging Customers in each of the Daily-Study Time Intervals



Certainly, customers' arrival times do not have the same degree of sensitivity or importance to them. As the sensitivity of arrival time increases, the closer the arrival time is to the peak hour, when most commuters are traveling to work or home.

On a scale of 1 for the sensitivity degree of the arrival time, the study time intervals were rated, based on how close a time period is to the peak hour. Where 1 is the highest degree of sensitivity of the four consecutive time intervals in each of the three peak hours. The sensitivity of a time interval gradually decreases by 0.25 degree, the further away it is from the closer peak hour, regardless whether it is before or after the peak hour, until the value of zero is reached representing a completely off-peak hour time interval (Appendix D, Figure D-I).

Based on the degree of sensitivity of the arrival time (S), the total number of reneging customers and the average values of affecting factors were calculated as Table 4.8 shows. The obtained results show that approximately half of reneging customers renege on the queue at the peak hour only, while the other half of them is distributed over the remaining workhours. In addition, reneging customers wait for a certain time before leaving the line at the peak hour, which is almost the same time when leaving during off-peak hours (Appendix D, Figure D-II). This behavior is explained by the average reneging customers' rank, the average queue intensity, and the degree of S, where the highest rank and highest queue intensity are at S = 0. Therefore, impatient customers do not have to wait for long times at high ranks, when facing high queue intensities at such a degree of S (Table 4.8).

The second lowest rank is at S = 1, where impatient customers give up their low rank at the peak hour to reach their destinations. Figure D-III in Appendix D shows that the greater the value of S than 0.5 degree, the lower the average rank an impatient customer is giving up.

Table 4.8

Degree of Sensitivity of Arrival Time and Corresponding Average Value of Affecting Factors

S	Number of Renegers	Relative Frequency	Average RWT (m)	RWT 85^{th} Percentile (m)	Average Rank	Average RWT/st	Average ρ
0	30	0.18	0.58	1.20	3.27	0.34	1.15
0.25	14	0.09	0.42	0.65	3.00	0.24	0.80
0.5	16	0.10	1.06	2.50	3.13	0.79	0.65
0.75	19	0.12	0.36	0.75	3.11	0.25	1.03
1	85	0.52	0.57	1.52	3.07	0.35	0.98

In addition, Figure D-IV in Appendix D shows that the highest average general queue intensity is at S = 0, which confirms the previously mentioned observation that customers order products that need more time to prepare at off-peak hours. This leads impatient customers to face high queue intensity rates at such hours, even with the lowest number of arrival customers. On the other hand, the lowest average general queue intensity is at

S = 0.5, where moderate arrival customers are expected to join the queue, but served with higher service rates. This explains the highest average RWT at S of 0.5 degree, at which impatient customers do not mind to wait for longer RWT at a moderate sensitive arrival time before reneging on a queue has a high departure speed. Even when impatient customers arrive at a semi-off peak time interval (S = 0.25), they wait for only 25 seconds on average before leaving the line at the lowest average rank, due to a relatively high average intensity rate.

In general, it is claimed that S of 0.5 is the degree an impatient customer starts to recognize how close the time of arrival is to the peak hour, as the more sensitive the arrival time is (for $S \geq 0.5$), the less the rank an impatient customer gives up, at a generally increasing queue intensity and a generally decreasing RWT.

In addition, it is concluded that S of 0.75 is the critical degree of sensitivity of arrival-time intervals, where reneging customers are willing to wait for the shortest average RWT. This is due to the relatively high intensity rate, but it is more related to the closeness of time intervals with this degree of sensitivity to the peak hour. At such a degree of sensitivity (S = 0.75), it is believed that reneging customers perceive and assess the time needed to catch their commitments; such as work, which mostly begins by the beginning of the morning peak hour. Therefore, an average RWT at S = 1 is higher than at S = 0.75, when reneging customers may do not mind waiting for extra time during peak hours, before leaving their lower ranks, as long as they are already late.

However, a non-linear regression model was built, using Excel software, for the relative frequency of the number of reneging customers based on the degree of sensitivity of the arrival time, as presented Equation (4.16).

$$RC_{RF} = -\,2.056S + 1.37e^{S} - 1.173 \tag{4.16}$$

Where:

RC_{RF} : Relative frequency of number of reneging customers

S : Degree of sensitivity of arrival time

The above equation represents the best estimation for the RC_{RF} based on the degree of S, with an R^2 of 0.945, adjusted R^2 of 0.889, and a standard error of only 0.060. Furthermore, the significance of the model was tested using ANOVA approach, on a confidence level of 90%, where the overall significance F is 0.055. Since the significance F is less than 0.10, it indicates that the model's coefficients are not jointly significant, which is a sufficient evidence to reject the null hypothesis that the model's coefficients have zero values. It should be noted that a confidence level of 95% leads not to reject the null hypothesis mentioned before. A comparison between relative frequencies of observed number of reneging customers, and expected ones obtained from Equation (4.16) is presented in Appendix D, Figure D-V. The regression statistics, the ANOVA results, and the regression analysis's outputs are shown in Appendix D, Tables D-VI, D-VII, and D-VIII, respectively.

With Equation (4.16), it becomes possible to determine any probability related to reneging behavior at any chosen degree of S, especially for peak hours, by intersecting the probability with the mentioned equation for the RC_{RF}. This includes the subsequent and aforementioned probabilities, such as the probability of a reneging event to occur, the relative frequency of reneging rate, and the probability of reneging at a specified range of an affecting factor when reneging behavior occurs.

4.12 Reneging Customers' Perceived Waiting Time

Reneging customers have a perception of how long they are supposed to wait in the queue, from the moment they join it based on their rank, until the moment they decide to give up the service and leave the line, based on their RWT. This time is suggested to be called Perceived Waiting Time (PWT), which is the time perceived and estimated by reneging impatient-customers waiting in a drive-thru line, according to their rank, RWT, and tolerated or previously estimated service time. The proposed equation for PWT in minutes is the following:

$$PWT = RWT + (Rank - 1)(\overline{ST} + \tau), \quad for\ Rank \geq 2 \qquad (4.17)$$

Where:

PWT	:	Perceived waiting time by reneging customers
RWT	:	Response waiting time
$\overline{ST}$	:	Average service time
τ	:	Average lost time

The average lost time was assumed to be 3 seconds (0.05 minutes), which is the time a waiting vehicle consumes to move from its current rank to the next one. While, the average service time was 2.01 minutes, as it was calculated before. Figure 4.10 shows the frequency of observed number of renegers corresponding to renegers' PWT, based on Table D-IX in Appendix D.

Figure D-X in Appendix D shows an irregular sinusoidal wave, which indicates that two or more functions are forming the wave, but at least one of them was confirmed as a trigonometric function, since the reneging customer's rank follows a Sine distribution; while, RWT/st ratio's distribution showed a dumping wave combined with the Weibull distribution.

Referring to Equation (4.7), reneging customer's PWT is divided by the service time (st) to obtain the estimated service rate (μ_E). Besides that, the resulting ratio represents also the PWT as multiples of the service time; this makes the ratio generalizable for other services, as presented in Equation (4.18) for PWT/st ratio.

$$\mu_E = \frac{PWT}{st} = \frac{RWT + (Rank - 1)(\overline{ST} + \tau)}{st}, \qquad for\ st > 0\ and\ Rank\ \geq 2 \tag{4.18}$$

The distribution of PWT/st ratio was tested against log-logistic distribution, assuming log (x = RWT/st) is logistically distributed, with the PDF shown in Equation (4.19) for the random variable x that follows a log-logistic distribution, with b and k parameters of 2.3 and 2.7, respectively. These two parameters were determined by keep changing their values; therefore, the minimum value of x is zero while the maximum value is 12.7. The P-value for $\chi^2_{0.05,3}$ is 0.397, indicating a moderate evidence not to reject the null hypothesis that PWT/st ratio is log-logistically distributed. If log (x) is logistically distributed, then x has a log-logistic distribution, with the following PDF:

$$f(x) = \frac{b^k k x^{k-1}}{(b^k + x^k)^2}, \qquad for\ x, b\ and\ k > 0 \tag{4.19}$$

Where:

b : Scale parameter

k : Shape parameter

In addition, PWT/st ratio was tested against log-normal distribution as well, using Equation (4.3), assuming that Ln(X=PWT/st) is normally distributed. Table D-XI in Appendix D shows the Chi-square goodness-of-fit test for log-normal distribution of PWT/st ratio (X), with a mean of 0.85, and a standard deviation of 0.6 for Ln(X). The P-value for $\chi^2_{0.05,3}$ is 0.781, which is much greater than the P-value obtained before when RWT/st ratio was tested against log-logistic distribution. This indicates a better fit with lognormal distribution, where it is strongly evident to conclude that PWT/st ratio is log-normally distributed. See Appendix D, Figure D-XII for the actual data compared with theoretical reneging customers' PWT/st ratio; that is, log-normally distributed.

4.13 Conditional Joint Probability Distribution of Factors Affecting Reneging

If f(X), h(Y) and g(Z) are the probability distribution functions of the main factors affecting the reneging behavior: renegers' rank, RWT/st ratio and the general queue intensity level, respectively, when reneging behavior occurs, then the joint probability distribution (JPD) of these factors could be calculated by intersecting their probabilities, using Equation (4.20). In other words, the resulting JPD represents the probability of reneging to occur under a certain range of each factor, when reneging behavior (R_b) is present.

$$P(X|R_b) \cap P(Y|R_b) \cap P(Z|R_b) = \int_X f_{X|R_b}(x)\,dx \int_Y h_{Y|R_b}(y)\,dy \int_Z g_{Z|R_b}(z)\,dz$$

$$= \int_X \frac{\pi}{2b} sin\left(\pi\,\frac{x}{b}\right) dx \int_Y \left(\frac{\beta}{\delta}\right)\left(\frac{y}{\delta}\right)^{\beta-1} exp\left[-\left(\frac{y}{\delta}\right)^{\beta}\right] dy \int_Z \frac{1}{z\omega\sqrt{2\pi}} exp\left[-\frac{(ln(z)-\theta)^2}{2\omega^2}\right] \quad (4.20)$$

Tables D-XIII, D-XV, and D-XVII; and Figures D-XIV, D-XVI, and XVIII in Appendix G show the JPD of each two factors by intersecting their probabilities. It is clear that most reneging events occur at low reneging customers' ranks with low RWT/st ratios, but at relatively high levels of the general queue intensity that are approaching 1 or slightly higher. In this regard, it was shown before that as intensity approaches 1, the time an impatient customer arrives to the queue is closer to the peak hour, when personal time of customers becomes more precious. Therefore, most renegers wait for low RWT/st ratio ≤ 0.5 before leaving the line at low ranks ≤ 2. Furthermore, the higher the reneging customer's rank is, the more the time an impatient customer waits in the queue before deciding to leave the line and giving up the service, as the probability of reneging is decreasing (Appendix D, Figure D-XIV). In general, regardless of the sensitivity of the arrival time, customers who join already observed long queues have a higher need for the service more than others; therefore, there is a low probability that they renege on the queue.

However, intensity levels higher than 1 mean longer queues or slower departure speeds of the queue (lower service rates). As mentioned before, the results show that excessive intensities were detected at off-peak hours, indicating they potentially were due to higher service times. In both cases for higher intensity levels, most impatient customers will not join the queue when its intensity is greater than its normal maximum ratio of 1, either due to a recognizable high-queue length exceeding their tolerated threshold, or due to a recognizable and/or previously perceived low speed departure of the queue. This explains the decreasing probability of reneging when intensity is increasing to levels more than 1.05, as both RWT/st ratio and reneging customers' rank are increasing (Appendix D, Figures D-XVI and D-XVIII). At such a range of intensity, less customers join the queue with a higher need for the provided service than others who do not, where joining customers can handle waiting for an extra time at higher ranks. Therefore, as the probability of reneging is decreasing for intensity levels more than 1, it is argued that more balking customers could be detected.

Figure D-XIX in Appendix D summarizes the JPD of X, Y, and Z; so, the resulting JPD represents the intersection of the three affecting factors when reneging occurs (Equation (4.20)). It is noted that the probability of reneging has the same general pattern between different ranges of the main affecting factors, forming a three-dimensional dumping sinusoidal wave.

It should be noted that $f_{X|R_b}(x)$, $f_{Y|R_b}(y)$, and $f_{Z|R_b}(z)$ are conditional probability distribution functions for the main factors affecting reneging, where the PDF of reneging behavior to occur in a certain range of number of time intervals was shown in Equation (4.6). Therefore, the probability of reneging behavior to occur in a range of R_b number of time intervals at a specific range of general queue intensity (Z) is the following, with the same process for the perceived queue intensity, RWT/st ratio, and reneging customers' rank:

$$\int_{Z}\int_{R_b} f_{Z,R_b}(z,R_b)\,dz\,dR_b = \int_{Z} f_{Z|R_b}(z)\,dz \int_{Z} f_{R_b}(R_b)\,dR_b \tag{4.21}$$

4.13.1 Probability of Reneging at a Specific Range of PWT/st Ratio, when Reneging Occurs

Since the random variable PWT/st ratio is log-normally distributed, then the lognormal PDF (Equation (4.22)) can be integrated to determine the probability of reneging at a specific PWT/st ratio when reneging occurs. In other terms, it is the probability to have a specific range of reneging customers' PWT/st ratio, when there is a reneging behavior (R_b).

$$f_{PWT,st|R_b}(PWT,st) = \frac{st}{PWT\omega\sqrt{2\pi}} exp\left[-\frac{\left(ln\left(\frac{PWT}{st}\right)-\theta\right)^2}{2\omega^2}\right], for\ st\ and\ PWT > 0 \tag{4.22}$$

As PWT cannot be determined without other factors, Equation (4.18) can be substituted into the lognormal PDF to determine the probability of reneging at a specified range of each affecting factor, when reneging behavior is present, as presented in Equation (4.23). Furthermore, service time can also be substituted for the queue intensity level, as Equation (4.24) shows.

$$f_{st,RWT,Rank|R_b}(st,RWT,Rank)$$

$$= \frac{st}{\omega\sqrt{2\pi}\left(RWT+(Rank-1)(\overline{ST}+\tau)\right)} exp\left[-\frac{\left(ln\left(RWT+(Rank-1)*(\overline{ST}+\tau)\right)-\ln(st)-\theta\right)^2}{2\omega^2}\right] \tag{4.23}$$

$$f_{\lambda,\rho,RWT,Rank|R_b}(\lambda,\rho,RWT,Rank)$$

$$= \frac{15\,\rho}{\lambda\omega\sqrt{2\pi}\left(RWT+(Rank-1)(\overline{ST}+\tau)\right)} exp\left[-\frac{\left(ln\left(RWT+(Rank-1)*(\overline{ST}+\tau)-ln\left(\frac{15\rho}{\lambda}\right)\right)-\theta\right)^2}{2\omega^2}\right] \tag{4.24}$$

It should be noted that $\overline{ST}$, τ, θ, and ω parameters are constants, which were estimated before.

Furthermore, Equations (4.22), (4.23), and (4.24) are conditional joint probability distribution functions. Similar to Equation 4.21, the probability of reneging behavior to occur in a range of R_b number of time intervals at a specific range of PWT and st is the following (Equation 4.25), with the same process for Equations (4.23) and (4.24):

$$\int\limits_{PWT}\int\limits_{st}\int\limits_{R_b} f_{PWT,st,R_b}(PWT, st, R_b)\, dR_b\, dst\, dPWT$$

$$= \int\limits_{PWT}\int\limits_{st} f_{PWT,st|R_b}(PWT, st)\, dst\, dPWT \int\limits_{R_b} f(R_b)\, dR_b \quad (4.25)$$

4.14 Queuing Theory in Case of Reneging Behavior

The literature did not address any attempt to incorporate the reneging behavior into the main equations of the queuing theory. In fact, one of the criticisms of the queuing theory is that it does not represent reality exactly, where the impatience factor is often ignored when studying the performance of queuing (8).

Referring to Appendix B, Table B- I, it is clear that all equations depend on the queue intensity, as it represents the extent to which the capacity of the service is utilized. For this reason, it is expected that the best incorporation for the reneging behavior into the queuing theory is to be based on the queue intensity, where all calculations will depend on it to measure the queuing performance taking into consideration the reneging behavior.

If impatient customers are normally to renege on the queue, then they should not be included when calculating the queue intensity. In fact, the calculation of the ordinary queue intensity assumes that the whole arrival customers will be served, ignoring the fact that some of them will leave the queue and abandon the service for various reasons. In such a case, the result is the queue intensity before reneging occurs. Equation (4.26) suggests the queue intensity after the occurrence of reneging.

$$\rho_R = (\lambda - R_R)/\mu \quad (4.26)$$

Where:

ρ_R : Queue intensity after the occurrence of reneging

λ : Average arrival rate (customers/time interval)

μ : Average service rate (customers/time interval)

R_R : Average reneging rate (customers/time interval)

The performance measures of the queue in Rio Café's drive-thru service were recalculated again, as shown in Appendix E, Table E-I, to determine the queue's characteristics after the occurrence of reneging behavior, using the queuing theory's main equations shown in Appendix B, Table B-XIV, and based on Equation (4.26) for the ρ_R. The obtained results show records much closer to what was observed in the field than what is shown in Table 4.2, especially for the average number waiting to be served when queue is present and the average waiting time. This is due to the reduction in the queue's intensity, where the queue's intensity based on all time intervals is reduced from 0.74 to 0.69, as well the intensity of the queue in the morning peak hour is reduced from 0.97 to 0.9, based on the general study period.

The reduction in the queue's intensity makes it possible to measure the performance of the queue in time intervals at which the reneging behavior was detected, after the reneging customers left the queue, with more realistic results. While before the occurrence of reneging, it is not possible to determine any of the queue's characteristics using the queuing theory's main equations, as long as the average queue's intensity is greater than 1.

To generalize Equation (4.26), two main options are proposed to determine the average reneging rate on the service's queue, assuming the average arrival and service rates are already known. The first one is generic, by finding the expected value of reneging rate per time interval ($E(R_R)$), based on Equation (4.5). Therefore, ρ_R is sugguseted to be calculated as Equation (4.27) shows.

$$\rho_R = \left(\lambda - \sum_{All\ R_R} R_R e^{-1.4078 R_R\ -\ 0.2326} \right) / \mu \tag{4.27}$$

The second option is related to the arrival time, by assuming or estimating the total number of reneging customers in a representative workday, and multiply it by the relative frequency of reneging customers (RC_{RF}) modeled in Equation (4.16) for the chosen degree of sensitivity of arrival time, assuming that RC_{RF} has the same distribution for any chosen representative workday. In this case, the queue intensity after the occurrence of reneging at a chosen degree of sensitivity of arrival time (ρ_{R_S}) is suggested as Equation (4.28) shows, where RC_{Time} is the total number of reneging customers in a chosen representative unit of time matches with the unit of the arrival rate and the degree of sensitivity of the arrival rate.

$$\rho_{R_S} = (\lambda - (-2.056S + 1.37e^S - 1.173)RC_{Time})/\mu \quad (4.28)$$

4.15 Managing Reneging Behavior

Irritation with factors causing delay is a prominent feature for queuing individuals during their waiting time for being served (8), which makes reneging a normal behavior to occur from queuers, especially by impatient customers, as an expression of their dis-satisfaction with delay or the queue's characteristics in general.

Referring to Appendix E, Table E-I, reneging behavior is sometimes not a problem but a solution, but to the extent in which it is tolerable for the service's provider against predicted profits. Dbeis and Al-Sahili (18) concluded that reneging was part of the solution to the queuing problem they investigated, when they assumed a reneging rate in analyzing the queues on different ATMs during the COVID-19 lockdown; a period in which maintaining safety instructions were more important than gaining profits.

On the other hand, reneging behavior has to be reduced to increase profits, by improving the performance of the queues, but at the expense of the cost. The previous sections mathematically discussed the PDFs of factors affecting reneging, where they can be used to understand how to reduce the probability of reneging to a tolerable extent. Since the arrival rate and the reneging customers' rank cannot be controlled by the service provider, reneging can be reduced by maintaining the queue's intensity less than 1 as far as possible, by reducing the service time. This will increase the service rate and RWT/st ratio, if the RWT stays constant. In fact, the higher the departure rate, the less the time impatient customers need to decide if they want to stay or leave the queue (less RWT). This does not mean that reneging customers will leave the queue faster in case of increasing the service rate, but it means the less time they need to assess the queue's performance, which leads to less PWT as a result.

Referring to Figure 4.7, most reneging events occurred at low RWT/st ratios, but at high levels of the queue's intensity (Figure 4.6 and Appendix C, Figure C-IX), and at high degrees of the sensitivity of arrival time (Appendix D, Figure D-II). Therefore, decreasing the queue's intensity level by increasing the service rate, will considerably reduce the probability of reneging. As a result of decreasing service time, estimated service rate will be increased too (PWT/st), but with a reduction in reneging customers' PWT due to decreasing their RWT (Equation (4.18)).

For example, if the average service time is decreased by only 15 seconds, then the average queue's intensity will be decreased to 0.65 instead of 0.74. A reduction of 9% in the average queue's intensity will reduce the probability of reneging for a general queue's intensity ≤ 0.65 by 21% (Equation (4.3)), when reneging occurs. If the average service time of a randomly selected time interval is the same as the average service time of all time intervals ($st = \overline{ST}$), then the reduced probability of reneging (0.3) occurs at PWT/st ratio ≤ 1.7 (Equation (4.18)). This is associated with a reduced reneging customers' PWT of approximately 3 minutes (with an average service time of 1.76 minutes), instead of 3.9 minutes when the probability of reneging for an average general queue's intensity ≤ 0.74 is 0.38 with the current average service time of 2.01 minutes. Assuming the reneging customers' rank is 2, then RWT is 1.84 minutes corresponding to a PWT of 3.9 minutes, while RWT is reduced to 1.19 minutes for a PWT of 3 minutes (Equation (4.17)).

Other factors, such as the sensitivity of arrival time, cannot be controlled by the service provider. In this case, retention strategies could be employed, as Kumar and Sharma (31) defined, to convince impatient customers to stay in the line. One of proposed strategies that can be used with or without reducing the service time is to offer a discount on the original provided goods' prices for customers willing to wait for extra times in peak periods. However, such a strategy has a negative effect that it may prompt customers to arrive to the queue at peak periods specially to have the offered discount, thus the arrival rate will increase causing a higher queue intensity rate and a longer queuing line as a result, which may increase balking in return.

In general, a drive-thru service provider should weigh the costs and profits of improving the performance of the queue. For instance, Rio Café's administration can reduce the number of reneging customers by adding a second window (double-channel queuing system) or by considerably reducing the service time. Both options might require hiring additional employees. Assuming that hiring one additional employee with a minimum wage salary of $510 per month ($17 a day), and an assumed average profit of $2 per reneging customer, then the cost/profit ratio is 0.88 for average reneging customers of 9.6 a day, neglecting the operational costs and the capital of adding a new window. Therefore, the reneging behavior is tolerable in Rio Café's drive-thru service, while it may be not for other services with different types and queue's characteristics.

However, regardless of its cost, characteristics, and the type of products it provides, automated service could be adopted as an extra window to reduce the utilization factor (U_f) presented in Equation (4.29), as May described (7) for multi-channel queuing systems (M/M/N).

$$U_f = \frac{\rho}{N} \tag{4.30}$$

Where:

ρ : Queue intensity

N : Number of channels

4.16 Spatial-Dynamic Application

As shown in previous sections, it is hard to manually employ probability distribution functions to determine the probability of reneging at different ranges of different factors. In fact, a service provider would need all those functions together at the same time to diagnose the reneging behavior on the drive-thru queue under management. To facilitate managing queues, taking into consideration the reneging behavior, it is suggested to use a spatial-dynamic mobile application to be developed for collecting, analyzing, and showing related data, based on the queuing theory's main equations and probability distribution functions and models related to the behavior of reneging, for both users: customers and service providers. The proposed application should not need more than the previously discussed PDFs and equations in its analysis process.

The role of the spatial aspect is to manage the demand on the queue, based on the location and the speed of the demanding customer traveling to the queue. As mentioned before, reneging behavior could be a part of the solution to the queuing problem. Therefore, taking into the account the reneging customers, or those who have the intention to renege, will help the service provider to manage the queue with more realistic queue's characteristics.

On the other hand, the application helps customers to make their decision, whether to join or leave the queue, by providing them with the queue's characteristics they are interested in; such as, its length, average waiting time, and their probable rank in the queue. If they are not satisfied with these characteristics, they might decide not to join the drive-thru line without incurring the travel to the queue. This will save the customers' precious time, especially at peak hours, from being wasted in waiting without being served. However, if they are satisfied with the queue's characteristics the application provided, reneging in error will be decreased, as it may happen if the impatient customer mistakenly reneges based on incorrect estimates (26).

In addition, the dynamic aspect of the application will allow the user to interact with real-time data. As a result, the application should provide probable profits or losses, based on the probability of reneging on the queue, whether due to its current characteristics, or due to the characteristics within the scope of the service provider's interest. For such a case, the dynamism of the application is needed, where outputs could be used as inputs to determine other characteristics related to any change and whether the related number of reneging is tolerable or not.

Figure E-II in Appendix E illustrates a summarized logical workflow to develop the basis of a spatial-dynamic application for drive-thru service's providers. This application should work in parallel with another application to be developed for customers. The workflow is built on collecting data on customers from two sources: calls from customers who are willing to join the queue, and queued customers. The application could be developed to allow customers in the queue to rate the service while they are waiting, which collects RWT from queued customers. However, the application should count the number of customers in the line, whether manually added by the service provider or automatically based on previously collected data and rates.

With every served customer, the service time variable will have a new value. The application should calculate the average service time for each queue based on the previously defined time interval, and calculate the general average service time. These two averages will be used in calculating the several probabilities of reneging after calculating the general or perceived queue intensity. Another role for the service time value is to define the impedance length or time the joining customer will travel for it to join the queue. This will be used in building a changing coverage area, based on the service time, the queue intensity, and the average traveling speed of customers.

Spatial dataset should be provided to the application to build the coverage area, based on the roads' network and their directions and maximum allowed speeds. It should be mentioned that the impedance length must not be used as a radius for circular coverage area. It is rather the length of each accessible road, considering its direction as well, from the center point of the drive-thru service, in an irregular polygon-shaped coverage area, as Dbeis and Al-Sahili (18) used ArcMap software to build the coverage area for related ATMs. Equation (4.30) shows the maximum impedance length needed to build a coverage area as the following:

$$M_{Max} = \frac{\bar{V}}{\mu(1-\rho)} \tag{4.30}$$

Where:

M_{Max}	:	Maximum impedance length (kilometers)
$\bar{V}$	:	Average traveling speed (kilometers per minute)
μ	:	Average service rate (customers per minute)
ρ	:	Average queue intensity

For the case study, using the average service rate and queue intensity calculated through the study period, with an assumed average traveling speed of 30 km/hr, the drive-thru service in Rio Café has a coverage area based on a maximum impedance length of 3.275 km.

In addition, the workflow shows that the application should use the location of the call from customers who selected to join the queue. The moment the customer enters the coverage area, the arrival rate corrects itself to take into consideration new arrivals or calls. Otherwise, the arrival rate will not add new calls unless the customer enters the coverage area. Furthermore, the application should consider the sensitivity of the arrival time, to use it in calculating the RC_{RF}, after calculating the R_R, as shown in Equations (4.16) and (4.5), respectively.

However, if the customer enters the coverage area without going to the drive-thru service, the application should track the customer's path for a certain-predefined time. If the customer goes in a wrong direction or stops for more than a predefined time, the call should be ignored. A coefficient of accuracy should be considered for customers who join the queue without using the application.

The application should use all these processes to calculate the probability of reneging and the queue's characteristics, with the reneging factor, and show results for the service provider. To attain the optimum of the application's performance, it is suggested to be run into two phases: beta and alpha. First phase, is to run a beta version of the application to collect base data and calculate constants for built-in mathematical equations, to use it in the second phase. The second phase; alpha, is to analyze collected data, collect new data, and correct outdated data. Therefore, it is recommended to build the application using machine learning techniques, where models can be trained to detect and analyze new patterns.

In general, developing such a spatial-dynamic application will assist service providers to manage their queues, especially a service provider with multiple drive-thru sites, who will better manage the system from a macroscopic perspective. In addition, customers can know the queue length and the expected waiting time before joining the queue, based on their locations and speeds. Therefore, they will know the queue's characteristics they are willing to join without the effort of travelling to experience the service by themselves. As a result, the number of trips will be reduced, and; therefore, the traffic congestion will be decreased, especially in peak hours.

Chapter Five
Conclusions and Recommendations

5.1 Summary

Reneging behavior of impatient customers joining the queue is rarely discussed mathematically in the literature. Besides that, the impatience factor is ignored in the queuing theory's main equations, which criticizes the theory as it does not exactly represent real-life queuing problems. This thesis fills in the gap, as an applied-mathematical investigation to reach a deeper understanding of this behavior by determining the probability distribution function it follows, and identifying the factors that affect its occurrence with their mathematical distributions, to incorporate the reneging behavior into the main equations of the queuing theory.

Through non-participant observations, the required data were manually collected using surveillance camera recordings from Rio Café's drive-thru service, located at Al-Huda Gas Station in Al-Bireh City. Approximately 123 workhours were analyzed; divided into 493-time intervals of 15 minutes for each, for a daily study duration of 7.25 hours for 17-fulltime workdays, focusing on the active periods of the selected drive-thru service. Reneging behavior was detected at 113-time intervals by 164 reneging customers out of 2713 customers who demanded the service.

Through this study, the probability distribution functions of the arrival rate, service time, and the time between successive reneging events were determined, as well as the probability distribution function that each of the factors affecting reneging behavior follows. In addition, linear and non-linear regression models were developed to predict the relative frequency of the reneging rate, and the relative frequency of reneging customers according to the sensitivity degree of the time they arrive at. The research goes beyond that by modifying the queuing theory's main equations using the developed regression models.

5.2 Conclusions

This thesis found that the exponential distribution is the probability distribution function of the time intervals between successive reneging events of at least one reneging customer per time interval. While contrary to expectations, the reneging rate does not follow the

Poisson distribution. These findings are for a First-In-First-Out (FIFO) service, with a single-channel queue that has an infinite length, in which both arrival and service rates are randomly distributed [M/M/1 (∞, FIFO)]. For the selected drive-thru service, it was found that the arrival rate and the service time follow Poisson and Log-Normal distributions, respectively.

In addition, it was concluded that the long waiting time was not the only factor causing impatient customers to renege on the queue, but other important factors played a pivotal role in affecting this behavior. The main affecting factors were the queue intensity; response waiting time over service time ratio (RWT/st); and reneging customer's rank, where each of them follows a specific probability distribution function, as follows:

- General queue intensity follows a Log-Normal distribution, while perceived queue intensity follows a Maxwell-Boltzmann distribution;
- RWT/st ratio follows a Weibull distribution;
- Reneging customer's rank follows a Sine distribution.

It was concluded that the probability of reneging is very sensitive to any change in service time. A 15-second reduction in the average service time led to decreasing the average intensity of the queue by 9%, which caused a reduction in the probability of reneging by 21%, when reneging behavior is present.

Beside these factors, it was concluded that reneging behavior is mainly affected by how far the customer's arrival time to the intended queue is close to the peak hour (degree of sensitivity).

Certainly, all these factors are related and equivalent to the time an impatient customer is waiting in the queue, but not in a direct way; contrary to what was concluded by Haight (26). However, it was found that these factors are integrated and jointly distributed, leading to the customer's decision to leave the queue without getting the service.

Finally, the proposed application is a methodology, as well as a theoretical application, which can be more developed using sensors (regardless of their types and costs). The concept of developed models and determined probability distribution functions can be applied for a variety of services, not just for drive-thrus, for other locations, and for both person and vehicular travel and queuing.

5.3 Recommendations

Based on the results of this thesis, it is recommended to modify the main equations of the queuing theory, especially the equation of the queue's average intensity level, by including the average reneging rate, as shown in Equations (4.26), (4.27), and (4.28). Through that, the queuing theory's calculations become more realistic, by taking into consideration the impatience factor of reneging behavior; an event that normally happens during waiting times in queuing systems.

Rio Café's administration is recommended to increase the capacity of the case study's queue, when queue is present, by at least two extra spaces, to accommodate five demanding customers at the same time, including customers intending to renege (four waiting customers, and one customer in the service).

In general, drive-thru service providers are encouraged to employ retention strategies to keep impatient customers in the line; such as offering a discount for customers willing to wait for an extra time, especially at peak hours. However, the service provider should weigh the costs and profits of any improvement or strategy in order to reduce the number of reneging customers.

Furthermore, it is recommended that future researches develop the required algorithms for the proposed spatial-dynamic mobile application, based on the determined probability distributions and developed models in this research, to facilitate managing queuing systems for drive-thru services, by optimizing temporal and spatial demands, and other characteristics affecting the queuing performance. This should assist the service providers and queuing customers in taking their actions and decisions, to meet their intersected interest: more served customers in a less waiting time.

In the near future, it is expected to have an expansion for full infrastructure drive-thru services in Palestine, especially that it is actively introduced for many types of services in the Middle East and regional countries. Therefore, decision makers and traffic agencies are encouraged to use the outcomes of this thesis to manage drive-thru queues and other queuing services susceptible to reneging, from a macroscopic perspective.

5.4 Limitations

One limitation of this research that it relies on data collected from one location, for one type of service, due to the low prevalence of fully-active integrated drive-thru services in Palestine and the large amount of data to be analyzed. Therefore, it is recommended in the future to test and verify the results obtained from this thesis, for other drive-thru locations and different types of services.

Furthermore, the study relied on a single channel queue. Therefore, the behavior and conclusions for multi-channel queuing services might vary.

List of Abbreviations

Abbreviation	Meaning
AASHTO	American Association of State Highway and Transportation Officials
ADT	Average Daily Traffic
AM	Ante Meridiem
ANOVA	Analysis of Variance
ATM	Automated Teller Machine
CLT	Central Limit Theorem
COVID	Corona Virus Disease
CPM	Customer per Minute
DF	Degree(s) of Freedom
DT	Drive-Thru
ESL	Effective Satisfaction Level
FIFO	First-In-First Out
HV	Hourly Volume
ICU	Intensive Care Unit
IQR	Interquartile range
ITE	Institute of Transportation Engineers
JPD	Joint Probability Distribution
LB	Lower Bound
MS	Mean Square
NL	Number of Lanes
OECD	Organization for Economic Co-operation and Development
PDF	Probability Distribution Function
P-K	Pollaczek–Khinchine
PM	Post Meridiem
PMF	Probability Mass Function
PWT	Perceived Waiting Time
RC	Reneging Customer(s)
RF	Relative Frequency
R_R	Reneging Rate
RWT	Response Waiting Time
SD	Standard Deviation
S	Sensitivity of Arrival Time
SS	Sum of Squares
ST	Service Time
TSC	Total Satisfaction Curve
UAE	United Arab Emirates
UB	Upper Bound
UK	United Kingdom
USA	United States of America

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Appendices

Appendix A

Case Study's Site

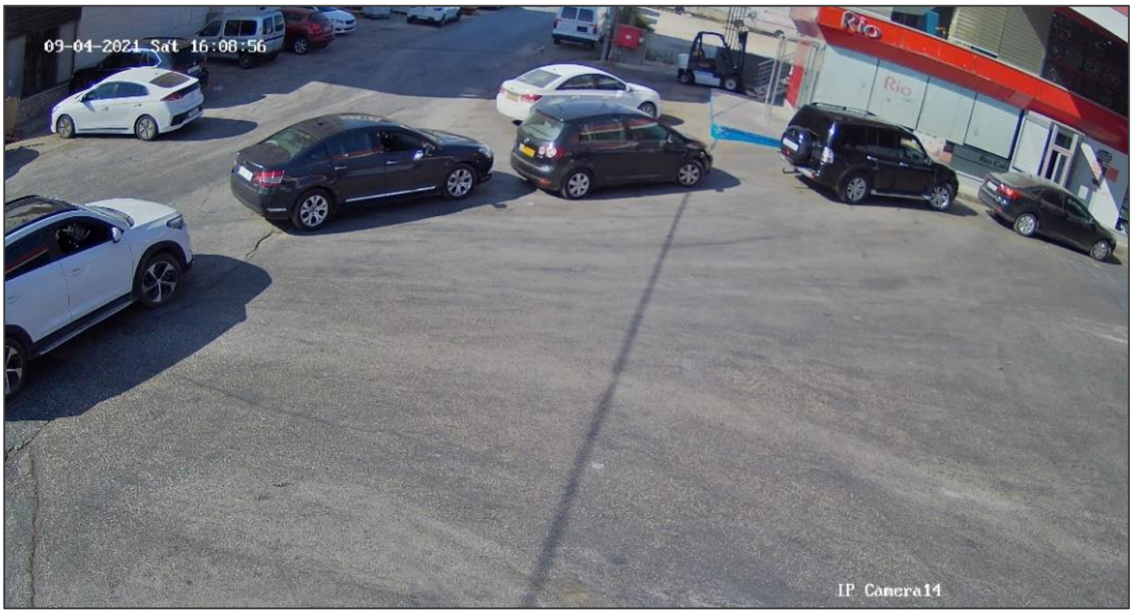
Figure A-I

Captured Image from Surveillance Camera Recordings for the Site

Source: Rio Café's Surveillance Camera Recordings

Figure A-II

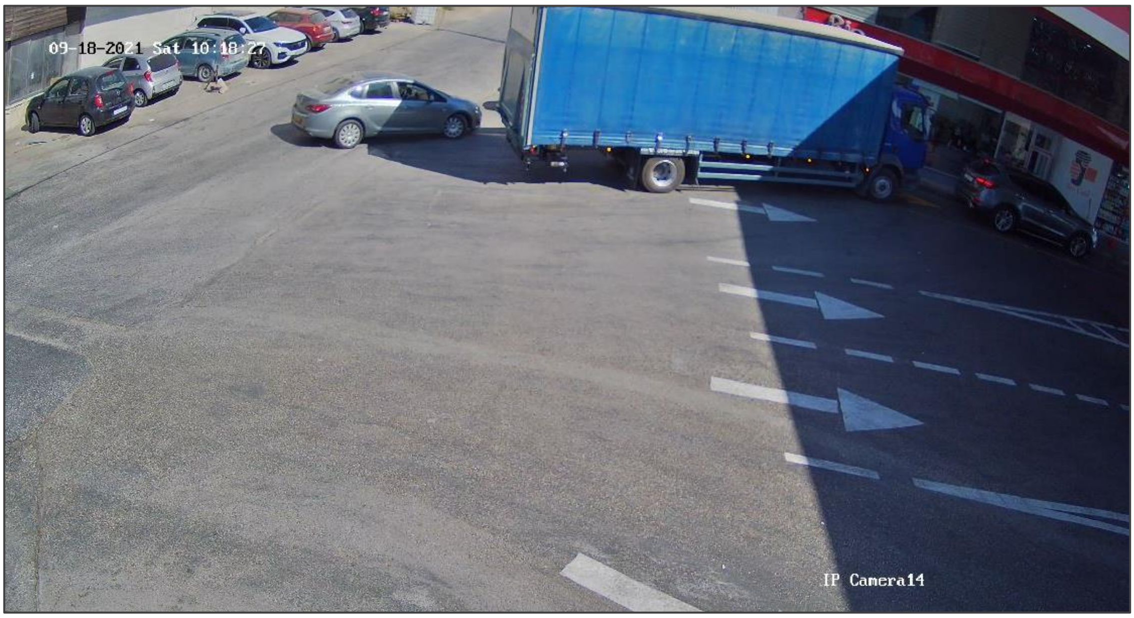
Captured Image from Surveillance Camera Recordings Showing Queued Vehicles



Source: Rio Café's Surveillance Camera Recordings

Figure A-III

Captured Image from Surveillance Camera Recordings Showing a Queued Single-Unit Truck



Source: Rio Café's Surveillance Camera Recordings

Appendix B

Analysis of Queuing Performance Measures

Table B-I

Chi-Square Goodness-of-Fit Test for Poisson Distribution of Arrival Rate

Arrival Rate (customers/time interval)	Observed Frequency (O)	$P(X \leq x)$	$P(X = x)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
0	2	0.004	0.004	2	0.000
2	43	0.088	0.084	41	0.098
4	131	0.358	0.270	133	0.030
6	162	0.686	0.328	162	0.000
8	99	0.894	0.208	103	0.155
10	36	0.975	0.081	40	0.400
12	17	0.996	0.021	10	4.900
$x \geq 14$	3	1.000	0.004	2	0.500
$\sum$	493	-	1.000	493	6.083
Right-Tailed Probability of $\chi_{0,6}^2$					0.414

Figure B-II

Poisson Distribution of Arrival Rate

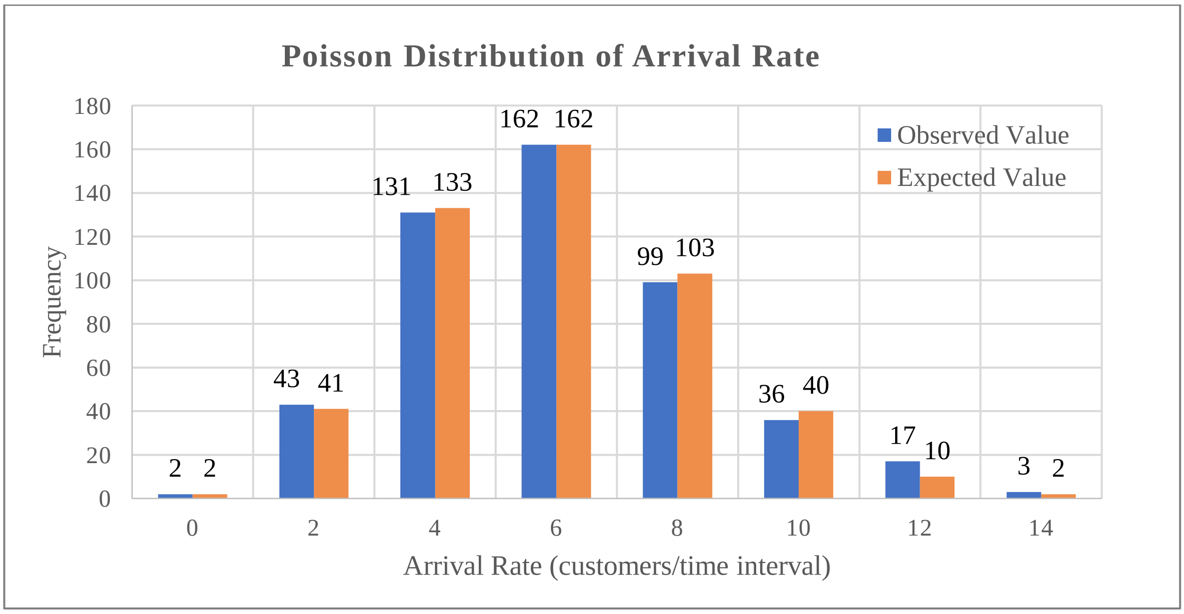


Table B-III

Chi-Square Goodness-of-Fit Test for Exponential Distribution of Service Time

Service Time in Minutes (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
0.5	2	0.002	0.002	1	1.000
1	40	0.096	0.094	46	0.783
1.5	114	0.336	0.240	118	0.136
2	121	0.580	0.244	120	0.008
2.5	102	0.754	0.174	85	3.400
3	50	0.861	0.107	52	0.077
3.5	28	0.922	0.061	30	0.133
4	18	0.956	0.034	17	0.059
4.5	7	0.975	0.019	9	0.444
5	5	0.986	0.011	5	0.000
5.5	1	0.992	0.006	3	1.333
6	1	0.995	0.003	2	0.500
6.5	0	0.997	0.002	1	1.000
x > 6.5	1	1.000	0.003	1	0.000
$\sum$	490	-	1.000	490	8.873
Right-Tailed Probability of $\chi^2_{0,11}$					0.634

Figure B-IV

Log-Normal Distribution of Service Time

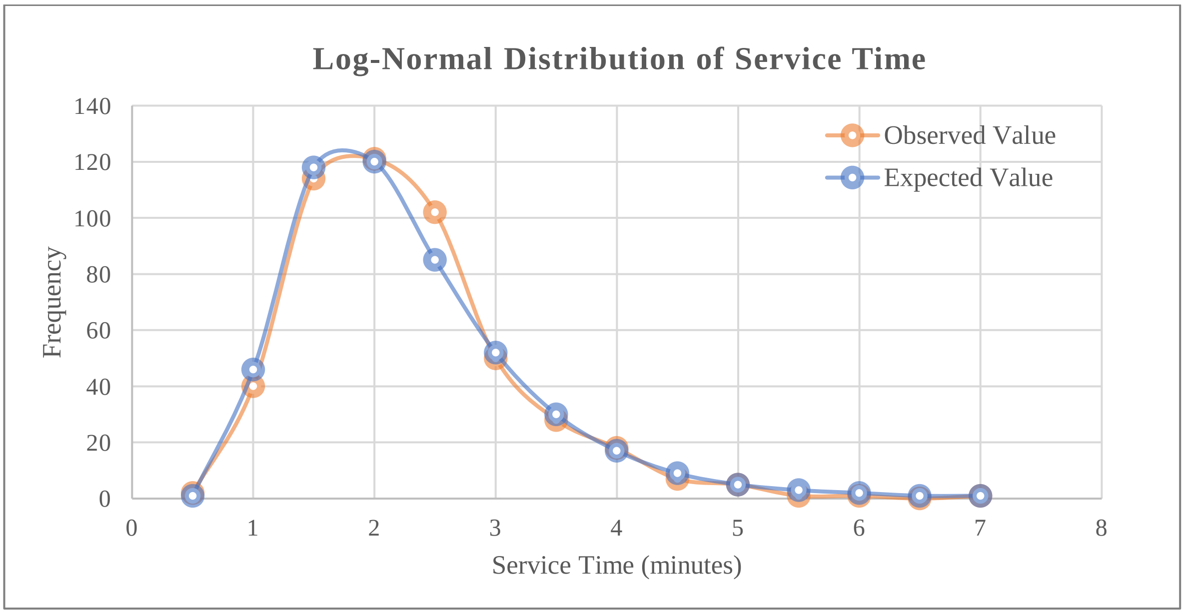


Table B-V

Computations of IQR Rule for Ln (st)

Variable	Value
Exclusive First Quartile (Q1)	0.324
Exclusive Third Quartile (Q3)	0.898
Interquartile Range (IQR)	0.573
Upper Bound (UB)	1.758
Lower Bound (LB)	-0.535

Table B-VI

Average Arrival Rate according to the Daily-Study Time Intervals and Workdays

Time Interval	Average Arrival Rate (customers/time interval)						
	Day						Average
	Sat	Sun	Mon	Tue	Wed	Thu	
7:00-7:15	2.7	3.7	4.3	2.5	5.0	3.7	3.6
7:15-7:30	4.0	6.3	6.0	5.0	5.3	8.0	5.8
7:30-7:45	2.3	6.7	9.0	9.5	8.0	9.0	7.4
7:45-8:00	3.7	9.7	6.7	7.5	8.3	8.0	7.3
8:00-8:15	6.0	9.3	9.7	7.5	10.0	10.7	8.9
8:15-8:30	5.3	5.7	7.3	10.5	7.7	10.0	7.8
8:30-8:45	3.7	7.0	5.7	5.0	6.7	5.7	5.6
8:45-9:00	6.0	6.0	7.3	9.0	5.3	6.3	6.7
9:00-9:15	2.3	5.0	4.7	7.0	5.7	6.0	5.1
9:15-9:30	5.7	5.0	7.3	4.0	8.3	7.0	6.2
9:30-9:45	4.3	6.0	7.0	4.0	6.7	4.3	5.4
9:45-10:00	5.0	5.7	7.3	6.0	4.0	5.0	5.5
10:00-10:15	6.3	6.0	4.3	4.0	5.0	5.0	5.1
14:00-14:15	4.3	6.0	4.3	5.5	7.0	5.0	5.4
14:15-14:30	6.3	6.0	6.0	5.0	6.0	5.3	5.8
14:30-14:45	3.7	3.0	7.3	5.0	4.0	5.3	4.7
14:45-15:00	6.0	4.0	6.7	3.0	4.0	4.7	4.7
15:00-15:15	5.7	6.0	5.3	6.0	6.0	4.7	5.6
15:15-15:30	4.3	5.3	6.0	4.5	4.7	7.3	5.4
15:30-15:45	8.0	5.0	5.0	6.0	2.3	4.3	5.1
15:45-16:00	5.7	4.0	4.3	4.0	4.0	4.0	4.3
16:00-16:15	6.3	3.3	2.7	3.0	7.3	4.7	4.6

16:15-16:30	5.0	4.7	4.3	2.5	4.3	5.3	4.4
16:30-16:45	5.0	5.7	4.7	5.0	5.0	5.0	5.1
16:45-17:00	5.0	2.7	6.3	3.0	2.7	5.3	4.2
17:00-17:15	4.3	3.3	3.3	5.5	4.0	6.7	4.5
17:15-17:30	5.7	4.0	5.3	2.0	3.7	8.3	4.8
17:30-17:45	5.3	7.0	6.0	2.0	5.3	5.7	5.2
17:45-18:00	4.3	4.7	5.0	5.5	4.0	6.0	4.9
Average	4.9	5.4	5.8	5.1	5.5	6.1	5.5

Table B-VII

Average Service Time according to the Daily-Study Time Intervals and Workdays

Time Interval	Average Service Time (minutes)						
	Day						Average
	Sat	Sun	Mon	Tue	Wed	Thu	
7:00-7:15	1.01	2.38	1.97	2.25	2.36	2.88	2.14
7:15-7:30	3.08	1.88	2.07	1.88	1.54	1.71	2.03
7:30-7:45	1.73	1.97	1.93	2.00	1.85	1.72	1.87
7:45-8:00	2.71	2.08	1.41	1.45	2.49	1.74	1.98
8:00-8:15	1.82	1.65	2.47	1.58	1.48	1.83	1.80
8:15-8:30	2.46	1.55	1.66	1.54	1.67	2.03	1.82
8:30-8:45	3.31	1.82	1.51	3.31	2.13	1.51	2.26
8:45-9:00	3.01	1.89	1.48	2.41	2.38	2.41	2.26
9:00-9:15	1.59	2.17	1.77	1.65	1.26	2.34	1.80
9:15-9:30	2.49	2.11	2.52	1.53	1.93	1.78	2.06
9:30-9:45	2.38	2.56	1.99	2.55	1.91	1.99	2.23
9:45-10:00	1.61	3.20	1.77	1.90	2.17	1.71	2.06
10:00-10:15	2.36	1.53	1.84	2.43	1.88	1.99	2.01
14:00-14:15	3.40	2.59	1.92	1.71	1.17	1.72	2.08
14:15-14:30	1.47	2.43	1.63	2.19	1.39	1.81	1.82
14:30-14:45	2.81	1.92	2.23	2.00	2.16	1.78	2.15
14:45-15:00	2.88	1.40	1.73	1.47	1.12	1.45	1.68
15:00-15:15	1.44	1.66	1.94	2.91	2.55	2.31	2.13
15:15-15:30	1.38	1.38	1.68	1.90	2.33	1.74	1.74
15:30-15:45	2.42	2.24	1.80	1.71	1.80	1.46	1.90
15:45-16:00	2.42	1.91	2.56	1.91	2.36	3.34	2.41
16:00-16:15	1.25	1.96	2.31	1.46	2.34	1.69	1.84

16:15-16:30	1.31	2.47	1.99	1.68	2.18	1.39	1.84
16:30-16:45	2.84	2.80	2.06	2.75	4.08	2.12	2.78
16:45-17:00	2.12	1.79	1.52	2.43	2.28	2.89	2.17
17:00-17:15	1.96	1.37	1.69	1.84	3.78	1.90	2.09
17:15-17:30	1.76	1.62	1.78	2.67	2.48	1.56	1.98
17:30-17:45	2.04	1.37	1.60	1.91	1.93	2.49	1.89
17:45-18:00	0.91	2.49	1.50	2.57	1.61	1.36	1.74
Average	2.14	2.01	1.87	2.05	2.09	1.95	2.01

Table B-VIII

One-Way ANOVA Table for the Means of the Arrival Rate, according to the Study Workdays

Source of Variation	SS	DF	MS	F	P-value	F crit
Between Groups	27.32	5.00	5.46	1.80	0.12	2.27
Within Groups	510.96	168.00	3.04			
Total	538.28	173.00				

Table B-IX

One-Way ANOVA Table for the Means of the Arrival Rate, according to the Study Workhours

Source of Variation	SS	DF	MS	F	P-value	F crit
Between Groups	227.14	28.00	8.11	3.78	0.00	1.55
Within Groups	311.14	145.00	2.15			
Total	538.28	173.00				

Table B-X

One-Way ANOVA Table for the Means of the Service Time, according to the Study Workdays

Source of Variation	SS	DF	MS	F	P-value	F crit
Between Groups	1.33	5.00	0.27	0.93	0.46	2.27
Within Groups	47.82	168.00	0.28			
Total	49.15	173				

Table B-XI

One-Way ANOVA Table for the Means of the Service Time, according to the Study Workhours

Source of Variation	SS	DF	MS	F	P-value	F crit
Between Groups	9.30	28.00	0.33	1.21	0.23	1.55
Within Groups	39.85	145.00	0.27			
Total	49.15	173				

Table B-XII

Correlation between Averages of Arrival Rate and Service Time, according to the Study Workhours

	Average Arrival Rate (customers/time interval)	Average Service Time (minutes)
Average Arrival Rate (customers/time interval)	1	
Average Service Time (minutes)	-0.80	1

Table B-XIII

Correlation between Averages of Arrival Rate and Service Time, according to the Study Workdays

	Average Arrival Rate (customers/time interval)	Average Service Time (minutes)
Average Arrival Rate (customers/time interval)	1	
Average Service Time (minutes)	-0.21	1

Table B-XIV

Queuing Performance Equations for M/M/1 (∞, FIFO) System

Symbol	Definition	Equation
ρ	Traffic Intensity, where λ is the arrival rate and μ is the service rate	$\frac{\lambda}{\mu}$
ρ (0)	Probability of empty system	$1-\rho$
ρ (n)	Probability of exactly n units in system	$\rho^n(1-\rho)$
E (m)	Average number waiting to be served	$\frac{\rho^2}{1-\rho}$
E (m/m>0)	Average number waiting to be served when queue is present	$\frac{1}{1-\rho}$
E (n)	Average number in system (waiting and service)	$\frac{\rho}{1-\rho}$
E (v)	Average time in system	$\frac{1}{\mu(1-\rho)}$
E (w)	Average waiting time only	$\frac{\rho}{\mu(1-\rho)}$

Appendix C

Analysis of Reneging Behavior

Table C-I

Natural Logarithm of Reneging Rate's Relative Frequency

Reneging Rate (renegers/time interval)	Frequency	Relative Frequency (RF)	Ln (RF)
0	380	0.771	-0.260
1	72	0.146	-1.924
2	32	0.065	-2.735
3	8	0.016	-4.121
4	1	0.002	-6.201
∑	493	1.000	-

Figure C-II

Reneging Rate vs Natural Logarithm of its Relative Frequency

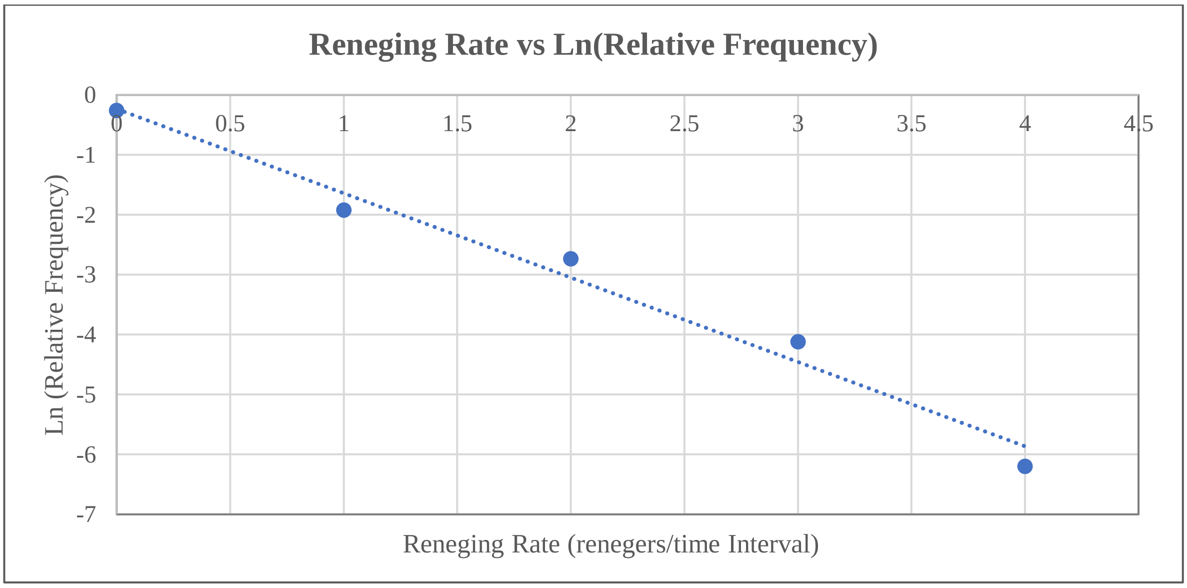


Table C-III

Regression Statistics for Reneging Rate's Equation

Statistic	Value
Multiple R	0.990
R Square	0.980
Adjusted R Square	0.973
Standard Error	0.367
Observations	5

Table C-IV

ANOVA for Reneging Rate's Equation

	DF	SS	MS	F	Significance F
Regression	1	19.818	19.818	146.835	0.001
Residual	3	0.405	0.135		
Total	4	20.223			

Table C-V

Regression Coefficients of Reneging Rate's Equation

	Coefficients	Standard Error	t Stat.	P-Value	Lower 95%	Upper 95%
Intercept	-0.233	0.285	-0.817	0.474	-1.138	0.673
Reneging Rate	-1.408	0.116	-12.118	0.001	-1.777	-1.038

Figure C-VI

Distribution of Reneging Rate's Relative Frequency

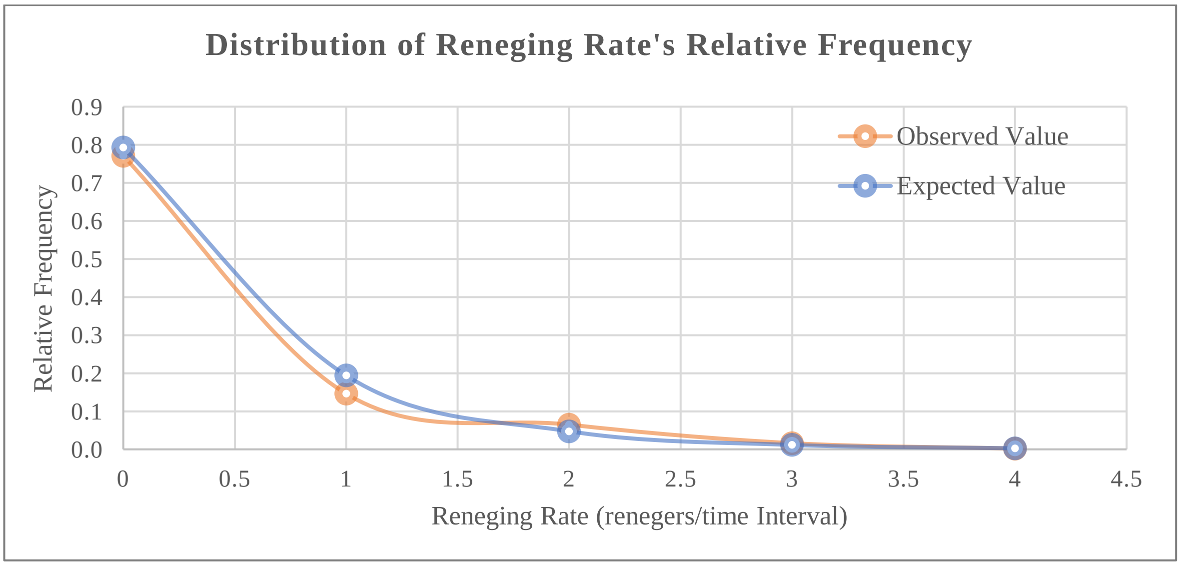


Table C-VII

Correlation between Factors Affecting Reneging Behavior

	Average R_R	Average RWT	Average Rank	Average L	Average λ	Average st	Average ρ
Average R_R	1.000						
Average RWT	-0.342	1.000					
Average Rank	0.268	-0.205	1.000				
Average L	0.188	-0.065	0.901	1.000			
Average λ	0.530	-0.311	0.453	0.452	1.000		
Average st	0.344	-0.280	-0.008	0.103	0.298	1.000	
Average ρ	0.542	-0.287	0.222	0.307	0.725	0.850	1.000

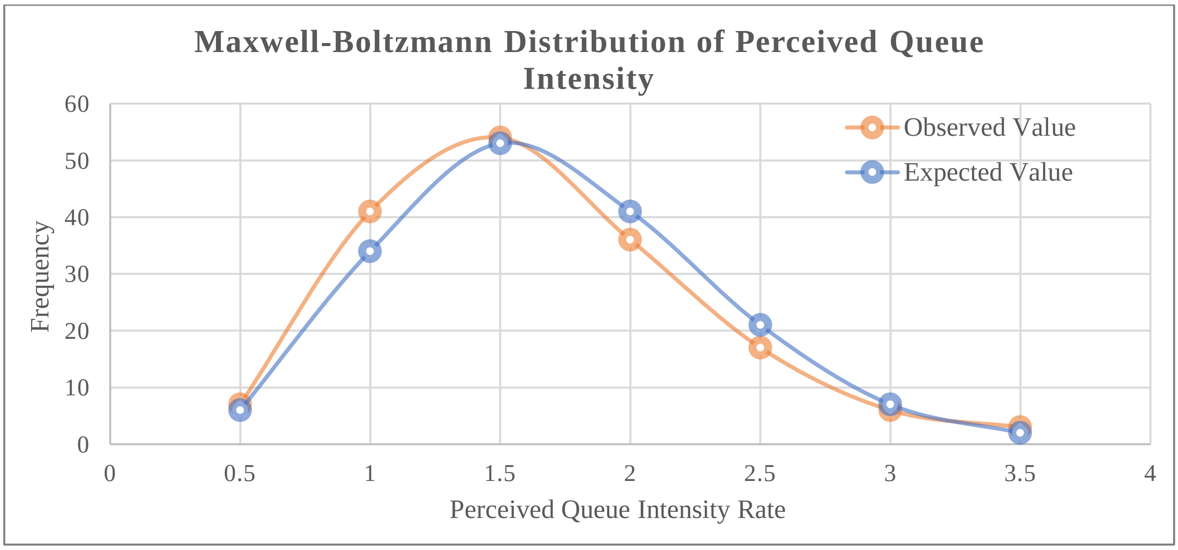
Table C-VIII

Chi-Square Goodness-of-Fit Test for Maxwell-Boltzmann Distribution of Perceived Queue Intensity

ρ_P (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
0.5	7	0.038	0.038	6	0.167
1	41	0.246	0.208	34	1.441
1.5	54	0.564	0.318	53	0.019
2	36	0.819	0.255	41	0.610
2.5	17	0.946	0.127	21	0.762
3	6	0.989	0.043	7	0.143
x > 3	3	1	0.011	2	0.500
∑	164	-	1.000	164	3.641
Right-Tailed Probability for $\chi_{0,5}^2$					0.602

Figure C-IX

Maxwell-Boltzmann Distribution of Perceived Queue Intensity, when there is a Reneging Event



Appendix D

Joint Probability Distributions of Reneging Behavior

Figure D-I

Degree of Sensitivity of Arrival-Time Interval

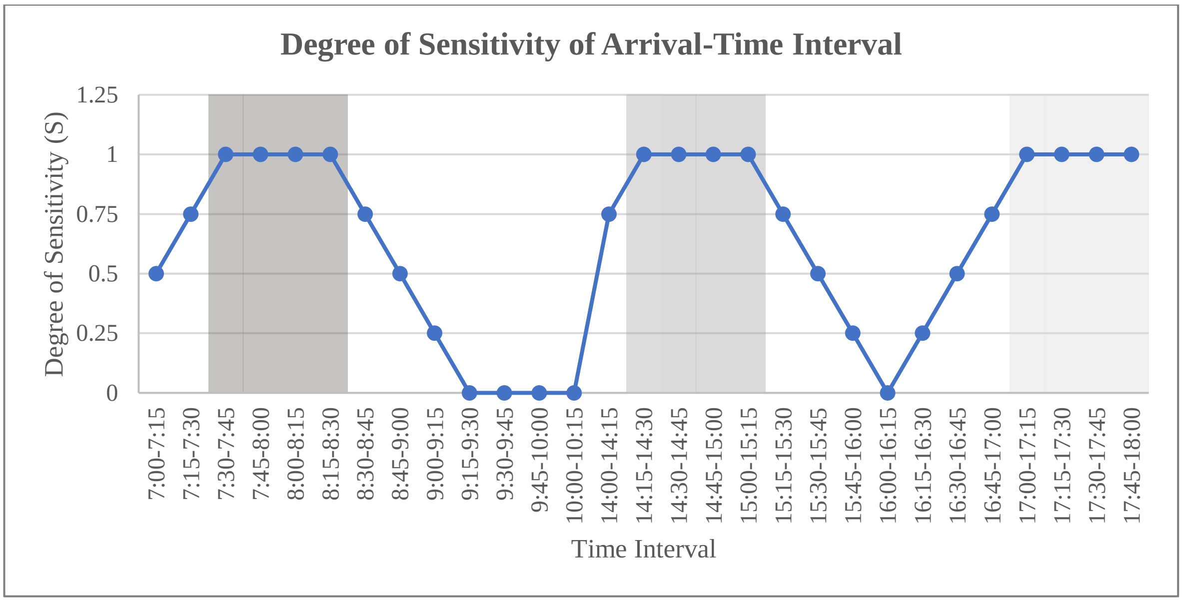


Figure D-II

Average RWT and Total Number of Renegers according to Degree of Sensitivity of Arrival Time

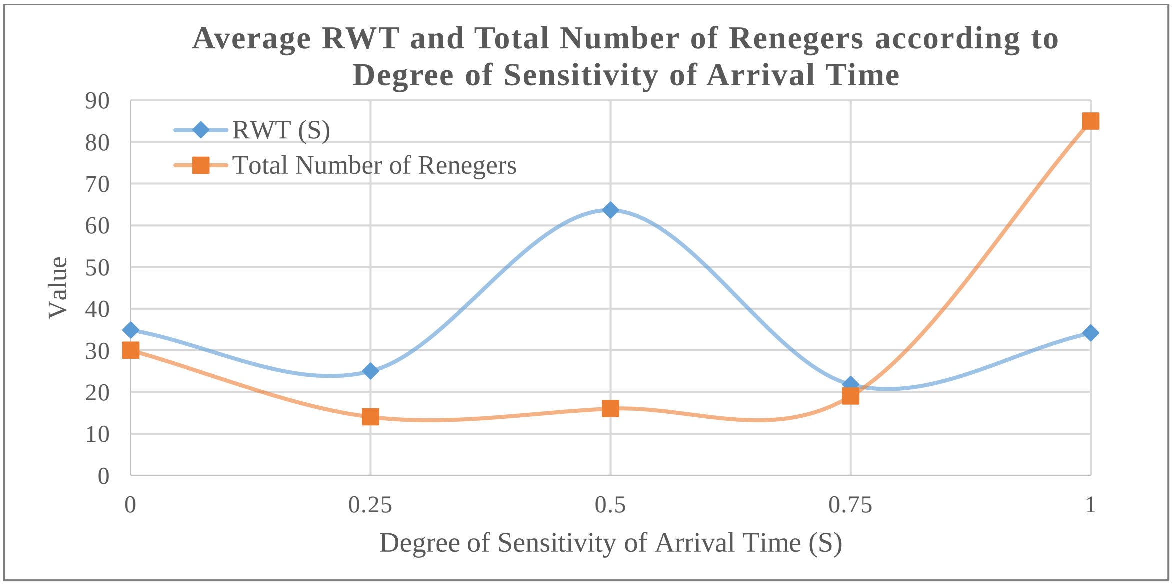


Figure D-III

Average Renegers' Rank according to Degree of Sensitivity of Arrival Time

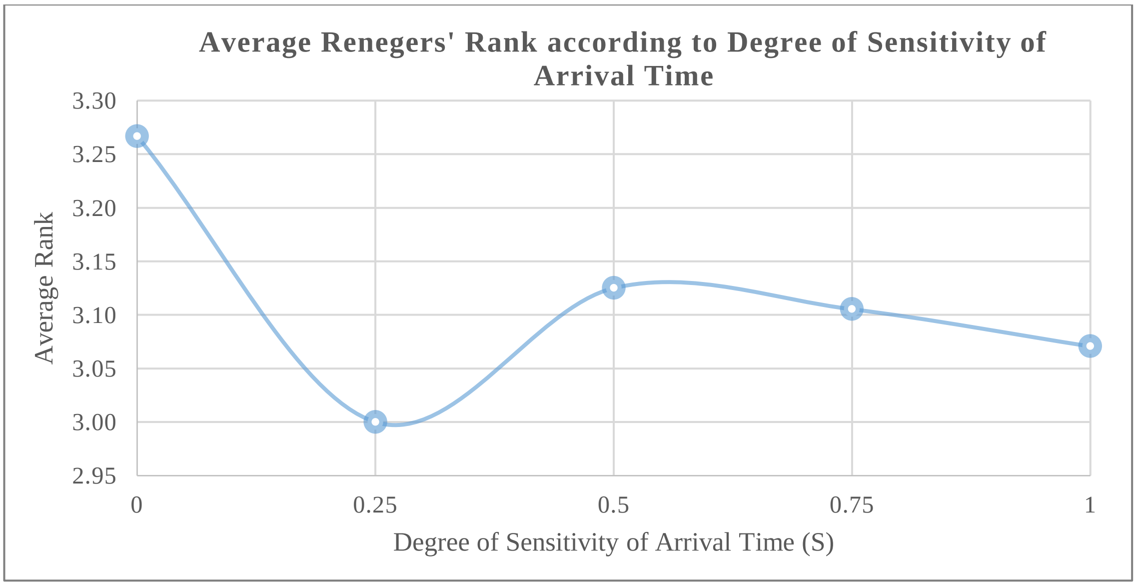


Figure D-IV

Average General Queue Intensity according to Degree of Sensitivity of Arrival Time

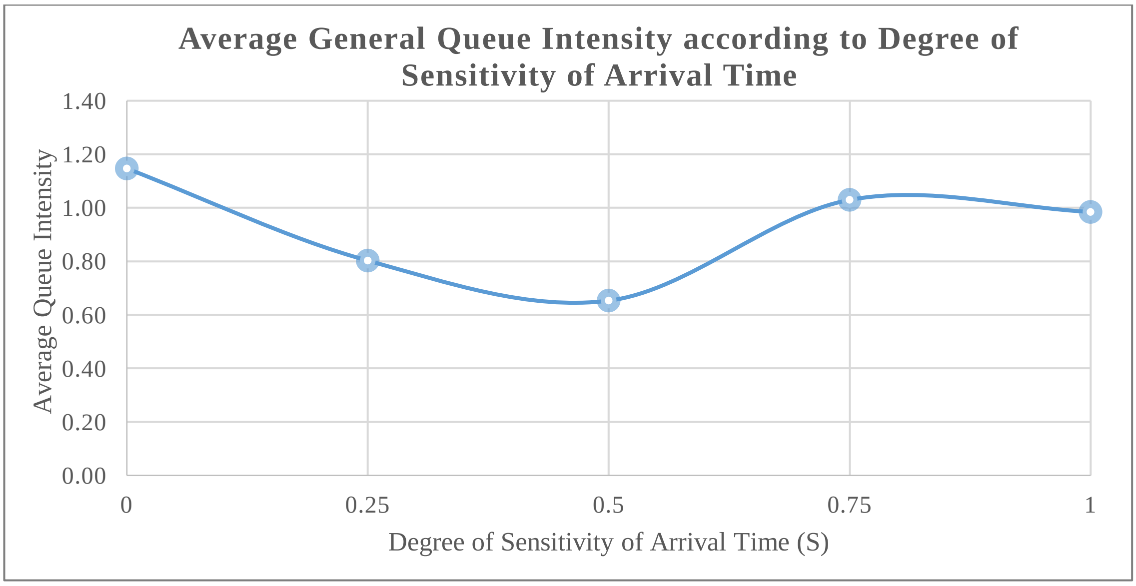


Figure D-V

Relative Frequency of Number of Reneging Customers according to Degree of Sensitivity of Arrival Time

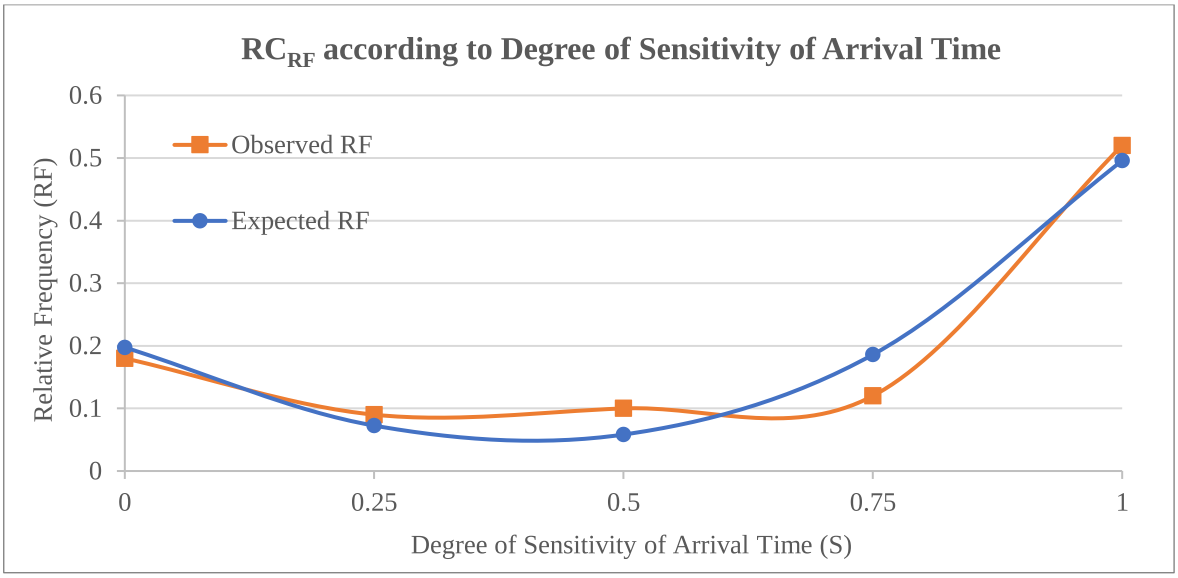


Table D-VI

Regression Statistics for Equation of Relative Frequency of Number of Reneging Customers

Statistic	Value
Multiple R	0.972
R Square	0.945
Adjusted R Square	0.889
Standard Error	0.060
Observations	5

Table D-VII

ANOVA for Equation of Relative Frequency of Number of Reneging Customers

	DF	SS	MS	F	Significance F
Regression	2	0.124	0.062	17.038	0.055
Residual	2	0.007	0.004		
Total	4	0.131			

Table D-VIII

Regression Coefficients of Equation of Relative Frequency of Number of Reneging Customers

	Coefficients	Standard Error	t Stat.	P-value	Lower 90%	Upper 90%
Intercept	-1.173	0.278	-4.217	0.052	-1.985	-0.361
e^{S*}	1.370	0.305	4.497	0.046	0.481	2.260
S	-2.056	0.526	-3.910	0.060	-3.592	-0.520

*Note: e^S is treated as a variable by itself, neglecting the coefficient of S variable

Table D-IX

Frequency of Observed Number of Renegers Corresponding to Renegers' PWT

PWT (minutes)	Observed Frequency	Cumulative Relative Frequency	Average Rank	Average RWT (minutes)
PWT ≤ 2.25	23	0.14	2	0.04
2.25 < PWT ≤ 3.50	17	0.24	2	0.66
3.50 < PWT ≤ 4.75	51	0.55	3	0.29
4.75 < PWT ≤ 6.00	9	0.61	2	2.45
6.00 < PWT ≤ 7.25	43	0.87	4	0.54
7.25 < PWT ≤ 8.50	16	0.97	5	0.84
8.50 < PWT ≤ 9.75	4	0.99	4	2.01
9.75 < PWT ≤ 11.00	1	1.00	5	2.30

Figure D-X

Frequency of Reneging Customers' PWT

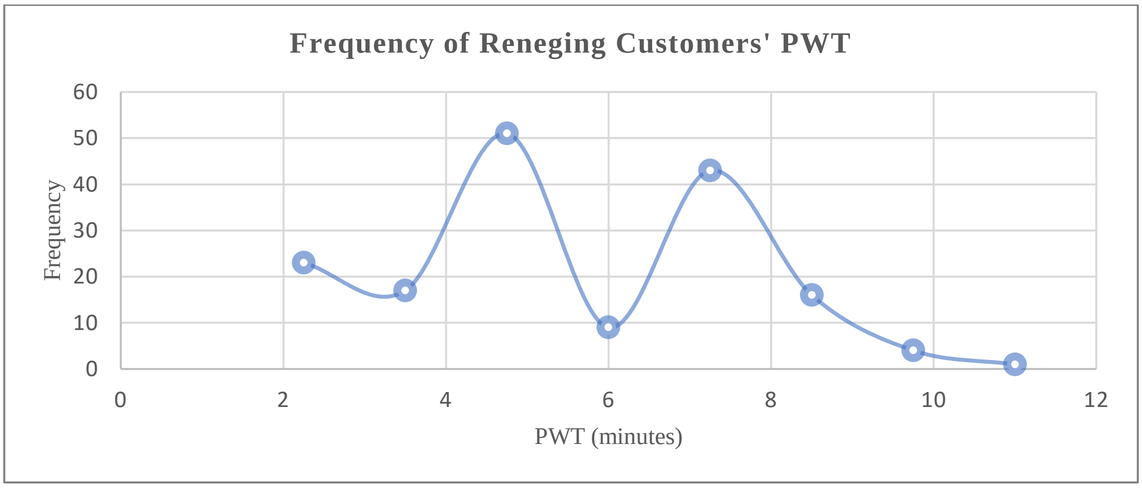


Table D-XI

Chi-Square Goodness-of-Fit Test for Log-Normal Distribution of Renegers' PWT/St Ratio, when there is a Reneging Event

PWT/st (X)	Observed Frequency (O)	$P(X \leq x)$	$P(x_{i-1} < X \leq x_i)$	Expected Frequency (E)	$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
2	62	0.397	0.397	65	0.138
4	73	0.814	0.417	69	0.232
6	20	0.942	0.128	21	0.048
8	5	0.980	0.038	6	0.167
10	3	0.992	0.012	2	0.500
x > 10	1	1.000	0.008	1	0.000
$\sum$	164	-	1.000	164	1.085
Right-Tailed Probability of $\chi^2_{0.05,3}$					0.781

Figure D-XII

Log-Normal Distribution of Renegers' PWT/St Ratio, when there is a Reneging Event

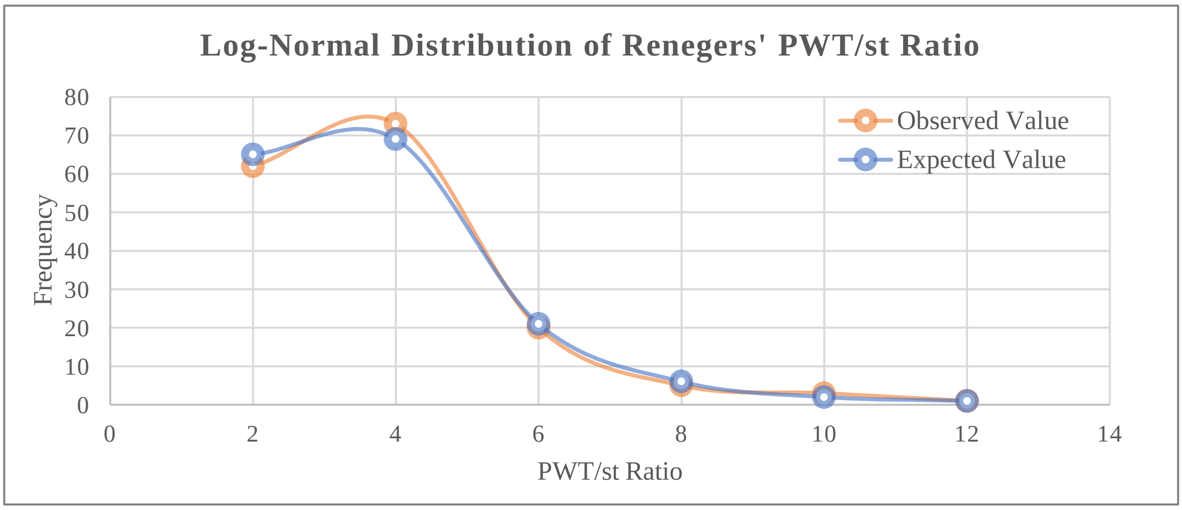


Table D-XIII

Joint Probability Distribution of Renegers' Rank and RWT/St Ratio, when Reneging Occurs

Renegers' RWT/st Ratio (Y)	Renegers' Rank (X)				Marginal Probability Distribution of Y
	x ≤ 2	2 < x ≤ 3	3 < x ≤ 4	x > 4	
y ≤ 0.5	0.274	0.246	0.199	0.076	0.795
0.5 < y ≤ 1	0.037	0.033	0.027	0.010	0.108
1 < y ≤ 1.5	0.015	0.014	0.011	0.004	0.044
1.5 < y ≤ 2	0.007	0.006	0.005	0.002	0.021
2 < y ≤ 2.5	0.004	0.004	0.003	0.001	0.012
2.5 < y ≤ 3	0.002	0.002	0.002	0.001	0.007
3 < y ≤ 3.5	0.001	0.001	0.001	0.000	0.004
y > 3.5	0.003	0.003	0.002	0.001	0.009
Marginal Probability Distribution of X	0.345	0.309	0.250	0.096	1.000

Figure D-XIV

Joint Probability Distribution of Renegers' Rank and RWT/St Ratio, when Reneging Occurs

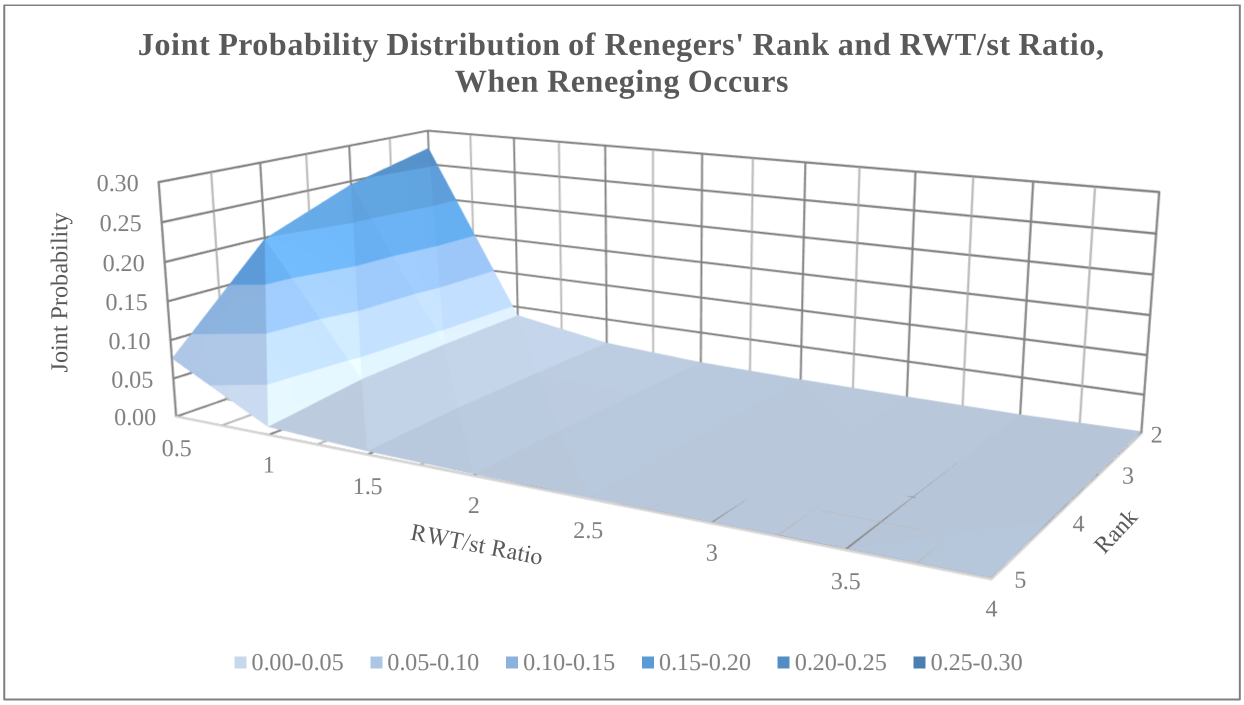


Table D-XV

Joint Probability Distribution of General Queue Intensity and Renegers' RWT/st Ratio, when Reneging Occurs

General Queue Intensity (Z)	Renegers' RWT/st Ratio (Y)								Marginal Probability Distribution of Z
	y ≤ 0.5	0.5 < y ≤ 1	1 < y ≤ 1.5	1.5 < y ≤ 2	2 < y ≤ 2.5	2.5 < y ≤ 3	3 < y ≤ 3.5	y > 3.5	
z ≤ 0.35	0.040	0.005	0.002	0.001	0.001	0.000	0.000	0.000	0.050
0.35 < z ≤ 0.7	0.233	0.032	0.013	0.006	0.004	0.002	0.001	0.003	0.293
0.7 < z ≤ 1.05	0.225	0.031	0.012	0.006	0.003	0.002	0.001	0.003	0.282
1.05 < z ≤ 1.4	0.137	0.019	0.008	0.004	0.002	0.001	0.001	0.002	0.172
1.4 < z ≤ 1.75	0.074	0.010	0.004	0.002	0.001	0.001	0.000	0.001	0.093
1.75 < z ≤ 2.1	0.039	0.005	0.002	0.001	0.001	0.000	0.000	0.000	0.049
2.1 < z ≤ 2.45	0.021	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.026
z > 2.45	0.027	0.004	0.001	0.001	0.000	0.000	0.000	0.000	0.033
Marginal Probability Distribution of Y	0.795	0.108	0.044	0.021	0.012	0.007	0.004	0.009	1.000

Figure D-XVI

Joint Probability Distribution of General Queue Intensity and Renegers' RWT/st Ratio, when Reneging Occurs

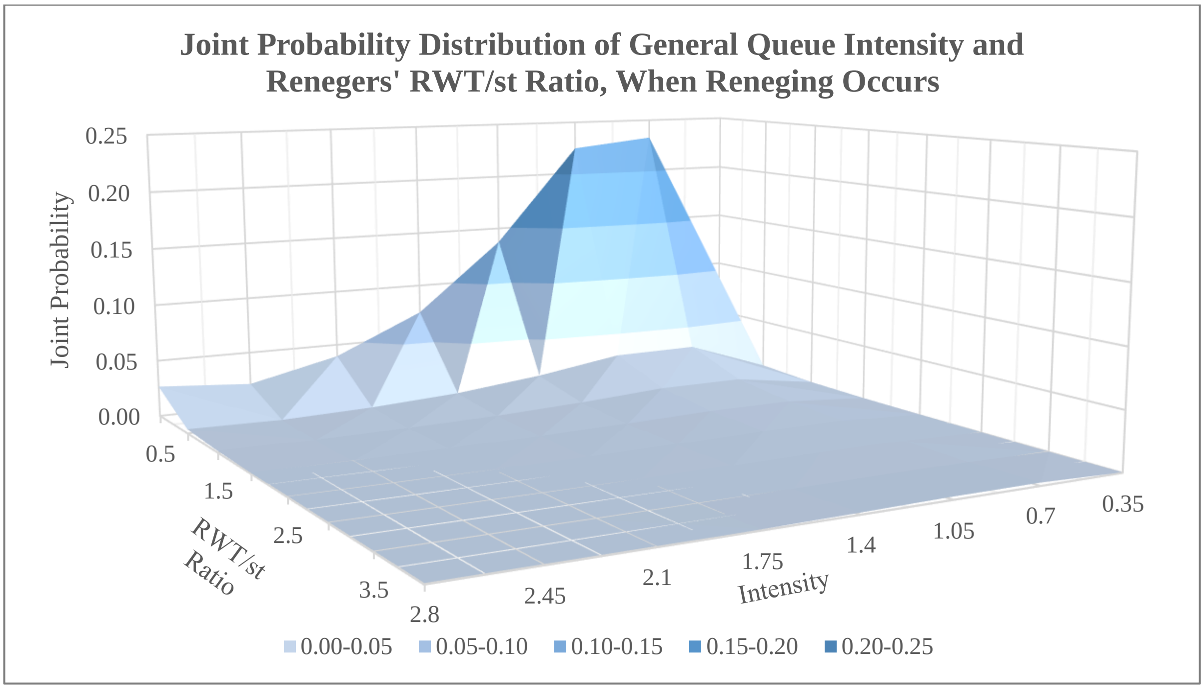


Table D-XVII

Joint Probability Distribution of General Queue Intensity and Renegers' Rank, when Reneging Occurs

General Queue Intensity (Z)	Rank of Renegers (X)				Marginal Probability Distribution of Z
	$x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4$	$x > 4$	
$z \leq 0.35$	0.017	0.016	0.013	0.005	0.050
$0.35 < z \leq 0.7$	0.101	0.090	0.073	0.028	0.293
$0.7 < z \leq 1.05$	0.097	0.087	0.071	0.027	0.282
$1.05 < z \leq 1.4$	0.059	0.053	0.043	0.017	0.172
$1.4 < z \leq 1.75$	0.032	0.029	0.023	0.009	0.093
$1.75 < z \leq 2.1$	0.017	0.015	0.012	0.005	0.049
$2.1 < z \leq 2.45$	0.009	0.008	0.007	0.003	0.026
$z > 2.45$	0.012	0.010	0.008	0.003	0.033
Marginal Probability Distribution of X	0.345	0.309	0.250	0.096	1.000

Figure D-XVIII

Joint Probability Distribution of General Queue Intensity and Renegers' Rank, when Reneging Occurs

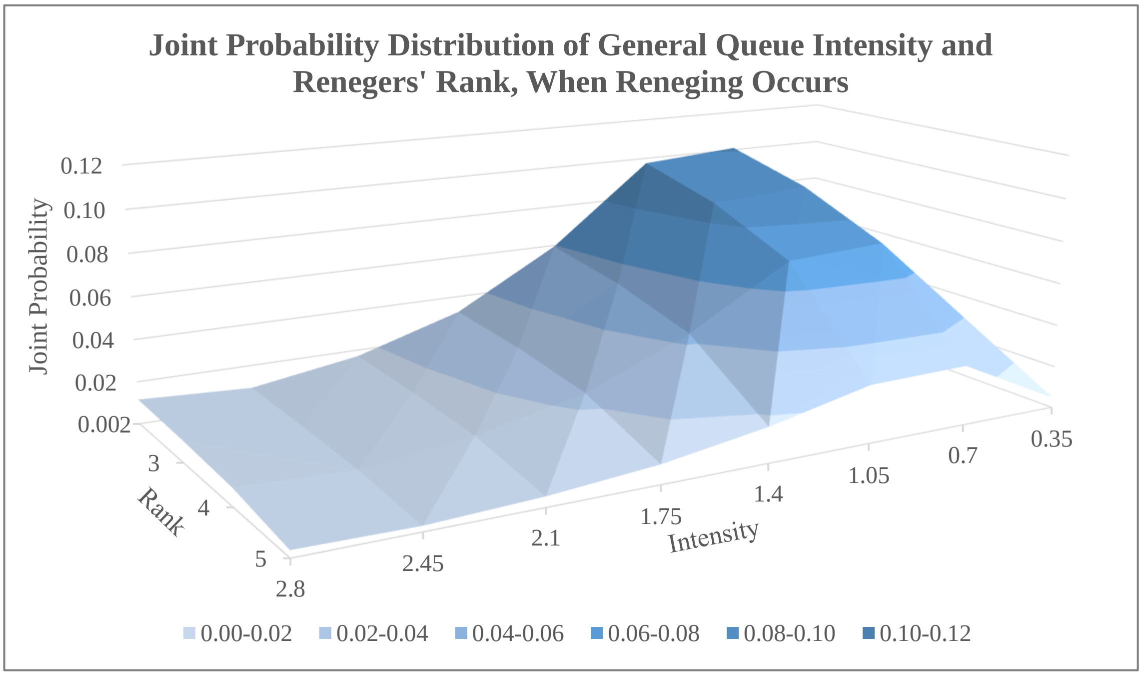
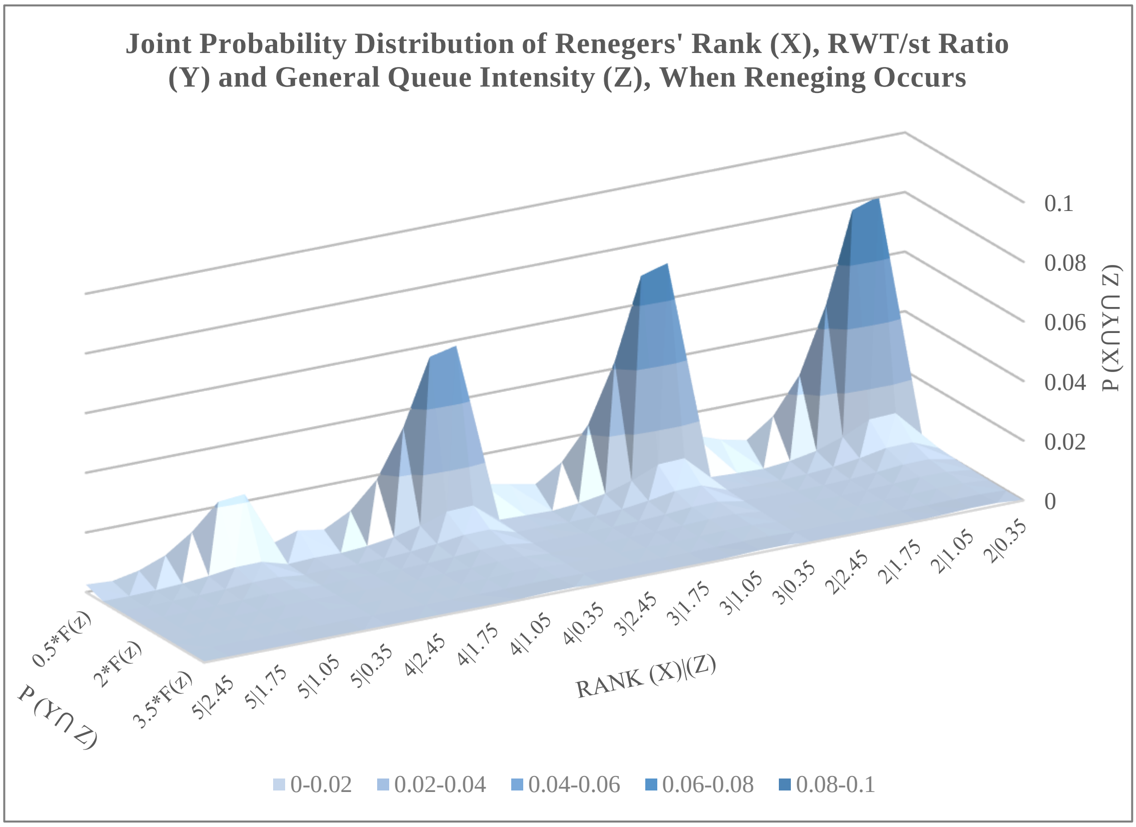


Figure D-XIX

Joint Probability Distribution of Renegers' Rank (X), RWT/st Ratio (Y) and General Queue Intensity (Z), when Reneging Occurs



Appendix E

Queuing Theory in Case of Reneging

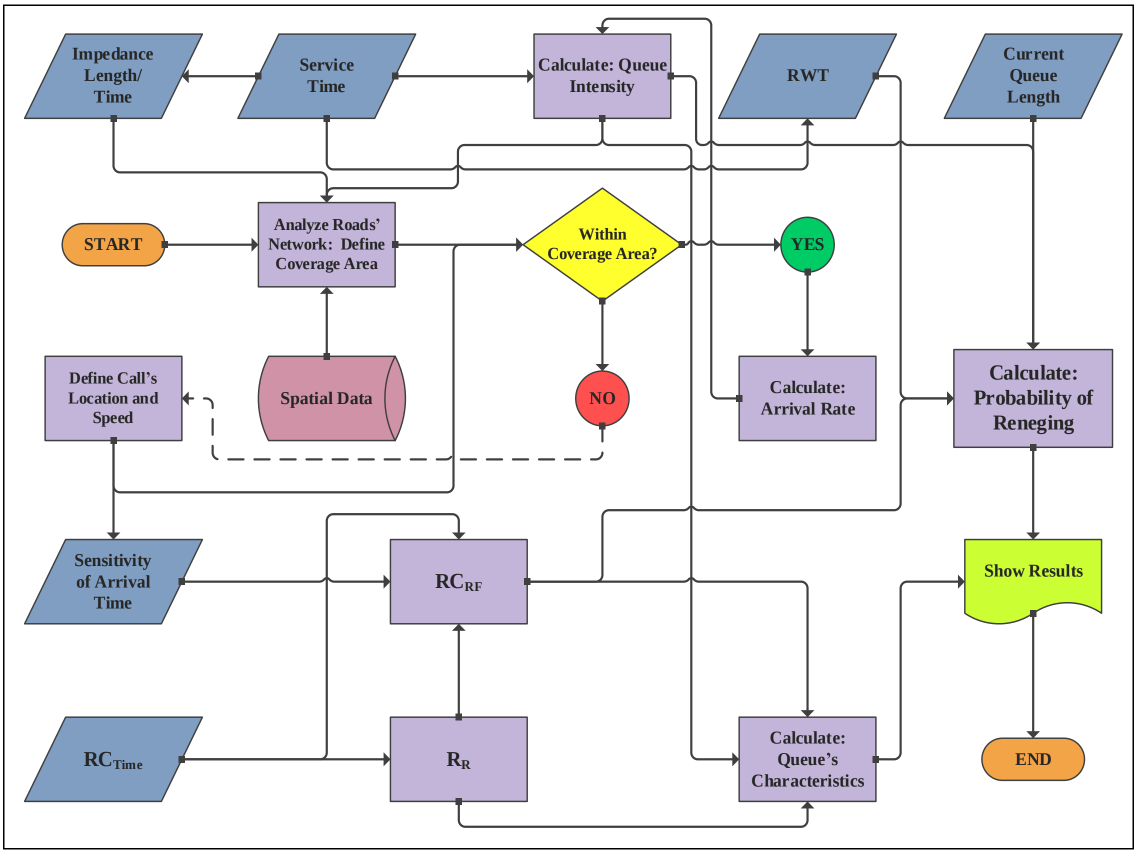
Table E-I

Characteristics of the Queue in Rio Café's Drive-Thru Service After the Occurrence of Reneging Behavior

Performance Measure	General Study Period		When Reneging was Detected	
	All Time Intervals	Morning PH Only	All Time Intervals	Morning PH Only
λ (customers/time interval)	5.5	7.8	7.45	9.58
R_R (customers/time interval)	0.33	0.59	1.45	1.67
$\lambda - R_R$ (customers/time interval)	5.17	7.21	6.00	7.91
μ (customers/time interval)	7.46	8.02	7.32	8.38
ρ_R	0.69	0.90	0.82	0.94
ρ (0)	0.31	0.10	0.18	0.06
ρ (1)	0.21	0.09	0.15	0.05
ρ (2)	0.15	0.08	0.12	0.05
ρ (3)	0.10	0.07	0.10	0.05
E (m) – customers	1.56	8.00	3.73	15.89
E (m/m>0) – customers	3.26	9.90	5.55	17.83
E (n) – customers	2.26	8.90	4.55	16.83
E (v) – minutes	6.55	18.52	11.36	31.91
E (w) – minutes	4.54	16.65	9.31	30.12

Figure E-II

Logical Workflow for Spatial-Dynamic Application for Drive-Thru Service Providers



جامعة النجاح الوطنية

كلية الدراسات العليا

سلوك الانسحاب من طوابير خدمة الطلب من المركبات: استقصاء رياضي تطبيقي قائم على حالة دراسية

إعداد

أيمن واصف محمد دبيس

إشراف

أ. د. خالد الساحلي

قدمت هذه الرسالة استكمالا لمتطلبات الحصول على درجة الماجستير في هندسة الطرق والمواصلات، من كلية الدراسات العليا، في جامعة النجاح الوطنية، نابلس–فلسطين.

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سلوك الانسحاب من طوابير خدمة الطلب من المركبات: استقصاء رياضي تطبيقي قائم على حالة دراسية

إعداد

أيمن واصف محمد دبيس

إشراف

أ. د. خالد الساحلي

الملخص

لم تكن يوما خدمة الطلب من المركبات (Drive–Thru) حكرا على مطاعم الوجبات السريعة، ولكنها لا تزال أكثر شيوعا فيها. خلال جائحة كورونا، وللحفاظ على التباعد الفيزيائي، امتدت خدمات الطلب من المركبات لتشمل القطاع الصحي أيضا. وعلى الرغم من أن هذه الخدمة بالكاد تتواجد في فلسطين بشكل متكامل وكامل الفعالية، إلا أنه من المتوقع انتشارها قريبا ببنى تحتية كاملة.

وعندما يتطلب الحصول على الخدمة الاصطفاف والانتظار، يظهر الزبائن المصطفون، وبشكل ملفت، رغبتهم بأن تقدم لهم الخدمة بأسرع ما يمكن؛ فتظهر سلوكيات الزبائن غير الصبورين بتصرفين رئيسيين: الانسحاب من الطابور، أو الامتناع عن الانضمام إليه. يعد الانسحاب من الطابور، وهو نطاق اهتمام هذه الدراسة، أحد السلوكيات المهملة عند تحليل الاصطفاف؛ مما يجعل من تحليل الطوابير وإدارتها لا يعكس واقع وحقيقة الأمر. ويعود السبب في ذلك إلى محدودية المعرفة حول هذا الحدث أو السلوك، والعوامل المسببة لحدوثه.

تهدف هذه الدراسة للوصول إلى فهم أعمق لنظرية الاصطفاف وخصائص الطابور؛ لتصبح النظرية أكثر انعكاسا وتفسيرا لمشاكل الاصطفاف الحقيقية، وذلك بتحليل سلوك المنسحبين من الزبائن غير الصبورين، وتحديد كيف تأثر قرارهم بخصائص الطابور.

من خلال ملاحظات مستقلة عن الحدث، جمعت البيانات المطلوبة باستخدام تسجيلات كاميرات المراقبة لخدمة الطلب من المركبات في مقهى ريو في مدينة البيرة. ولأغراض هذه الدراسة؛ حللت البيانات رياضيا لتحديد دالة التوزيع الاحتمالي لسلوك الانسحاب من الطابور، وكذلك للعوامل المؤثرة على قرار الزبائن المنتظرين. بالإضافة إلى ذلك، طورت نماذج انحدار خطية وغير خطية لتوقع معدل الانسحاب من الطابور والتكرار النسبي لعدد الزبائن المنسحبين.

أظهرت الدراسة أن التوزيع الأسي هو دالة التوزيع الاحتمالي للفترات الزمنية الواقعة بين انسحابات متتالية من الطابور لزبون واحد على الأقل خلال مدة زمنية. ومن ناحية أخرى، فإن معدل الانسحاب من الطابور لا يتبع توزيع بواسون (Poisson Distribution).

خلصت الدراسة إلى أن وقت الانتظار الطويل ليس العامل الوحيد المتسبب في انسحاب الزبائن غير الصبورين من الطابور، بل هناك عوامل أخرى تؤثر على هذا السلوك. هذه العوامل متكاملة فيما بينها وموزعة رياضيا بشكل مشترك، مما يقود الزبون في نهاية المطاف لاتخاذ قراره بمغادرة الطابور.

الكلمات المفتاحية: سلوك الانسحاب من الطابور، زبائن غير صبورين، استبقاء الزبائن، نظرية الاصطفاف، توزيع الاحتمالات المشترك، نمذجة مكانية ديناميكية.