An-Najah National University
Faculty of Graduate Studies

Analysis of The Logistic Distribution Use in The Suppression Technique for Scalability in Multicast Routing

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To:

My Supervisor Dr. Mohammad Najib Ass'ad

My Father Ali Khalil Hamad

For their indispensable help

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Abstract

The immense growth of the computer-supported communication systems, especially the internet, made it imperative to design protocols that have to be efficient and scalable to support the work of the networks' infrastructure. By scalable is meant the ability of the protocol to cope with the requirements of groups of the communicating processes when they grow very large in size.

The ever increasing demand on communication and the high capability of modern networks call continuously for efficient solutions to problems of communication. Among these solutions was the introduction of multicast routing and also the use of periodic unacknowledged messaging.

Related to these two solutions of the problem of scalability, certain techniques were used to overcome this problem, including the suppression technique.

This study deals with utilizing probabilistic distribution functions (pdfs) in the suppression technique with the aim of improvement of scalability of multicast routing in communication networks.

The two most employed distributions in the suppression techniques are the uniform and the exponential distributions, the first outperforms the second in the performance time metric, while the exponential excels in the performance metric of extra messages.

This study introduces a modified form of the logistic distribution as a candidate for use in the suppression technique and compares it with the two other above mentioned distributions. The MATLAB software was used in calculating the values of the performance metrics and in drawing the corresponding figures for comparing the results.

The logistic distribution was proved to excel or compete with the other two pdfs in time performance metrics and to have a comparable performance in the overhead metrics.

Chapter One

Background

1.1 Introduction

This chapter presents the concepts and conditions that underlie the communication in computer-supported networks, related to suppression and scalability which constitute the main subject of this study.

The chapter contains the following topics:

- Computer Networks
- Multicasting
- Algorithms
- Multicast Routing Constraints
- Multicasting Algorithms & Protocols
- Scalability & Multicast Protocols
- Probability Distribution Functions

1.2 Computer Networks

During the 20th century the key technology was information gathering, processing and distribution. Among other developments we witnessed the birth and immense growth of computer industry (Tanenbaum, 2002, p.1). The merging of computer and communications has had a profound influence on the way computer systems are organized. Those systems are called computer networks, the design and organization of which constitute the domain of study of this section.

In 1983 networks were used by the universities and large businesses; in 1996, computer networks, especially the internet, had become a daily reality for millions of people. Networks are classified in accordance with two dimensions:

a) *Transmission Technology:* broadcasting, unicasting, and multicasting

b) *Scale:* Local Area Network (Lan) in buildings and campuses, Metropolitan Area Network (Man) in cities, Wide Area Network (Wan) in country or continent and Internet (Planet). In the 1980s, many kinds of Lan and Wan existed, but now, the internet is dominating.

1.2.1 Network Software

It was difficult to standardize networks with regard to their hardware, so they were standardized with regard to software. There are two standardizing methods, each considering the network as a stack of layers or levels; these layers with the protocols that define communication between the corresponding layers in different appliances, constitute the architecture of the network.

There are two important network architectures: the Open System Interconnection (OSI) reference model and the Transmission Control Protocol/Internet Protocol (TCP/IP) reference model.

Although the protocols, of the OSI model are rarely used now, the features of each layer in this model are still very important; the opposite is with the TCP/IP model. Here, it suffices to give a brief description of the OSI model.

1.2.2 Network Layers

The OSI depends on proposals by International Standards Organization (ISO), and has seven layers Figure 1.1 each built over the one below it, and each offers certain services to the higher layer; when a layer(A), in a certain appliance uses its protocols to interact with the corresponding layer (A1) on another appliance, it does this by sending the message to the layer directly down it, in its own appliance, till it reaches the physical layer; and when the message is transferred to the other appliance, it goes upwards till it reaches the corresponding layer (A1).

OSI	TCP/IP	
7- Application	Application	
6- Presentation		Not present
5- Session		In the model
4- Transport	Transport	
3- Network	Internet	
2- Data Link	Host to network	
1- Physical		

Figure (1.1): *The OSI and TCP/IP Models*

The seven layers are:-

1- The Physical Layer

It is concerned with transmitting raw bits over a communication channel; the design issues, here, deal mainly with mechanical, electrical and timing interfaces.

2- The Data Link Layer

This layer includes the Media Access Control (MAC) which is a sub layer that is responsible for the control of media access. In the internet (TCP/IP) model, this sub layer is part of the network access component.

3- The Network Layer

The main task of the network layer is to provide unique network addressing and to route data efficiently toward its destinations in the global network.(Wittmann, 2001, pp.45-48).

4- The Transport Layer

The task of this layer is to support data exchange between the communication partners. It accepts data from above, passes them correctly to the network layer; it also determines what type of service to provide to the session layer, and ultimately, to the user of the network.

5- The Session Layer

It allows users on different machines to establish sessions between them.

6- The Presentation Layer

It is concerned not with moving data around as in lower layers, but with the syntax and semantics of the information transmitted, transforming the entering or leaving code systems of the data.

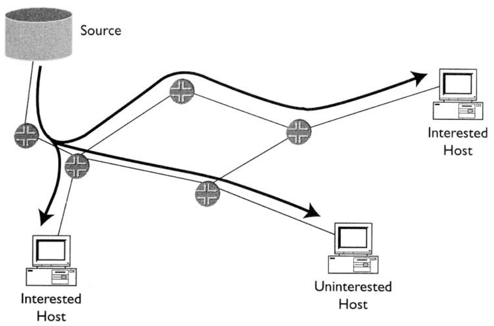
7- The Application Layer

It contains a variety of protocols, including those in e- mail and others on which the world wide web (www) depends.

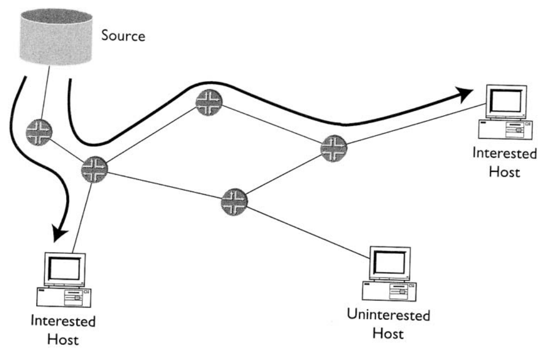
1.3 Multicasting

With the spread of diverse communication networks, like LAN, WAN, and the largest network of networks i.e. the INTERNET, various methods of communication were devised between and among them, to cope with the differing nature of the appliances and the different needs of their users.

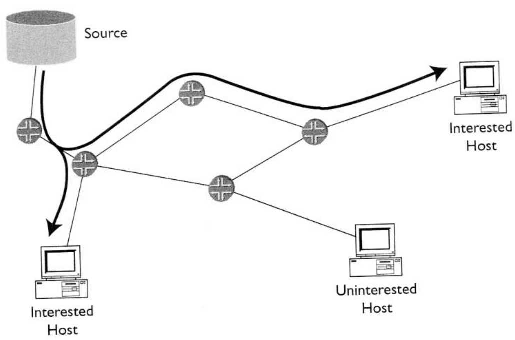
Three main methods of communication were used: broadcasting, unicasting and multicasting Figure 1.2 .



Broadcast



Unicast



Multicast

Figure(1.2)*: Broadcast, Unicast and Multicast*

Broadcasting, e.g. radio and TV systems, is a too general method of data delivery: a copy of the message by the sender is received by every possible receiver in the systems network.

Unicast is a too individual method of data delivery, a separate copy of the message is delivered to the receiver to which the address corresponds e.g. small networks.

Multicasting a set of interested receivers are assigned one address and a copy of any message is delivered to each one of the set (Comer, 2001, p.635).

Compared to broadcasting, multicasting is an efficient method of communication; and though, in both, a single sender exists, however, with broadcast there is no restriction with respect to the group of receivers, data is sent to all potential receivers; any one who is equipped with the required device is capable of receiving the data, with the only restriction being whether the device is activated (Wittman, 2001, p.5); also the interfaces in multicasting do not automatically forward the frames to the CPU – hence wasting resources as in broadcasting - but instead, the hardware of the interface are programmed with specifications of which multicast frames to accept and which to reject (Comer,2001, pp.126 –127).

Also in comparing unicasting with multicasting we observe clearly the advantage of multicasting in group communication, Figure 1.2. And though most communications in the Internet, for example, is done by unicasting, where a file is transferred from the server to a computer; if many persons want the same file, bandwidth becomes important and the economy provided by multicast becomes necessary.

Multicasting system consists of four components:

1- Definition of multicast host groups by multicast address;

2- A mechanism for joining and leaving the host group;

3- Routers with routing protocols to handle the duplication of multicast as needed and to handle issues surrounding group management and;

4- Application protocols for creating and managing the data that are distributed in a multicasting session.

The applications of multicasting, in IP, are diverse and include multimedia applications, like video conferencing, and internet audio, replication; and data applications, like stock quotes, news feeds; interactive gaming, information delivery, database replication and software distribution.

Despite the advantages that multicasting has in comparison with unicasting and broadcasting (mainly decrease of the network load in certain situations), it suffers from certain problems (Kineriwala, 1999):

1- Joining and leaving a group;

2- Time sensitive delivery of multicast traffic;

3- Scalability: the potential increase of the members of a group.

4- Security matters

The word “routing” has, in general, two identities:

1- routing protocols which have the task of identifying the state of the network, its available resources and distributing this information through the network;

2- routing algorithms which use this information to compute the most economic paths (Kuipers, 2001).

In the following three sections we'll discuss these two identities of routing with concentration on multicast routing.

1.4 Algorithms

An algorithm represents a set of steps designed to achieve a complex mathematical operation, each step carrying the operation forward by one small increment, and with, perhaps, a built-in repetition of one or more of the steps until certain conditions are reached (Encyclopedia Britannica).

1.4.1 Computational Complexity

Certain problems may have different algorithms for their solutions; hence, a way for comparing algorithms becomes eminent, and certain guidelines will be useful. Among these we have the natural size of the data (N) to be treated in the problem. The parameter N might be the degree of a polynomial, the size of a file to be sorted or searched, the number of nodes in a graph, etc. Depending on the parameter N, most algorithms have running time proportional to one of the following functions:

1: This is the case when all of the instructions of a program are executed once or few times only.

Log N: This is the case when a problem is solved by transforming it into a smaller problem, cutting the size by some constant fraction.

N: This is the case when a small amount of processing is done on each element of the input.

N log N: Similar to log N, but with smaller subproblems of the original problem.

N^2: This is the case in processing all pairs of data items, perhaps in a double nested loop.

N^3: Similar to N^2.

2^N: These problems which grow exponentially are not practical (with brute force solution).

The running time of a particular problem may be some constant times one of the above functions (e. g. N^3), in that case, we say the computational complexity is of the order of that function (e.g. $o(N^3)$).(Sedgewick,1988).

P/NP- Problems

Problems themselves may be subdivided in accordance with the computational complexity of the available algorithms that are used in solving them.

In regard to the practicality of the running time of program: Problems are divided into two types (Sedgewick, 1988, pp. 634,635):

a) P– Problems

These are the problems that can be solved by deterministic algorithms in polynomial time.

"Deterministic", here, means that actual computers don't face many choices in moving, from on step of the algorithm to another; and "polynomial" means that change of computer affects the running time by only a polynomial factor.

Of the P- problems we may mention the sorting problem, where running time is proportional to N^2.

b) NP - Problems

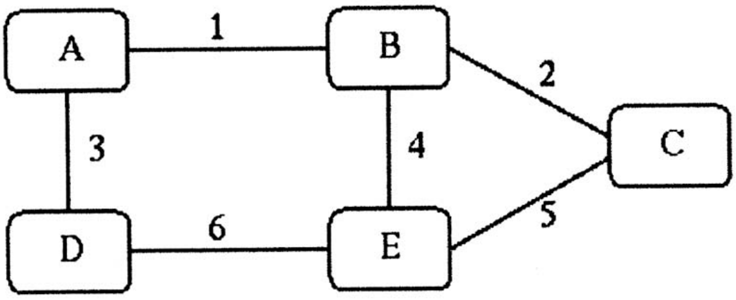
These are the problems that can be solved by non–deterministic algorithms in polynomial time.

"Non determinism" means that the algorithm, when faced by a choice of several options after a certain step, has the power to "guess" the right option and also to verify the correctness of the solution.

The relation between P–problems and NP– problems is not clear; but a certain subset of NP–problems is distinct by being easy to solve on a non–deterministic computer, but none of them has an efficient algorithm for solution on a conventional computer, this subset of NP– problems is called NP–complete Problems (Sedgewick, 1988, p.636,639).

1.5 Multicast Routing Constraints

In order to better understand the multicast routing problem, one may utilize the most commonly used terms from Graph theory, where a graph G is undirected and without loops, and nodes (v) in the graph Figure 1.3 represent hosts, and edges, (E) represent network links, N(v) denotes the set of neighbors of $v \in V$, $\delta(v)$ the number of such neighbors. (Oliveira & Pardalos, 2003).



Figure(1.3)*: Graphs, Edges and Nodes*

With each edge (i,j) ∈ E, functions are associated that represent characteristics of the network links including:

1- Capacity e(i,j) i.e. the maximum amount of data that can be transmitted between nodes i and j;

2- Cost w(i,j) i.e. the cost of using the link (i,j) including leasing, maintenance, etc;

3- Delay d(i,j) i.e. the time needed to transmit information between the nodes i, and j.

Capacity is related to the problem of congestion, where congestion on a link is defined as the difference between capacity and usage. Though multicast routing is built on the idea of reducing bandwidth used in transmission of data, difficulties may arise in certain cases, and enhanced measures and procedures ought to be taken.

The cost of a path p or w(p) is defined as the sum of the costs of all edges that constitute the path. Optimizing the cost is an objective of multicast routing, and a problem arises when there is a conflict between need of large resources and satisfying this objective.

Real time multicast is characterized by the fact that messages in it should be received by all destinations within a specified delay bound. The exact amount of delay bound depends on particular applications, but one can define delay bound for a node (Widyono, 1994, p.5), to be “the longest delay at it that a packet can tolerate without missing its end–to–end deadline”.

Delay constraint may be defined by “link delay” where delay means the delay that packets experience on that link, including queuing, transmission, and propagation. (Chakraborty, 2003, p.5).

The path delay is an additive function, i.e. it equals the sum of delays from source to destination, for all destinations; hence the problem of finding the path is solvable in polynomial time shortest path algorithms, such as Dijkstra, can be used to achieve this objective. But adding the constraint of delay to the original problem of multicast routing makes the problems either delay-constrained Steiner tree, or delay-shortest path tree, both of which are NP-complete problem (Moqbel, 1999, p.6).

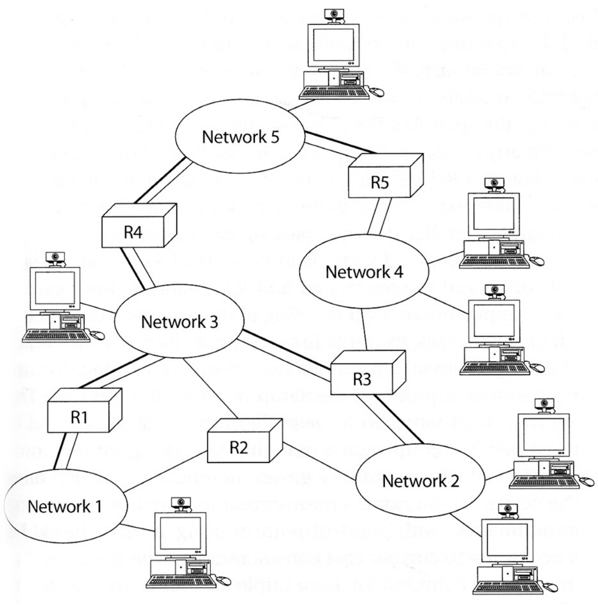
Many multimedia real-time applications such as audio and video conferencing, collaborative environments and multiplayer games, need multicast for efficient transmission, and need also Quality of Service (QoS) guarantees to run correctly, these guarantees are related to bandwidth, delay, jitter (delay variation), congestion and reliability.

But guaranteeing QoS and source utilization are conflicting objects and require a trade-off (Kuipers, 2001); historically, conventional multicast routing protocols such as CBT (Core Based Trees) and PIM (Protocol-Independent Multicasting) are not QoS aware and are designed for delivery of best-effort traffic (Dai, 2002).

The main goal of QoS multicast routing is to construct a feasible multicast tree with sufficient resources to satisfy the link constraints (e.g. bandwidth) and tree constraints (e.g. end-to-end delay bound). In fact, this proved to be a NP-complete problem when there are multiple routing objectives to satisfy, and cannot be solved by efficient algorithms (Dai, 2002).

1.6 Multicasting Algorithms and Protocols

Data is not forwarded to individual receivers in the network, but to a group of receivers generally. This requires the establishment of multicast trees, and consequently routing algorithms and routing protocols Figure 1.4 (Wittmann & Zitterbrat, 2001, p. 53).



***Figure(1.4)**: Networks and Routers (Wittman & Zitterbrat, p. 79)*

Network Protocols are used to exchange routing information in the network and build routing tables. Information consists of packets which are of two types:

i) Control Packets that are sent by routing protocols for the purpose of exchanging information between routers about how to deliver data packets through the network

ii) Data Packets which use the network to communicate data between hosts.

Routing is defined as the process by which a router calculates a forwarding table by using its knowledge of network taken from local configuration and dynamic routing protocols. (Edwards, 2002, p.320).

A router is a network layer device that typically has two or more interfaces on different networks Figure 1.4 and enables forwarding of packets between those networks. Routers perform their function depending on routing tables, which provide information on how data is to be formulated in a router. With static algorithms, this table does not change during operation (Wittmann & Zitterbrat, 2001, Pp 53-54).

1.6.1 Routing Algorithms

There are two basic types of routing algorithms: *static routing algorithms* and *adaptive (dynamic) routing algorithms* Figure 1.5 .

In the *static routing algorithms*, the routing table is initialized during the system set-up, and remains without change during operations. In fact, this type of routing algorithms is limited to static groups, and is not useful for practical data in networks: dynamic groups with members leaving or new members participating, and also for cases of overloaded links or intermediate systems.

On the contrary, *adaptive routing algorithms* are able to adapt their routing information dynamically to current data in the network, and hence they are used with most applications.

Adaptive routing algorithms are subdivided into: centralized algorithms and distributed algorithms.

With centralized algorithms, a central entity makes the decision, in the networks; this is accompanied with the advantage of having complete knowledge of the state of the network, but it also has the disadvantage of the possibility of bottlenecks and breakdown of the whole system.

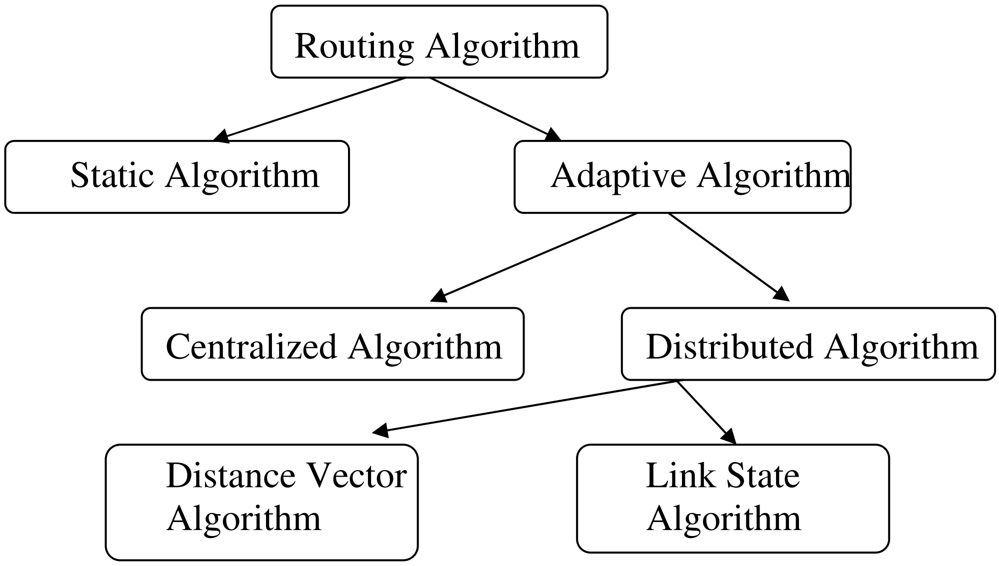
Practically, distributed algorithms are currently used in networks, where each router independently makes a routing decision based on the information available to it.

Distributed routing algorithms include two basic types: Distance- vector algorithms and link state algorithms.

Distance-vector algorithms have the objective of determining the shortest distance to a communication partner, and they generally utilize the Bellman-Ford algorithm in their calculations.

Link state algorithms assume that each network node has a map of the network, and can consequently calculate the optimal path to every other system in the network; in link state algorithm, the Dijkstra algorithm is usually employed in the calculations.

The above typology of routing algorithms is summarized in Figure 1.5



***Figure(1.5)**: Classification of Routing Algorithm*

(Wittmann & Zitterbrat*, p.77)*

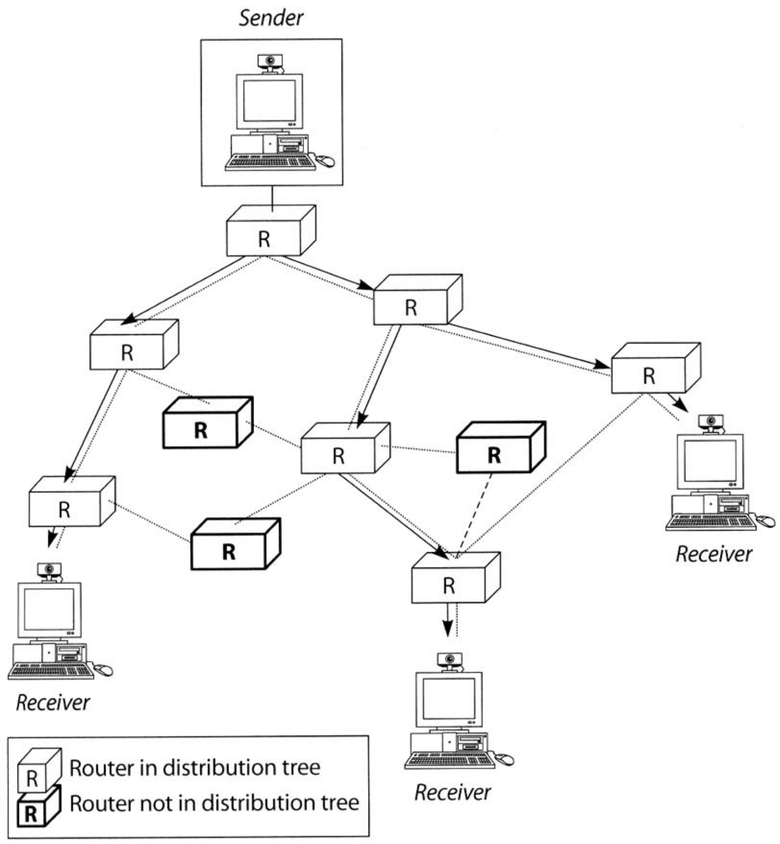
1.6.2 Distribution Trees

In contrast to the single path between sender and receiver in unicast routing, some form of distribution tree is required for multicast routing.

A distribution tree is a path for delivering data to interested listeners created by joining and pruning branches of delivery, also referred to as multicast data path (Edwards et al, 2002, p 310).

Several algorithms are available today for constructing distribution trees Figure 1.6, one of the simplest of these is the flooding algorithm, in which a router, when receiving data, sends it to all interfaces joined to it with the exception of the interface from which the data was received.

An alternative type of algorithms is that of spanning trees, this type includes three basic techniques (Wittmann and Zitterbart, 2001,p.80):



Figure(1.6)*: Distribution Tree (Wittman, p.77)*

a) Source- based Routing

This is based on the assumption that the receiver initiates the calculation of routing information, which implies the creation of a spanning tree for each source.

b) Steiner Trees

This type of tree aims at a global optimization of the cost in establishing a spanning tree, and not necessarily between any certain pair of nodes. The Steiner tree problem is NP- Complete and heuristics are proposed for the construction.

c) Rendezvous- points trees

In contrast to Steiner trees these trees consider multiple senders and receivers, and in contrast to source-based trees they do not suffer from the initial network wide flooding of data units.

Compared to Steiner trees, only the selection of an optimal rendezvous point presents an NP-complete problem. Simple heuristics are generally used for the selection of rendezvous points.

1.6.3 Multicasting Protocols

Here, we discuss protocols related to the internet; these are based on source-based routing or trees with rendezvous points.

Choosing a multicast routing protocol may depend on the particular environment in which it is to be used. Some environments have plentiful bandwidth, and multicast group members are densely distributed throughout the network (i.e. many of the subnets contain at least one group member); this type of environment is called “dense- mode”. In this case the following protocols are used: Distance Vector Multicast Routing Protocol (DVMRP), Multicast Open Shortest Path First (MOSPE), and Protocol-Independent Multicast-Dense Mode (PIM- DM) (Maufer & Semeria, 1997, p.26).

The opposite case (Sparse-Mode) is when the Multicast groups are widely dispersed (not necessarily small) and a high bandwidth is not available; in such cases, using the flood technique is wasteful (in opposition to the dense mode); the sparse-mode routing protocols include Core- Based Trees (CBT) and Protocol-Independent Multicast Sparse Mode(PIM-SM).

1.7 Scalability and Multicast Protocols

The immense growth of the computer-supported communication systems, especially the internet, made it imperative to design protocols that have to be efficient and scalable to support the work of the networks' infrastructure. By scalable is meant the ability of the protocol to cope with the requirements of groups of the communicating processes when they grow very large in size.

By large groups is meant "those groups that consist of several hundred or thousand members" (Wittmann, 2001, p.19). An example of large groups is distributed games which may consist of very large members of users. Among the problems of large groups is the heavy burden placed on group management due to the highly dynamic nature of such groups, and also the additional data exchange created within the group.

Scalability of a system can be measured along three different dimensions. ***First***, a system can be scalable w.r.t. its size, meaning that we can easily add more users and resources to the system. ***Second***, a geographically scalable system is one in which users and resources may lie far a part. ***Third***, a system may be administratively scalable, meaning that it can still be easy to manage even if it spans many independent administrative organizations. Unfortunately, a system that is scalable in one or more of these dimensions often exhibits some loss of performance as the system scales up. (Tanenbaum and Van Steen, 2002, p.10).

Another important aspect related to scalability is the aspect of reliability which means that "all data is delivered to the receiver in the correct order without any errors and without any duplication" (Wittmann, 2001, p. 23). Reliability may be achieved through the exchange of control data between the communication processes. This places an additional burden on bandwidth and demands special mechanisms to meet the problem.

Other aspects related to scalability or group size is the group topology in the form of geographical distribution and the heterogeneity of the group members; heterogeneity is related to the technical possibilities that apply to members (e.g. the networks being high-speed or over slow, error-prone wireless connections).

Multicast protocols were designed to face the problem of scalability; hence, they deal with more scalability issues than other protocols, including the problem of restricting the transmission of special hosts, which means that entire subsets might not have any receivers. These protocols include algorithms for routing, quality of service, real-time transport, distributed directories and domain name systems (Schooler, et al, 2001).

Among the techniques used for solving the problem of scalability of multicast protocols, we have the three fundamental techniques or micro algorithms: suppression, Announce-listen, and leader election. These techniques share the property of reducing the number of messages that are transmitted by a group of communication processes (Schooler, 2001, p.4).

1.7.1 Announce-Listen

Instead of sending acknowledgments (ACKs) by the receiver of a message to denote its reception, or Negative-acknowledgment (NACKs) for loss of a message, in Announce-listen a sender process disseminates information to a group of processes by sending, periodically, multicast announcement, and receiver passively listens for these announcements Figure 1.7. A listener process infers information about the global state of a system from the periodic receipt or loss of message from announcer processes. This method largely reduces the number of messages, in contrast with the (ACK) and (NACK) which may allow implosion in certain cases (Schooler, 2001, p.4).

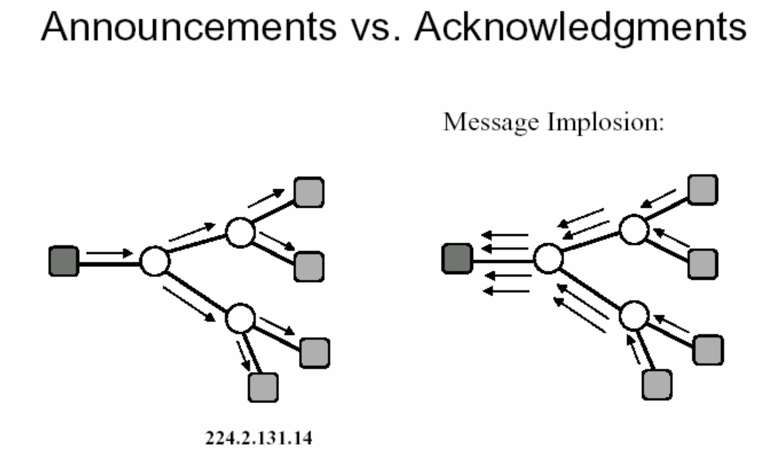


Figure (1.7)*: Announcement and Acknowledgement*

1.7.2 Leader-Election

In this technique, the number of messages is reduced by identifying a single leader, to act on behalf of a group; and, in fact, this technique helps a group of processes to behave as if it were one process.

1.7.3 Suppression

The first uses of suppression technique for delaying of messages were known decades ago with the ALOHA protocol and the Ethernet protocol, in both cases messaging is of contentious nature which led to the possibility of collision of messages during transmission (Schooler, 2001, p.30).

In the suppression technique, a process waits for a certain delay period, chosen randomly in accordance with a particular probability distribution function (pdf) before sending a message. In the delay period, if it receives a message it suppresses its own massage; otherwise the announcement is sent as intended, This method of unacknowledged announcement where announcement is resent periodically ad infinitum and the network is not permanently disconnected, denote that the system is reliable.

Suppression reduces the number of messages, like other techniques; it promotes scalability because it spreads out simultaneous transmissions over a given interval, while at the same time allowing the receipt of earlier messages to suppress later messages containing identical content (Schooler, et al, 2001), and it has special importance in cases when processes arrive into a system concurrently e.g. replying to the same message simultaneously which is the usual situation in teleconferencing.

In suppression, we differentiate between:-

1. Lossy and nonlossy conditions i.e. when messages may be lost in the network and the opposite case.
2. Fixed delay and non-delay conditions i.e. when the transmission delay is fixed and when the delay is zero.

Suppression is a random technique which depends on certain pdf's in its application (See section 1.7), the effectiveness of which is measured by two performance metrics: *time elapsed*, which is the minimum time selected by any process, and *extra messages* which are unnecessary messages sent by processes other than the one which sent the earliest message owing to delay in the time of transmission of this message.

The problem facing the research in suppression technique is that optimizing one of these two metrics affects adversely the optimization of the other metric; and hence it becomes imperative to make a trade off between the two.

1.8 Probability Distribution Functions

Probability distribution functions depend on the notion of the random variable.

A random variable x, is a function where the domain is any partition of the sample space, and the codomain is the real set of numbers R. (Encyclopedia Britannica).

It is clear from the definition of the random variable that any value x_i of a finite random variable has a corresponding probability $p(x_i)$; hence, the values the random variable takes with their corresponding probabilities form a function called the probabilistic distribution function , $f(x_i)$, such that:

i) $f(x_i) \geq 0$, for all x_i

ii) $\sum f(x_i) = 1$

In continuous random variables, a probability density function serves to represent a probability distribution in terms of integrals. Any function that is everywhere non-negative and whose integral from $-\infty$ to $+\infty$ is equal to 1 is a probability density function. If a probability distribution has density f(x), then intuitively the infinitesimal interval [x,x+dx] has probability f(x)dx.

Also, we notice the two following conditions for the pdf:

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

In the case of the continuous distribution function of a random variable the pdf is defined as the derivative if it exists of that function, and it is usually calculated in two ways:-

a) finding the probability of x lying in an interval [a,b]:

$$P(a \le x \le b) = \int_a^b f(x)dx$$

b) finding the probability of x being less than or equal to a certain value (Cumulative Distribution Function)

$$P(x \le a) = \int_{-\infty}^a f(x)dx$$

For explaining the relation between the random variable and its pdf, we notice in the accompanying Figure 1.8 that the total area under the curve is one unit, y = f(x) represents pdf of the random variable x; the probability of x being in the interval (a,b) equals the shaded area or

$$P(a < x < b) = \int_a^b f(x)dx$$

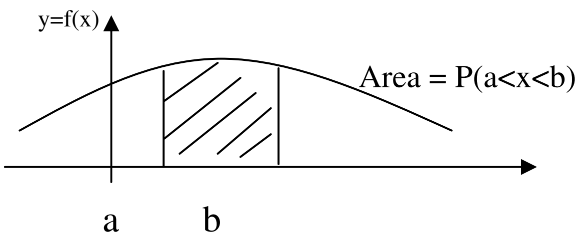
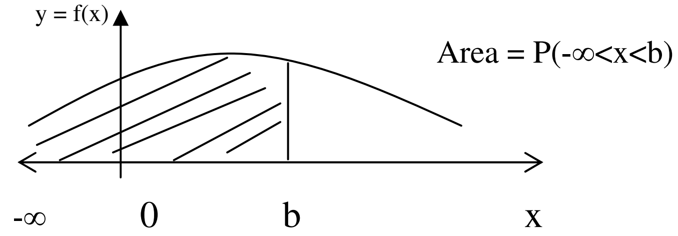


Figure (1.8)*: a Probability Density Function*

The probability of x being in the interval (-∞,b) Figure 1.9 equals the shaded area or

$$P(-\infty < x < b) = \int_{-\infty}^b f(x)dx$$



***Figure(1.9)**: a Probability Density Function*

There are many distributions of the random variables, but only some of them are used in multicast protocols.

1.8.1 The Uniform Distribution

The uniform (rectangular) distribution function, is defined by u(α,β), with the probability density p(t) $= \dfrac{1}{\beta-\alpha}$, and the domain $\alpha < x < \beta$,

Where $-\infty < \alpha < x < \beta < \infty$

The mean of u(α,β) = $(\dfrac{\alpha+\beta}{2})$ and the variance = $\dfrac{(\alpha-\beta)^2}{12}$, (Sugakkai, 1980)

In multicast protocols where the time (t) replaces the x variable, the domain becomes $\alpha < t < \beta$ where $0 < \alpha < \beta < \infty$, Figure 1.10 illustrates the case of the uniform distribution where 0 < t < T , if T=10 then

$$p(t) = \frac{1}{(T-0)} = \frac{1}{(10-0)} = \frac{1}{10}$$

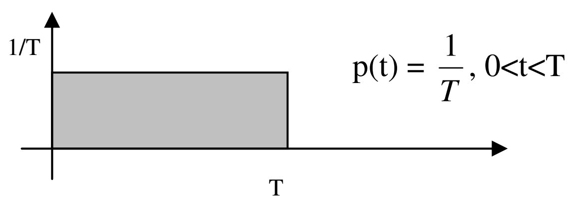


***Figure (1.10)**: The Uniform Density Function*

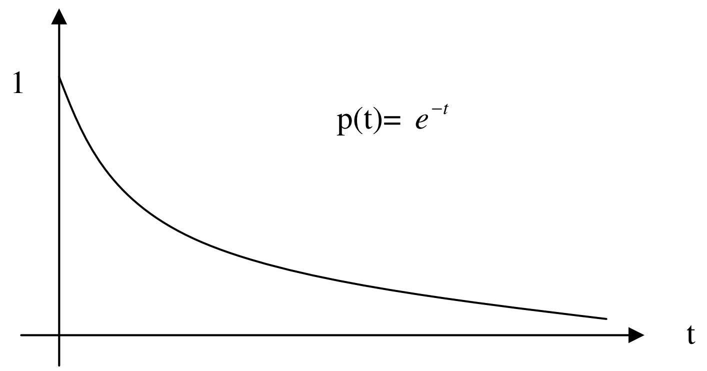
1.8.2 The Decaying Exponential Distribution

The general exponential distribution function is denoted by e(μ,σ), with pdf = $\frac{1}{\sigma}e^{\frac{-(x-\mu)}{\sigma}}$ and the domain μ <x < ∞. Where $-\infty < \mu < x < \infty$, σ > 0.

The mean is μ and the variance = σ^2 .

In multicast protocols, where the time (t) replaces the x variable, we have the adapted decaying exponential distributions with μ = 0, 0 < t < ∞, its pdf p(t) = $\frac{1}{\alpha}e^{\frac{-t}{\alpha}}$

Figure 1.11 illustrates the case of the decaying exponential distribution with α = 1



Figure(1.11): *The Decaying Exponential Distribution with α=1*

1.8.3 The Logistic Distribution

The logistic distributed function has had a lengthy history in classical statistics, specially in population studies by Verhulst (1804-1849) and others. Verhulst's equation is formalized by the differential equation

$$\frac{dP}{dt} = rP(K - P)$$

This equation represents the type of growth called logistic, in which a quantity **P** grows in proportion to both its present size and its distance from an upper limit **K** (Berresford, 1996, p.666)

P represents population size and **t** represent time, the constant **r** defines the growth rate and **K** is the carrying capacity i.e. the maximum value that **P** may have. The general solution to this equation is a logistic function. In ecology, species are sometimes referred to as r-strategist or K-strategist depending upon the selective processes that have shaped their life history strategies. (Wikipedia, the logistic function)

The solution of this logistic differential equation is the logistic function

$$P = \frac{K}{1 + ce^{-rKt}}$$

(appendix I).

In statistics, the logistic distribution function plays a leading role in the methodology of logistic regression, where it makes an important contribution to the literature on classification. The logistic distribution function has also appeared in many guises in neural network research. In early work, in which continuous time formalisms tended to dominate, it was justified via its being the solution to a particular differential equation. In later work, with the emphasis on discrete time, it was generally used more heuristically as one of the many possible smooth, monotonic functions that map real values into a bounded interval. More recently however, with the increasing focus on learning, the probabilistic properties of the logistic function have begun to be emphasized. This emphasis has led to better learning methods and has helped to strengthen the links between neural networks and statistics. (Jordan, 1995).

Logistic distribution functions are good models of biological population growth in species which have grown so large that they are near to saturating their ecosystems, or of the spread of information within societies. They are also common in marketing where they chart the sales of new products over time; in a different context, they can also describe demand curves. (Math 120).

In addition to the above mentioned qualities and uses, the similarity of certain forms of the logistic distribution function to the normal distribution and relative ease of treatment (Ass'ad, 1988, p.4.14) make it a candidate for study in our work.

The pdf of the logistic distribution is

$$f(x) \quad = \frac{a}{\sigma}\frac{e^{-\frac{a}{\sigma}(x-\lambda)}}{(1+e^{-\frac{a}{\sigma}(x-\lambda)})^2}$$

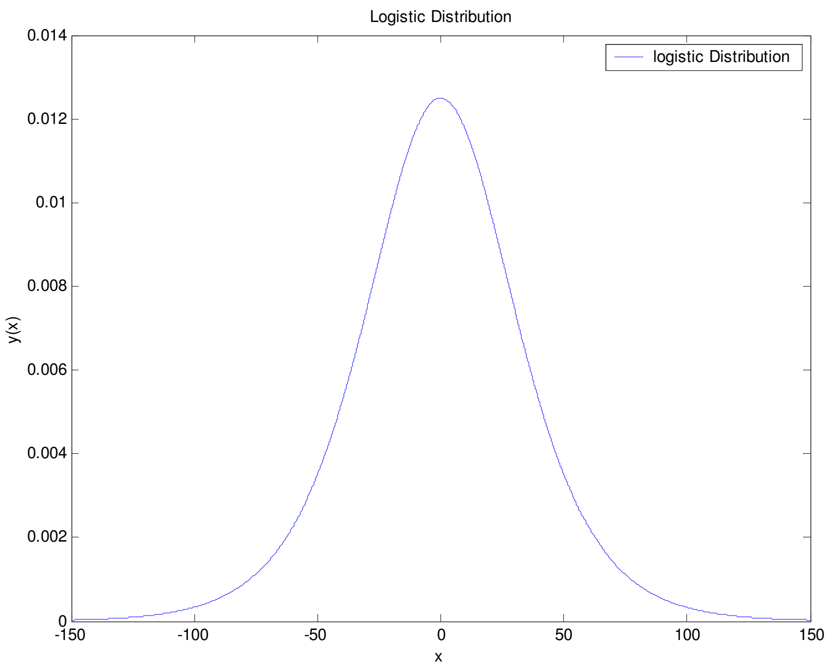


***Figure (1.12)**: The Logistic Distribution* where $-\infty < x < \infty$, $\sigma > 0$ (Wikipedia).

The standardized logistic distribution has the mean $\lambda = 0$, variance $\sigma^2 = 1$, and $f(x) = \frac{ae^{-ax}}{(1+e^{-ax})^2}$, see Figure 1.12.

In multicast protocols where the time variable (t) replaces the x variable, the modified function of the pdf of the logistic distribution function will be

$$f(t) = \frac{(2)ae^{-at}}{(1+e^{-at})^2}$$

where $0 \le t < \infty$

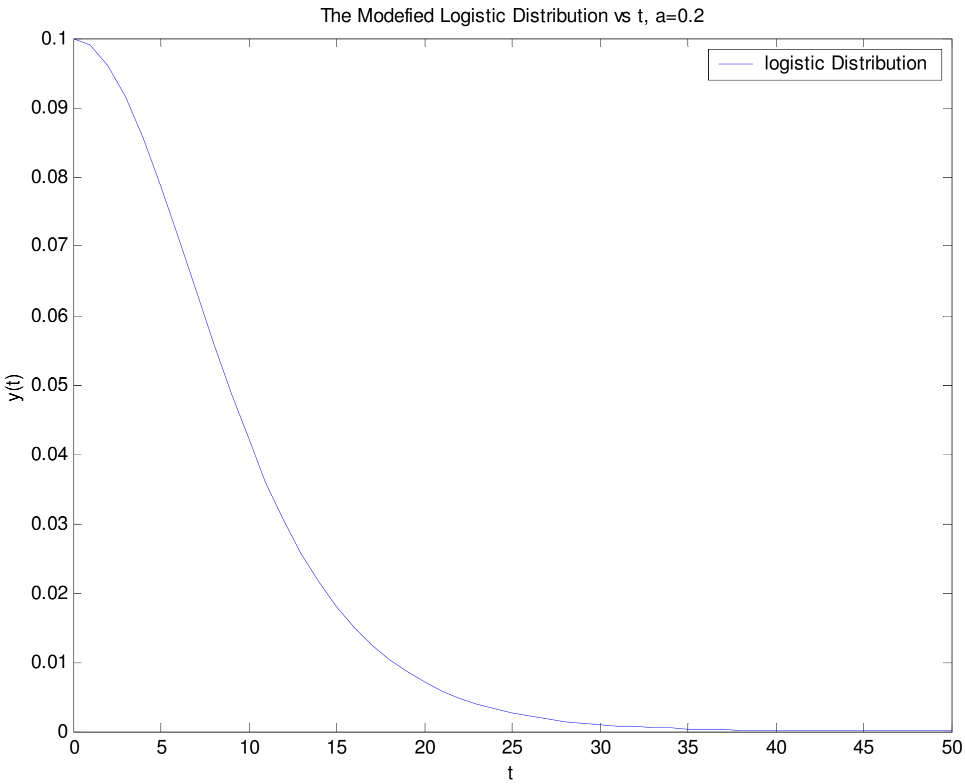


***Figure (1.13)**: The Modified Logistic Distribution*

The mean value for the logistic distribution function p(t) = $\frac{(2)ae^{-at}}{(1+e^{-at})^2}$ equals $\frac{2}{a}\ln 2$. The mean and the variance of the logistic distribution may be calculated from the formulas $\int_0^\infty t.f(t)dt$, $\int_0^\infty (t-\mu)^2 P(t)dt$, (appendices B,C).

Chapter Two

Time Elapsed & Extra Messages

In Non Lossy Communication

2.1 Introduction

In this chapter, we study the suppression technique (see: 1.7.3) under two conditions:

a- No messages are lost (non lossy case)

b- Delay transmission time (Δ) is not equal to zero, it will be considered here as constant between all pairs of nodes in the network.

Two performance metrics in the suppression technique will be studied, namely: time elapsed, or the delay incurred by a process that utilizes the suppression technique, and extra messages, or the corresponding messaging overhead.

In studying these two performance metrics, a number of parameters are utilized, which are defined, with their realistic values as follows:

1) **N**, the number of processes participating in the algorithm, this lies in the range of O(1) to O(100,000)

2) **T**, the upper bound of the suppression interval, or it is the interval at which announcement messages are sent periodically, which lies in the range of O(1) second to O(5) minutes.

3) **Δ**, the transmission delay, which depends on the domain of communication, it lies in the range of O(0.1) msec to O(10) seconds.

We assume that $T \geq 1$, $N \geq 1$ and $\Delta \leq T$; $\frac{\Delta}{T} \leq 1$; $\frac{\Delta}{T}$ lies generally in the range $(3\times10^{-6},0.5)$ and $\frac{T}{N}$ lies in the range $(1\times10^{-5},300)$ (Schooler, 2001, p.20).

4) α of the exponential distribution is supposed equal to **T**.

5) **a** of the logistic distribution is supposed equal to 2/**T**. (see 2.2.1.3).

2.2 The Suppression Algorithm and Its Performance Metrics

In suppression, a process waits (sleeps) in an interval of time without sending a message. The time interval is chosen randomly depending on an already given pdf and a delay time **T**; if a message arrives during the sleeping time, the process suppresses its own message, otherwise it sends the message.

The suppression algorithm is represented by the following pseudocode:-

```
SUPPRESSION(p,T)
    1  t = random(p,T)
    2  sleep(t)
    3  if no_message_received( ) then
    4  send_message( )
```

where t is a random delay time chosen by a process in the system depending on a procedure *random(p,T)* where p(t) is the pdf of the time distribution and T is a parameter of the distribution (Schooler, 2001, p.12).

We shall treat the performance metrics for the three pdfs, of the study (the uniform, the decaying exponential, and the logistic distribution functions) separately, and then compare the different values of their performance analytically and/ or with graphs.

2.2.1 Time Elapsed

The time elapsed, chosen by any process *i*, measures the sleeping time selected by the process; in other words, it is the minimum delay, that the process *i* allows the other processes before becoming suppressed (and also the algorithm before being completed).

In the accompanying graph Figure 2.1, for the uniform distribution function (any other pdf will do), if p(x) represents the value of the pdf on which the process *i* depends on choosing the minimum time (t_{min}) then:

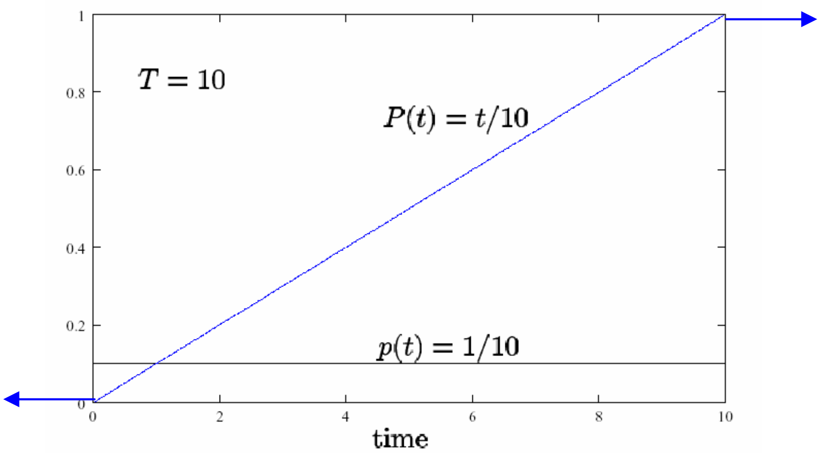


***Figure (2.1)**: Cumulative Distribution for the Uniform Distribution*

$P_i(t)$ (Shaded area) represents the probability the process *i* may choose time less then *t*; and ($1-P_i(t)$) represents the probability that the process *i* may choose time greater than *t*.

$P_i(t)$ is called the cumulative distribution function, and may be calculated by the value of $\int_0^t p(x)dx$

Having N processes to deal with, our metric for time elapsed, will be $E(t_{min})$, or the expected value of (t_{min}) which is the minimum elapsed time before one of the processes wakes up and sends a message, using the general formula:

$$E(t) = \int t.p(t)dt$$

we have

$$E(t_{\min}) = E\left[\min_{0 \le i < N} t_i\right] = \int_0^T t\left[\sum_{0 \le i < N} \overline{P}_i(t)\right]dt \qquad (2.1)$$

Where $\overline{P}_i(t)$ is the probability that t_i is the minimum. This event occurs if *i* takes the minimum value *t* with probability p(t) and all other processes have times greater than t, i.e. each with probability (1-P(t))

$\overline{P}_i(t) = p(t)(1 - P(t))^{N-1}$ (independent events)

$$E(t_{\min}) = \int_0^T tNp(t)(1 - P(t))^{N-1} dt$$

$$\int tNp(t)(1 - P(t))^{N-1} dt = -\int td(1 - P(t))^N \qquad (2.2)$$

using integration by parts

$$\int udv = u.v - \int v.du$$

u = t, $dv = d(1 - P(t))^N$

so for the equation (2.2) we have

$$-\int td(1 - P(t))^N = -\left(t(1 - P(t))^N - \int (1 - P(t))^N dt\right)$$

$$= \int_0^T (1-P(t))^N dt - t(1-P(t))^N \Big|_0^T$$

$$= \int_0^T (1-P(t))^N dt - \left[T(1-1)^N - (0)(1-0)^N\right]$$

$$= \int_0^T (1-P(t))^N dt$$

Where N is the number of processes, P(t) is the cumulative distribution function, and will be calculated separately for each one of the pdf's of the study from the formula P(t) = $\int_0^t p(x)dx$

2.2.1.1 Time Elapsed for the Uniform Distribution Function

The Uniform distribution function is the most applied in suppression algorithm; it has also inherent properties which encourage its use: mainly, its simplicity and effectiveness; and in fact it is used as a reference with which other distribution functions are compared.

For the uniform distribution:-

$$p(t) = \frac{1}{T}, \quad 0 \le t \le T$$

$$P(t) = \int_0^t \frac{1}{T} dx = \frac{1}{T} x \Big|_0^t = \frac{1}{T}(t-0) = \frac{1}{T} t = \frac{t}{T}$$

$$E(t_{\min}) = \int_0^T (1-P(t))^N dt = \int_0^T \left(1-\frac{t}{T}\right)^N dt$$

Let $u = 1 - \frac{t}{T} \Rightarrow \frac{du}{dt} = \frac{-1}{T} \Rightarrow dt = -Tdu$

$$\therefore E(t_{\min}) = \int u^N (-T) du = -T\,\frac{u^{N+1}}{N+1}$$

substitute for u

$$E(t_{\min}) = \frac{-T}{N+1}\left(1-\frac{t}{T}\right)^{N+1}\Bigg|_0^T = \frac{-T}{N+1}\big((1-1)-(1-0)\big) = \frac{+T}{N+1}$$

(2.3)

This relation was used by Nonnenmacher to estimate the value of N, given T, and $t_{received} = t_{min} + \Delta$ in one round in the form

$$N = \frac{T}{t_{received} - \Delta} - 1 \quad \text{(Nonnenmacher, 1999)}$$

2.2.1.2 Time Elapsed for the Decaying Exponential Distribution Function

The decaying exponential distribution has been used in collision – avoidance algorithms in Ethernet and packet radio and shared media networks generally.

For the exponential distribution:-

$$p(t) = \frac{1}{\alpha} e^{\frac{-t}{\alpha}}$$

$$P(t) = \int_0^t p(x)dx = \int_0^t \frac{1}{\alpha} e^{\frac{-x}{\alpha}} dx$$

$$\frac{1}{\alpha}\cdot\frac{e^{\frac{-x}{\alpha}}}{\frac{-1}{\alpha}}\Bigg|_0^t = -\frac{\alpha}{\alpha} e^{\frac{-x}{\alpha}}\Bigg|_0^t = -e^{\frac{-x}{\alpha}}\Bigg|_0^t = -\left(e^{\frac{-t}{\alpha}} - e^0\right) = 1 - e^{\frac{-t}{\alpha}}$$

$$E(t_{min}) = \int_0^\infty (1-P(t))^N dt = \int_0^\infty \left(1-(1-e^{\frac{-t}{\alpha}})\right)^N dt = \int_0^\infty \left(e^{\frac{-t}{\alpha}}\right)^N dt$$

$$= \frac{e^{\frac{-Nt}{\alpha}}}{\frac{-N}{\alpha}}\Bigg|_0^\infty = \frac{-\alpha}{N} e^{\frac{-Nt}{\alpha}}\Bigg|_0^\infty = \frac{-\alpha}{N}(0-1) = \frac{\alpha}{N} \qquad (2.4)$$

2.2.1.3 Time Elapsed for the Logistic Distribution Function

The logistic distribution function has many applications in many fields like statistics, neural networks, economics, and education (Jordan, 1995). Its similarity to the normal distribution and relative ease of treatment (Ass'ad, 1988) make it a candidate for study in our work.

For the modified logistic distribution function

$$\text{p(t)} = \frac{2ae^{-at}}{(1+e^{-at})^2}$$

$$\text{P(t)} = \int_0^t \frac{2ae^{-ax}}{(1+e^{-ax})^2}\,dx$$

Let u = 1 + e^{-ax}

$$\frac{du}{dx} = -ae^{-ax}$$

$$dx = \frac{du}{-ae^{-ax}}$$

so $$2\int \frac{ae^{-ax}}{u^2}\frac{du}{-ae^{-ax}} = -2\int \frac{du}{u^2} = -2\int u^{-2}du = -2\frac{u^{-1}}{-1} = 2u^{-1} = \frac{2}{u}$$

substitute the original values

$$\text{P(t)} = \int_0^t \frac{2ae^{-ax}}{(1+e^{-ax})^2}dx = 2(1+e^{-ax})^{-1}\Big|_0^t$$

$$= 2\left(\left(\frac{1}{1+e^{-at}}\right) - \left(\frac{1}{1+1}\right)\right)$$

$$= \frac{2}{1+e^{-at}} - 1 \qquad (2.5)$$

The cumulative area under the logistic distribution function starting at t = 0 and ending at infinity must be equal to one

$$P(\infty) = \int_0^{\infty} \frac{2ae^{-at}}{(1+e^{-at})^2} dt$$

$$P(\infty) = \left. \frac{2}{(1+e^{-at})} \right|_0^{\infty}$$

$$= \left(\frac{2}{1+e^{-\infty}} \right) - \left(\frac{2}{1+1} \right) = 2-1=1$$

We notice here that the total area under the curve of the pdf is equal to (1), which means that the modified formula of the logistic function satisfies the probability distribution function requirements.

To solve for the expected minimum time for suppression we use the formula

$$E(t_{min}) = \int_0^{\infty} (1-P(t))^N dt$$

$E(t_{min})$ for the logistic function equals

$$E(t_{min}) = \int_0^{\infty} \left(1 - \left(\frac{2}{1+e^{-at}} - 1 \right) \right)^N dt$$

$$= \int_0^{\infty} \left(2 - \left(\frac{2}{1+e^{-at}} \right) \right)^N dt$$

$$= 2^N \int_0^{\infty} \left(1 - \left(\frac{1}{1+e^{-at}} \right) \right)^N dt$$

let $\quad u = 1-\left(\frac{1}{1+e^{-at}}\right) = 1-\left(1+e^{-at}\right)^{-1}$

$$\frac{du}{dt} = \left(1+e^{-at}\right)^{-2}\left(-ae^{-at}\right)$$

$$dt = \frac{du}{-ae^{-at}\left(1+e^{-at}\right)^{-2}}$$

and also $1-u = \frac{1}{\left(1+e^{-at}\right)}$

$$\Rightarrow \left(1+e^{-at}\right) = \frac{1}{1-u}$$

$$\Rightarrow e^{-at} = \frac{1}{1-u} - 1 = \frac{u}{1-u}$$

$$E(t_{min}) = 2^N \int (u)^N . \frac{du}{-ae^{-at}\left(1+e^{-at}\right)^{-2}}$$

$$= \frac{2^N}{-a}\int u^N . \frac{(1-u)du}{u(1-u)^2}$$

$$= \frac{2^N}{-a}\int \frac{u^{N-1}}{1-u} . du$$

Since u = $1-\left(\frac{1}{1+e^{-at}}\right)$, and t takes values between zero and ∞,

1- u = $\frac{1}{1+e^{-at}}$ satisfies the inequality:

$\frac{1}{2} \le 1-u < 1$, using this result in $E(t_{min})$ leads to:-

$$E_1(t_{min}) = \frac{2^N}{-a}\int u^{N-1} du = \frac{-2^N}{a}\frac{u^N}{N}, \quad 1-u \to 1$$

$$E_2(t_{min}) = \frac{2^N}{-a}\int 2u^{N-1}du = \frac{2^{N+1}}{-a}\int u^{N-1}du = \frac{-2^{N+1}}{a}\frac{u^N}{N},\ 1-u \to \frac{1}{2}$$

Substitute u $= 1-\left(\frac{1}{1+e^{-at}}\right)$ in E_1 and E_2 leads to

$$E_1(t_{min}) = \frac{2^N}{-aN}\left(1-\frac{1}{1+e^{-at}}\right)^N\Bigg|_0^\infty = \frac{-2^N}{aN}\left((1-1)^N-\left(1-\frac{1}{2}\right)^N\right)$$

$$= \frac{-2^N}{aN}\left((0)^N-\left(\frac{1}{2}\right)^N\right) = \frac{2^N}{aN.}\frac{1}{2^N} = \frac{1}{aN}$$

$$E_2(t_{min}) = \frac{2^{N+1}}{-aN}\left(1-\frac{1}{1+e^{-at}}\right)^N\Bigg|_0^\infty = \frac{-2^{N+1}}{aN}\left((1-1)^N-\left(1-\frac{1}{2}\right)^N\right)$$

$$= \frac{-2^{N+1}}{aN}\left((0)^N-\left(\frac{1}{2}\right)^N\right) = \frac{2^{N+1}}{aN.}\frac{1}{2^N} = \frac{2}{aN}$$

$$\therefore \frac{1}{aN} < E(t_{\min}) < \frac{2}{aN}$$

To prove the convergence of the integral in $E(t_{min})$ we notice that:

1) The functions $\frac{u^{N-1}}{1-u}$, $2u^{N-1}$, are both continuous in any interval [a,b], ($u \neq 1$); hence they are integrable

2) $2u^{N-1} \geq \frac{u^{N-1}}{1-u} \Rightarrow 2u^{N-1}$ dominates $\frac{u^{N-1}}{1-u}$, and $2u^{N-1}$ is convergent $\Rightarrow \frac{u^{N-1}}{1-u}$ is convergent also in accordance of the Domination Test for convergence of improper integrals (Thomas-Finney, 1999, p.525).

To check the validity of these approximation formulas Table 2.1 in calculating $E(t_{min})$ for the logistic distribution function, we compared the graphs of the two bounding formulas E1 and E2 with the graph of the original formula, depending on the trapezoidal rule for evaluating the integral, see Figure 2.2; the comparison showed that the graph of the integral formula lies between the graphs of E1 and E2 as expected, and is almost identical with E2 (the upper bound).

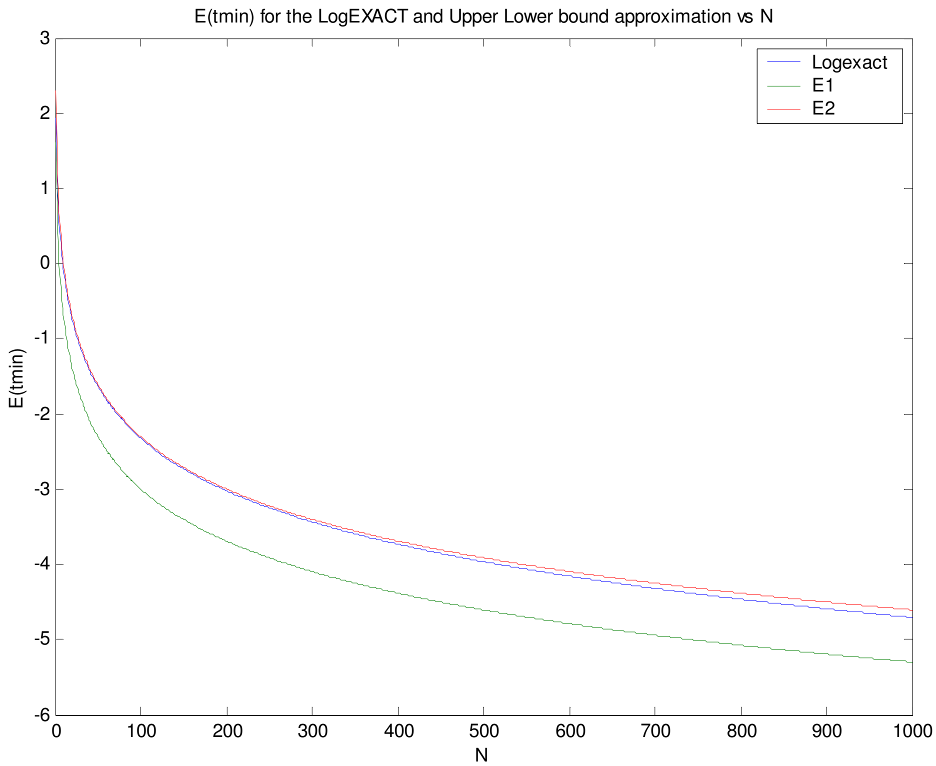


***Figure (2.2)**: Comparison: E(t_{min}) vs N*

To specify the parameters α and a, we give equal values to p(t) at t = 0, for the three distribution functions.

Taking T = 10,

For the uniform pdf:

$$p(0) = \frac{1}{T} = 0.1$$

For the decaying exponential pdf:

$$p(t) = \frac{1}{\alpha} e^{\frac{-t}{\alpha}}$$

$$p(0) = \frac{1}{\alpha} * 1 = 0.1 \rightarrow \alpha = 10$$

For the logistic pdf:

$$p(t) = \frac{2ae^{-at}}{(1+e^{-at})^2}$$

$$p(0) = \frac{2a*1}{(1+1)^2} = \frac{a}{2} = 0.1 \rightarrow a = 0.2$$

With these values of the parameters, we may compare time elapsed metric $E(t_{min})$ for the three distribution functions, as given in the Table 2.2 .

Table (2.1)*: $E(t_{min})$ for the three distribution functions*

Name of Distribution function	pdf	mean	$E(t_{min})$
Uniform	$p(t) = \frac{1}{T}$	$\frac{\alpha+\beta}{2} = \frac{T}{2}$	$\frac{T}{N+1}$
Exponential	$p(t) = \frac{1}{\alpha} e^{\frac{-t}{\alpha}}$	α	$\frac{\alpha}{N}$
Logistic	$p(t) = \frac{2ae^{-at}}{(1+e^{-at})^2}$	$\frac{2}{a}\ln 2$	$\frac{1}{aN} < E(t_{\min}) < \frac{2}{aN}$, $E(t_{\min}) \approx \frac{2}{aN}$

Table (2.2)*: $E(t_{min})$ for the three distribution functions (T=10)*

Distribution Function	$E(t_{min})$
Uniform	$= \frac{10}{N+1}$
Decaying exponential	$= \frac{10}{N}$
Logistic	$< \frac{2}{0.2N} \; or < \frac{10}{N}, \; (in \quad E_2(t_{\min}))$

From Table 2.1 and Table 2.2, we conclude:

1- For large values of N, time elapsed is inversely proportional to the number of processes N, and tends to zero where N is large enough (see Figure 2.3) .

2- Time elapsed is directly proportional to the mean for constant N in all the three distributions.

3- Comparing the values of $E(t_{min})$ for the three distributions:

   It is clear from the Table 2.2 that the logistic distribution function excels the other two distribution functions (which are almost equal) in terms of $E(t_{min})$, since the smaller the time elapsed is, the better.

Figure 2.4a also shows this result in comparison of the performance metric, time elapsed, of the three distributions, for different values of N (number of processes).

2.3 $E(t_{min})$: Comparison for The Three pdfs

Realistic values of the variables and parameters T, N, α, a were adopted in all the following graphs, as presented in Table 2.2 .

2.3.1 Variation of $E(t_{min})$ for the Logistic pdf vs N

To illustrate the relationship between $E(t_{min})$ and N, we draw the graph relating $E(t_{min})$ vs N for different values of N, where N ranges from 1 to 1000 processes and T = 10, see Figure 2.3 .

Using the log scale on the y axis for $E(t_{min})$ vs N, we find that $E(t_{min})$ decreases continuously with a slower rate with increase of N.

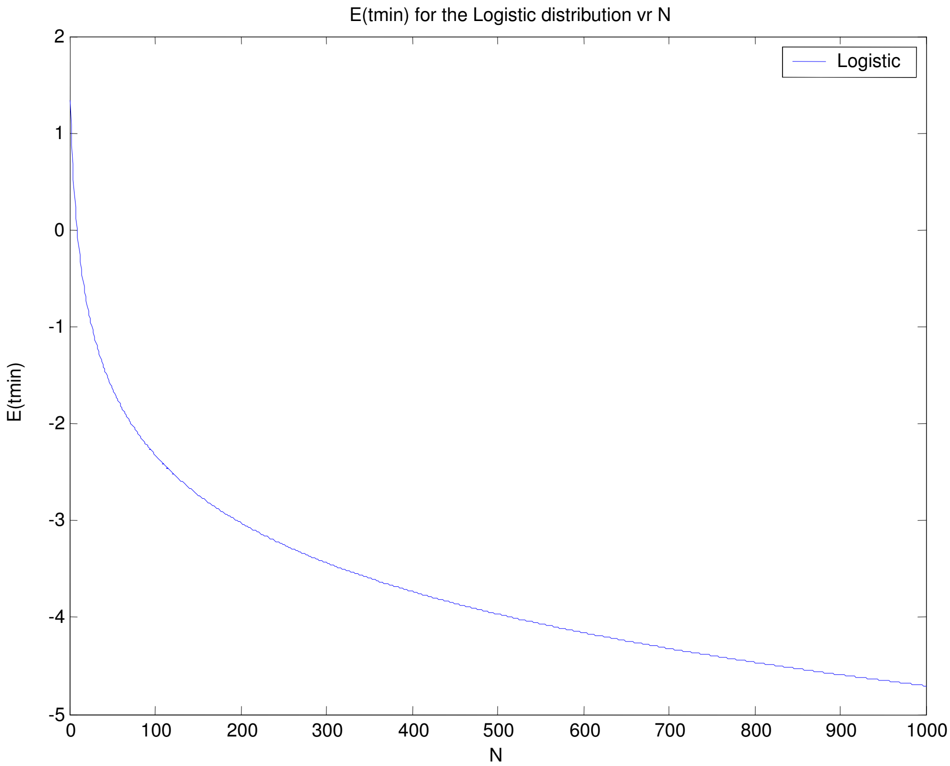


Figure (2.3): *$E(t_{min})$ vs N*

2.3.2 $E(t_{min})$ for the three pdfs vs N

To compare the relationships between $E(t_{min})$ and N, for the three distribution functions we draw the graphs Figures 2.4a,b relating $E(t_{min})$ vs N for different values of N, where N ranges from 1 to 1000 processes.

Using the log scale on the y-axis for $E(t_{min})$, we find from the graphs of the three distributions: the uniform, the exponential and the logistic as seen in Figure 2.4a that the logistic distribution function outperforms the two other distribution functions in general, this is markedly clear for large values of N as is shown in Figure 2.4b, while the uniform distribution function slightly outperforms the exponential distribution in general.

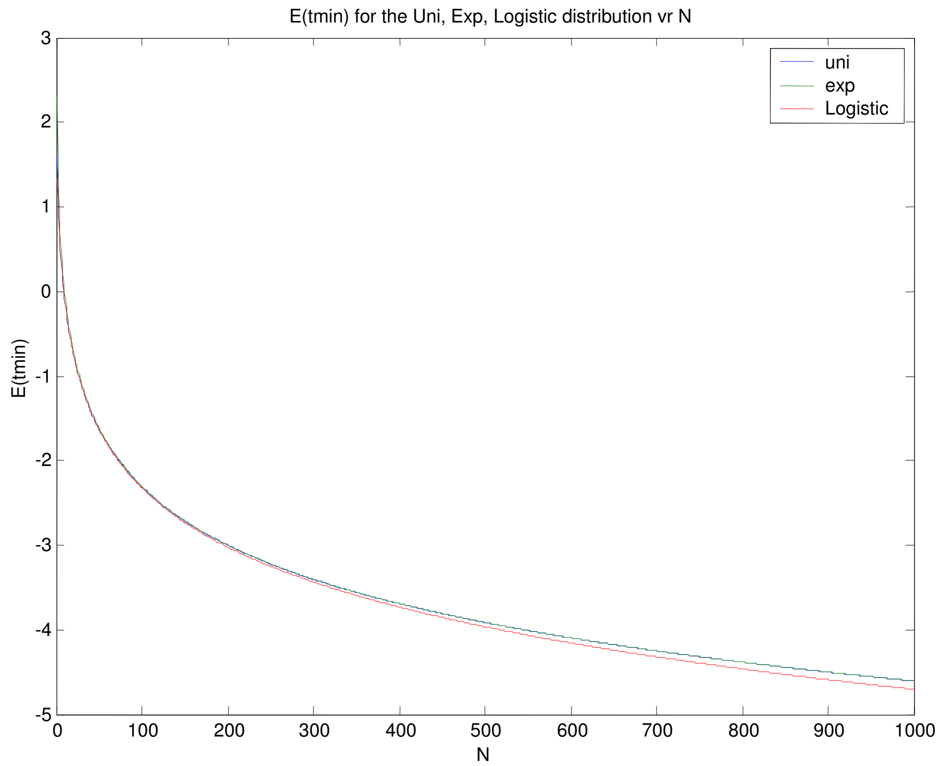


Figure (2.4a)*: Comparison: $E(t_{min})$ vs N*

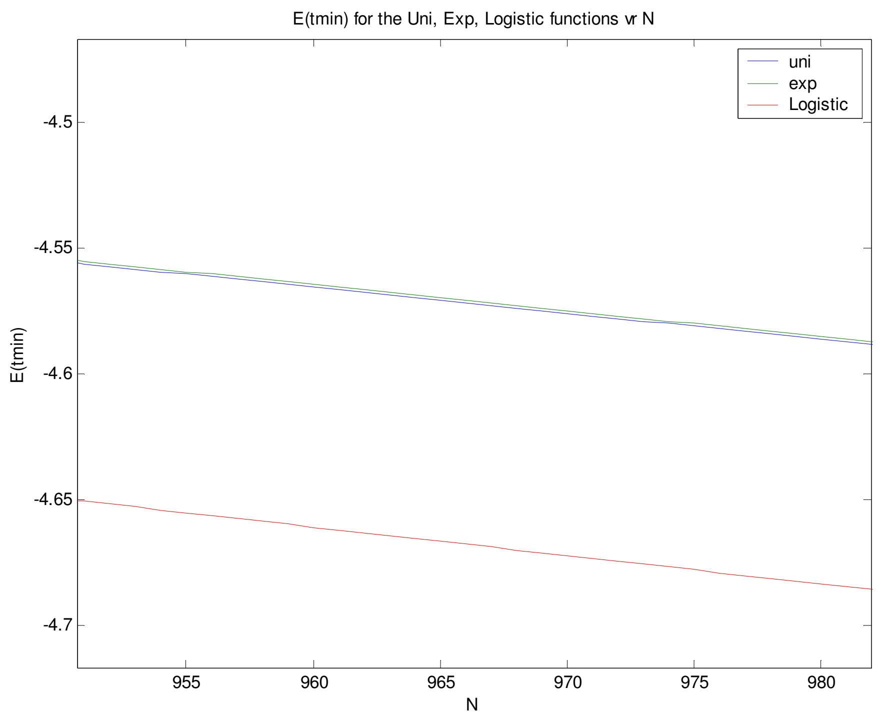


Figure (2.4b)*: Comparison: $E(t_{min})$ vs N, in large scale*

2.4 Extra Messages *in Non-lossy Networks*

2.4.1 Introduction

Extra messages represent the other performance metric with which we measure the efficiency and effectiveness of pdfs used in the suppression algorithm, in addition to time elapsed which we studied before.

In this section we will study extra messages in a lossless network with delay, where delay (Δ), the message transmission time, is greater than zero.

The reason for occurrence of extra messages is that the earliest message sent takes time (delay) to reach other processes; and hence they might send messages if their suppression times admit. Let the number of the participating processes be N (0,1,2 … N-1), each choosing its own time elapsed; these values of time elapsed may be arranged in ascending order (t_0, t_1, $t_{2,}$.. t_{N-1}); naming the processes in accordance with the time vector (t_0, t_1, $t_{2,}$.. t_{N-1}) such that process P_0 corresponds to t_0, P_1 corresponds to t_1 and so on. In Figure 2.5, let all processes begin together, each with its own suppression time elapsed, P_0 being the process with the least suppression time (t_{min}) = t_0, P_0 sends its message which reaches the other processes in time = $t_0 + \Delta$.

In Figure 2.5, Since the suppression time of P_k is less than $t_0 + \Delta$, P_k awakens and sends a message (i.e. extra message); on the other hand, the message sent by P_0 reaches P_i before it awakens and hence P_i suppresses its own message.

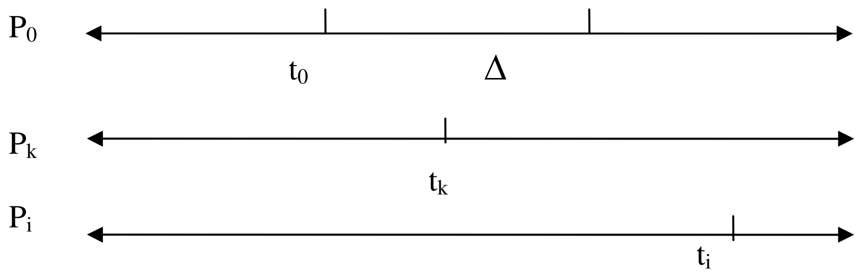


***Figure (2.5)**: Comparison: Extra messages and delay* (Δ)

The condition that a certain process k whose suppression time is t_k sends an extra message, any message beyond the earliest message whose suppression time is t_{min}, is:

$$t_{min} < t_k < t_{min} + \Delta$$

It is obvious that this relation is meaningful only if Δ is greater than zero.

2.4.2 Expected Value of the Number of Extra Messages

From the preceding inequality:

$$t_{min} < t_k < t_{min} + \Delta \qquad (2.6)$$

We may derive the following inequalities:

$$t_{min} < t_k \qquad (2.7)$$

$$t_k < t_{min}+\Delta \qquad (2.8)$$

$$t_k-\Delta < t_{min} \qquad (2.9)$$

From (2.7) and (2.9) we have:

$$t_k-\Delta < t_{min} < t_k \qquad (2.10)$$

Any process that may send an extra message must satisfy inequality (2.10); hence we may calculate the probability that t_{min} lies between t_k-Δ and t_k by determining the probability that each process $i \neq k$ chooses a time greater than t_k-Δ, and subtracting from it the probability that every process chooses a time greater than t_k.

$$\text{E[\#extra]} \quad = \sum_{0 \le k < N} \Pr[t_k - \Delta < t_{\min} < t_k]$$

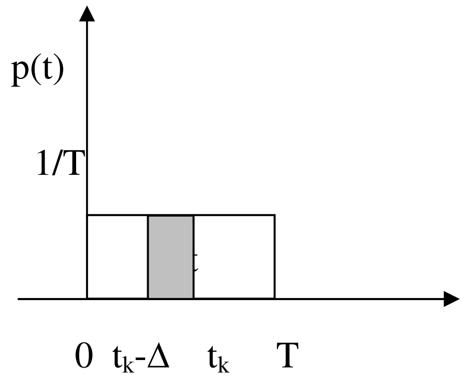


***Figure (2.6)**: E[#extra]*

To find E[#extra] we notice that a process x chooses a suppression value x ≥ t, with probability $(1 - P(t))$ and chooses a suppression value x ≥ t-Δ, with probability $(1 - P(t - \Delta))$.

For process x:

$$\text{E[\#extra]} \quad = \int_{\Delta}^{T} p(t)(1 - P(t - \Delta))^{N-1} dt - \int_{\Delta}^{T} p(t)(1 - P(t))^{N-1} dt$$

For N process:

$$\text{E[\#extra]} \quad = N\int_{\Delta}^{T} p(t)(1 - P(t - \Delta))^{N-1} dt - \int_{\Delta}^{T} Np(t)(1 - P(t))^{N-1} dt$$

$$= N\int_{\Delta}^{T} p(t)(1 - P(t - \Delta))^{N-1} dt - \int_{\Delta}^{T} d(1 - P(t))^{N}$$

$$= N\int_{\Delta}^{T} p(t)(1 - P(t - \Delta))^{N-1} dt - (1 - P(t))^{N}\Big|_{\Delta}^{T}$$

$$= N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt - \left((1-1)^N - (1-P(\Delta))^N\right)$$

$$= N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt - (1-P(\Delta))^N$$

using the approximation formula for small values of x

$$(1-x)^N \approx 1 - Nx$$

The result will be:

$$= N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt - (1-NP(\Delta))$$

$$\text{E[\#extra]} = N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt \qquad (2.11)$$

Where

N: number of processes,

Δ: message transmission delay,

T: the maximum amount of time a process waits before issuing a message,

P(Δ): the cumulative probability P(t) when t = Δ .

2.4.3 Extra Messages for the Uniform Distribution Function

For the Uniform distribution

$$p(t) = \frac{1}{T}$$

P(Δ) $= \frac{\Delta}{T}$

P(t-Δ) = $\frac{t-\Delta}{T}$

E(#extra) $= N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)(1 - P(t-\Delta))^{N-1}\,dt$

$$= N.\frac{\Delta}{T} - 1 + N\int_{\Delta}^{T} \frac{1}{T}\left(1 - \frac{t-\Delta}{T}\right)^{N-1} dt$$

$$= \frac{N\Delta}{T} - 1 + \frac{N}{T}\int_{\Delta}^{T} \left(1 - \frac{t-\Delta}{T}\right)^{N-1} dt$$

$$= \frac{N\Delta}{T} - 1 + \frac{N}{T}\int_{\Delta}^{T} \left(1 - \frac{t}{T} + \frac{\Delta}{T}\right)^{N-1} dt$$

let u $= \left(1 - \frac{t}{T} + \frac{\Delta}{T}\right) \Rightarrow \frac{du}{dt} = \left(-\frac{1}{T}\right) \Rightarrow dt = -Tdu$

E(#extra) $= \frac{N\Delta}{T} - 1 + \frac{N}{T}\int (u)^{N-1}(-T)du = \frac{N\Delta}{T} - 1 - N\frac{u^N}{N}$

Substituting in the original value of u will give

E(#extra) $= \frac{N\Delta}{T} - 1 - \frac{N}{N}\left(1 - \frac{t}{T} + \frac{\Delta}{T}\right)^N \Bigg|_{\Delta}^{T}$

$$= \frac{N\Delta}{T} - 1 - \left(\left(1 - 1 + \frac{\Delta}{T}\right)^N - \left(1 - \frac{\Delta}{T} + \frac{\Delta}{T}\right)^N\right)$$

$$= \frac{N\Delta}{T} - 1 - \left(\left(\frac{\Delta}{T}\right)^N - 1\right)$$

$$=\frac{N\Delta}{T}-1-\left(\frac{\Delta}{T}\right)^{N}+1$$

$$=\frac{N\Delta}{T}-\left(\frac{\Delta}{T}\right)^{N} \qquad (2.12)$$

2.4.4 Extra Messages for the Exponential Distribution Function

For the exponential distribution:-

$$\text{p(t)} \quad =\frac{e^{\frac{-t}{\alpha}}}{\alpha}$$

$$\text{P(t)} \quad = 1-e^{\frac{-t}{\alpha}}$$

$$\text{P}(\Delta) \quad = 1-e^{\frac{-\Delta}{\alpha}}$$

$$\text{E(\#extra)} \;=\; N.P(\Delta)-1+N\int_{\Delta}^{T} p(t)\big(1-P(t-\Delta)\big)^{N-1}dt$$

$$=\; N.(1-e^{\frac{-\Delta}{\alpha}})-1+N\int_{\Delta}^{\infty}\frac{e^{\frac{-t}{\alpha}}}{\alpha}\left(1-(1-e^{\frac{-(t-\Delta)}{\alpha}})\right)^{N-1}dt$$

Solving for the integration first

$$N\int_{\Delta}^{\infty}\frac{e^{\frac{-t}{\alpha}}}{\alpha}\left(1-1+e^{\frac{-(t-\Delta)}{\alpha}})\right)^{N-1}dt$$

$$N\int_{\Delta}^{\infty}\frac{e^{\frac{-t}{\alpha}}}{\alpha}\left(e^{\frac{-(t-\Delta)}{\alpha}}\right)^{N-1}dt$$

let $u=e^{\frac{-(t-\Delta)}{\alpha}} \Rightarrow \frac{du}{dt}=-\frac{1}{\alpha}e^{\frac{-(t-\Delta)}{\alpha}}$

$$dt = -\alpha e^{\frac{t}{\alpha}} e^{\frac{-\Delta}{\alpha}} du$$

substituting for dt

$$N\int \frac{e^{\frac{-t}{\alpha}}}{\alpha} (u)^{N-1} (-\alpha) e^{\frac{t}{\alpha}} e^{\frac{-\Delta}{\alpha}} du$$

$$Ne^{\frac{-\Delta}{\alpha}} \int (u)^{N-1} du = -Ne^{\frac{-\Delta}{\alpha}} \frac{u^N}{N} + c$$

substitute for u by $e^{\frac{-(t-\Delta)}{\alpha}}$ then the integration will be

$$= -\frac{N}{N} (e^{\frac{-\Delta}{\alpha}})(e^{\frac{-(t-\Delta)}{\alpha}})^N \Big|_{\Delta}^{\infty}$$

$$= -e^{\frac{-\Delta}{\alpha}} \left((e^{\frac{-\infty}{\alpha}})^N - (e^{\frac{-(\Delta-\Delta)}{\alpha}})^N \right) = -e^{\frac{-\Delta}{\alpha}} \left((0)^N - (e^0)^N \right)$$

$$= -e^{\frac{-\Delta}{\alpha}} (0-1)$$

$$= e^{\frac{-\Delta}{\alpha}}$$

substituting $e^{\frac{-(\Delta)}{\alpha}}$ for the integral in the formula for E(#extra) we get

$$\text{E(\#extra)} = N.(1 - e^{\frac{-\Delta}{\alpha}}) - 1 + N \int_{\Delta}^{\infty} \frac{e^{\frac{-t}{\alpha}}}{\alpha} \left(1 - (1 - e^{\frac{-(t-\Delta)}{\alpha}}) \right)^{N-1} dt$$

$$\text{E(\#extra)} = N - N e^{-\frac{\Delta}{\alpha}} - 1 + e^{-\frac{\Delta}{\alpha}}$$

$$= (N-1)(1 - e^{-\frac{\Delta}{\alpha}}) \tag{2.13}$$

2.4.5 Extra Messages for the Logistic Distribution Function

For the logistic distribution:-

$$\text{E(\#extra)} = N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt$$

Where

N: number of processes,

Δ: message transmission delay,

T: the maximum amount of time a process waits before issuing a message.

Substituting for the logistic distribution we have

$$\text{E(\#extra)} = N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)\left(1 - P(t-\Delta)\right)^{N-1} dt$$

$$= NP(\Delta) - 1 + N\int_{\Delta}^{\infty} \frac{2ae^{-at}}{\left(1+e^{-at}\right)^2}\left[1 - \left(\frac{2}{1+e^{-a(t-\Delta)}}\right) - 1\right]^{N-1} dt \qquad (2.8)$$

2.4.6 Graphical Representation

Realistic values for the variables and parameters T, N, α, and a were adopted in all the following graphs as presented in Table 2.2 .

For the time delay Δ, we used values ranging from 0.1 *msec* - 10 *sec*.

2.4.6.1 Variation of E(#extra) for the Logistic Distribution Function versus Δ

To illustrate the relationship between E(#extra) and Δ, with a = 0.2, we calculate the E(#extra) messages for certain values of N, giving Δ values ranging from (0 – 1), we use the Trapezoidal rule to find the value of integration, given in the general formula for extra messages.

Form Figure 2.7 (small values of N) and Figure 2.8 (large values of N), we find that E(#extra) increases with increase of Δ and also it increases with increase of N. This result is expected since increase of Δ allows more processes to release messages which are extra messages because they do not suppress other processes, suppression is caused only by the first sent message.

The first message arrives at any other process before other (extra) messages because the time it takes is $t_0 + \Delta$, where

$$t_0 < t_i \quad , i > 0$$

and hence it causes suppression of the other process.

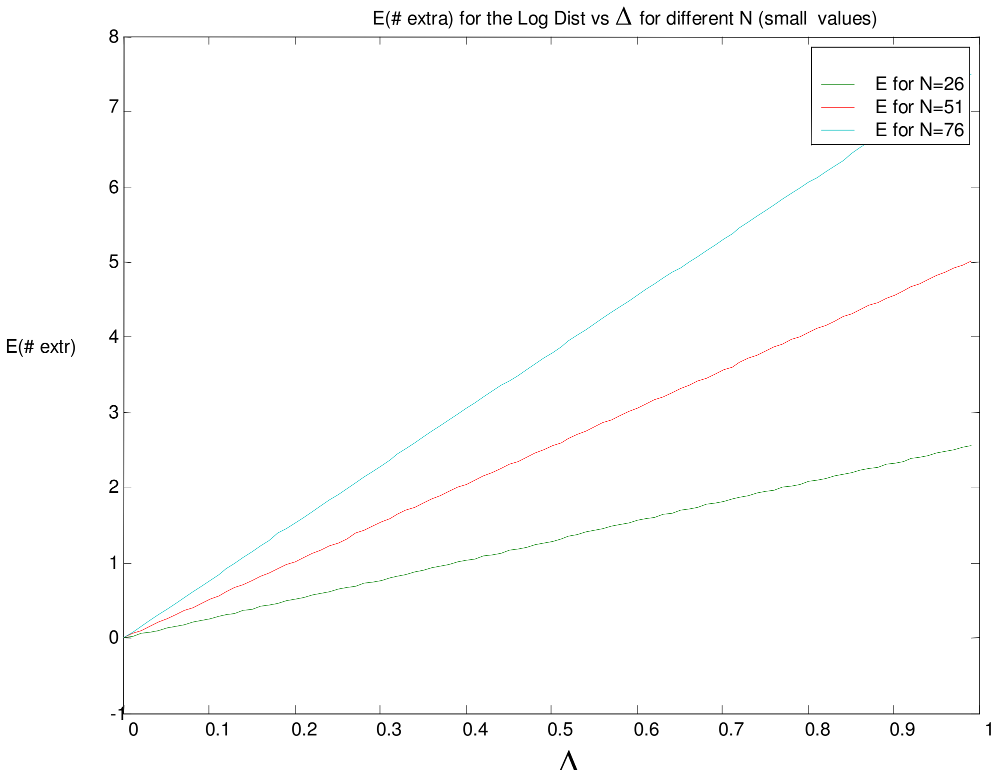


***Figure (2.7)**: E(#extra) vs Δ*

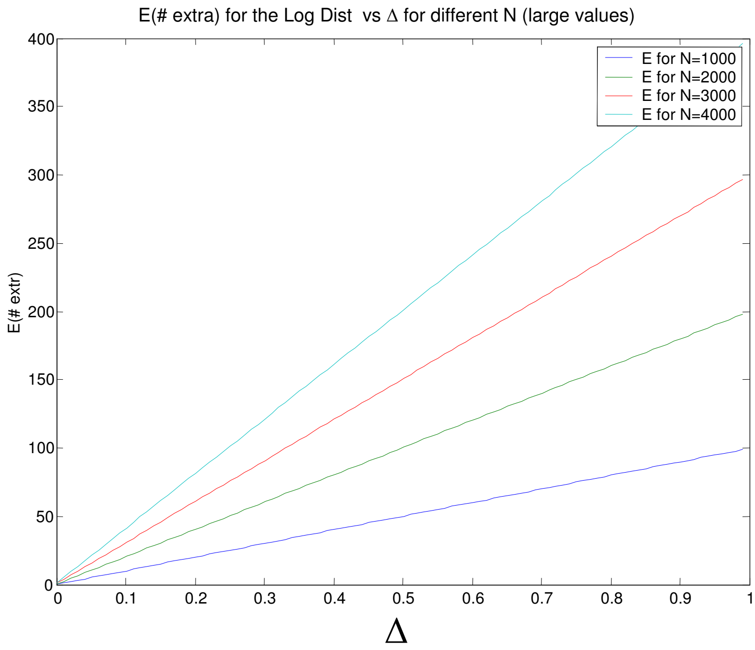


Figure (2.8)*: E(# extra) vs* Δ

2.4.6.2 Variation of E(#extra) for the Logistic Distribution Function versus N

To illustrate the relationship between E(#extra) and N for the logistic distribution function, we draw the graph relating E(#extra) versus N, where N ranges from (0-200). From Figures 2.9a, b we notice that, E(#extra) for the logistic distribution increases with increase of N for any value of Δ, and it also increases with increase of values of Δ.

This result is expected since increase of N (for the same value of Δ) allows more processes to release extra messages.

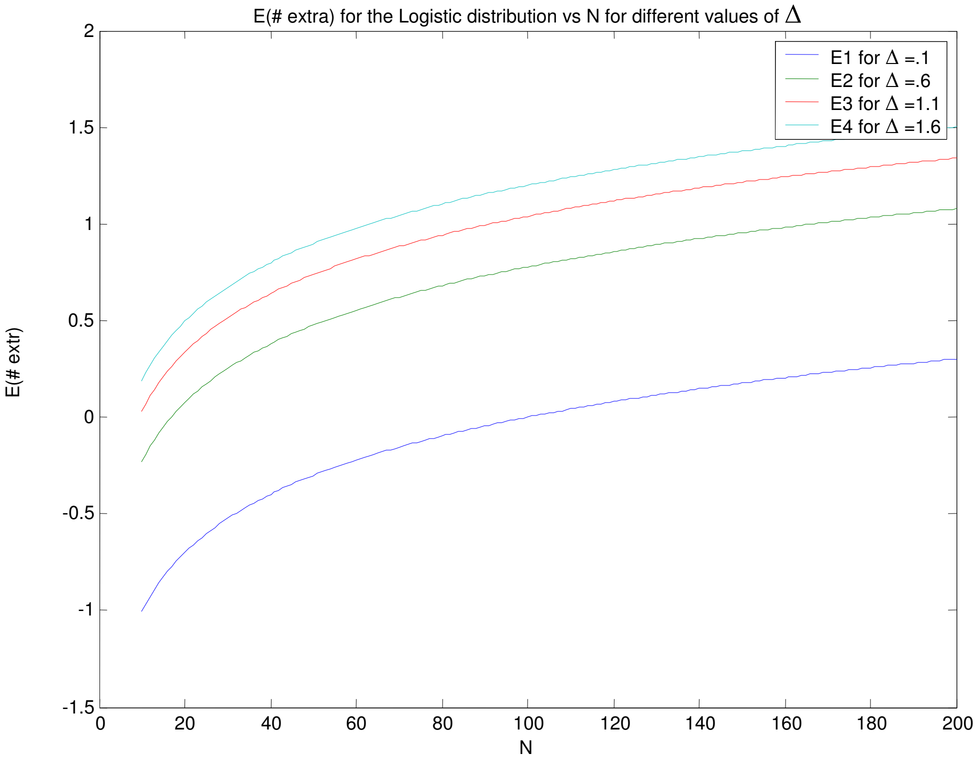


***Figure (2.9a)**: E(# extra) vs N*

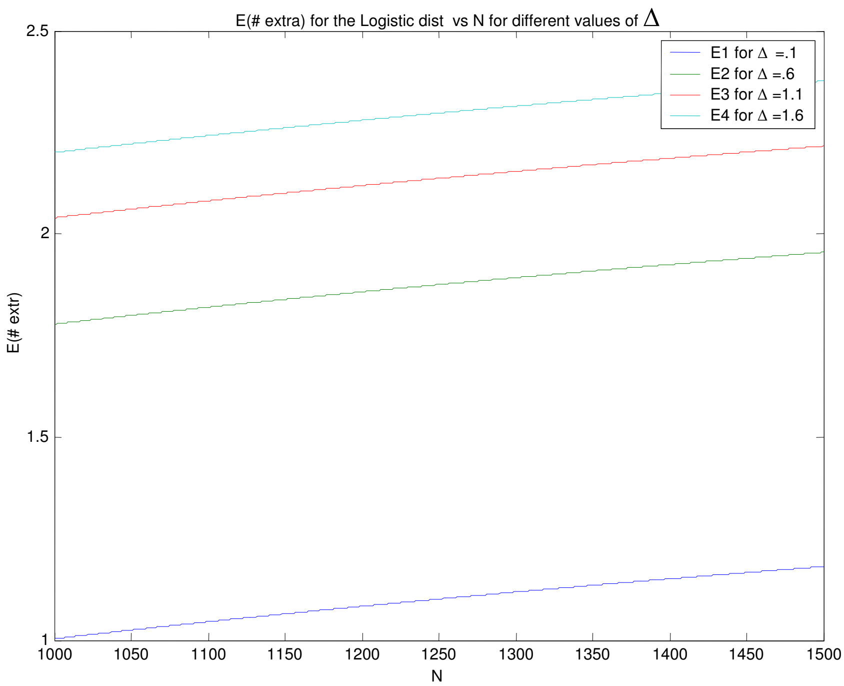


***Figure (2.9b)**: E(#extra) vs* N

2.4.6.3 E(#extra) for the Uniform, Exponential, and Logistic Distributions versus Δ

To compute the relationships between E(#extra) and Δ, for the three distribution functions, we draw the graphs relating E(#extra) versus Δ for different values of Δ, where Δ ranges from (0.1 – 1.0 *sec)*, and N = 100. From Figure 2.10 we notice that the logistic distribution function outperforms the other two distributions for all values of Δ up to delta ≈0.85sec.

By comparing Figure 2.10 with N = 100 and Figure 2.11a with N = 500, we notice that the logistic distribution function outperformed the other two pdfs in the first case for values of Δ (0.1- 0.85 sec) and in the second case for values of Δ(0.1–0.35), which suggests that the logistic distribution outperforms the other two pdfs in the extra messages metric for smaller values of Δ as N increases.

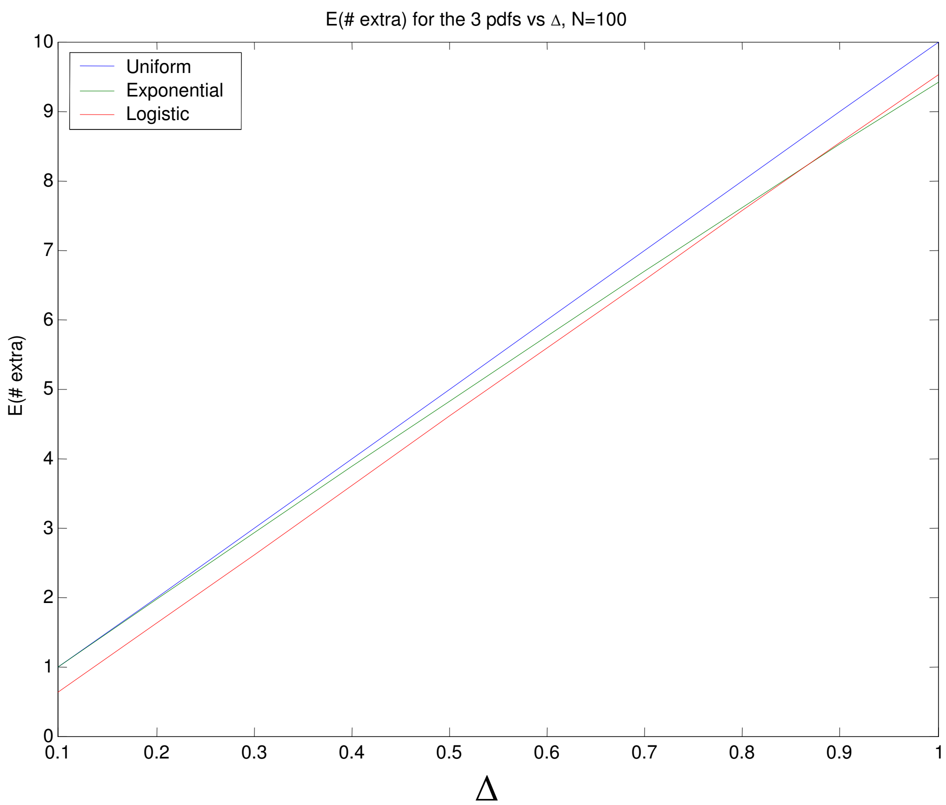


***Figure (2.10)**: E(# extra) vs Δ*

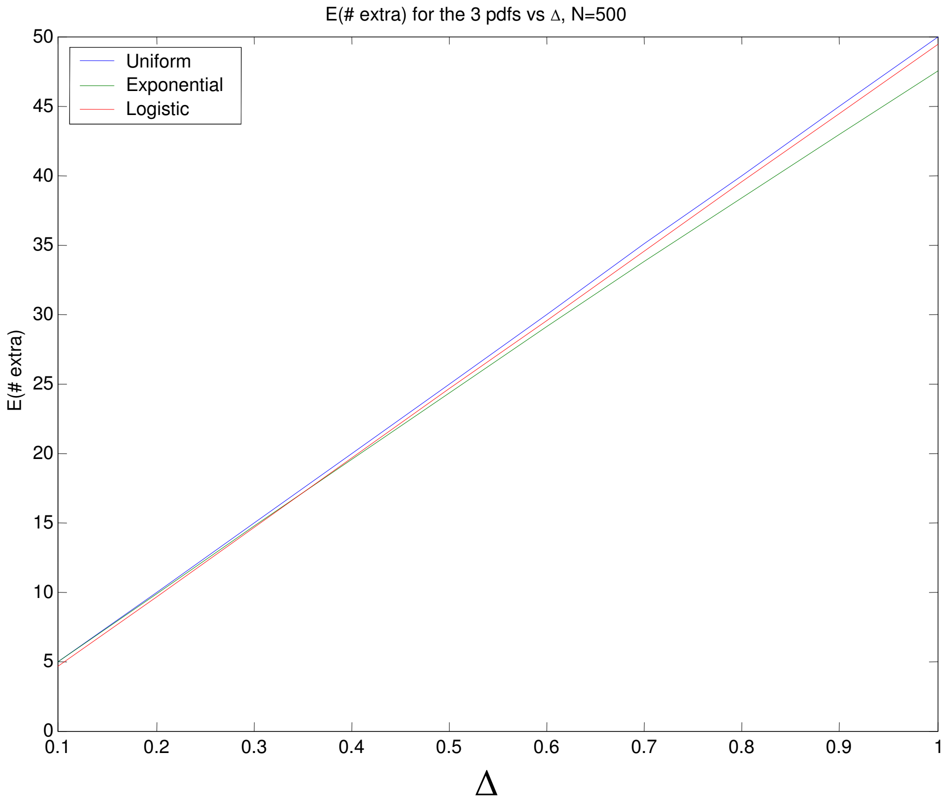
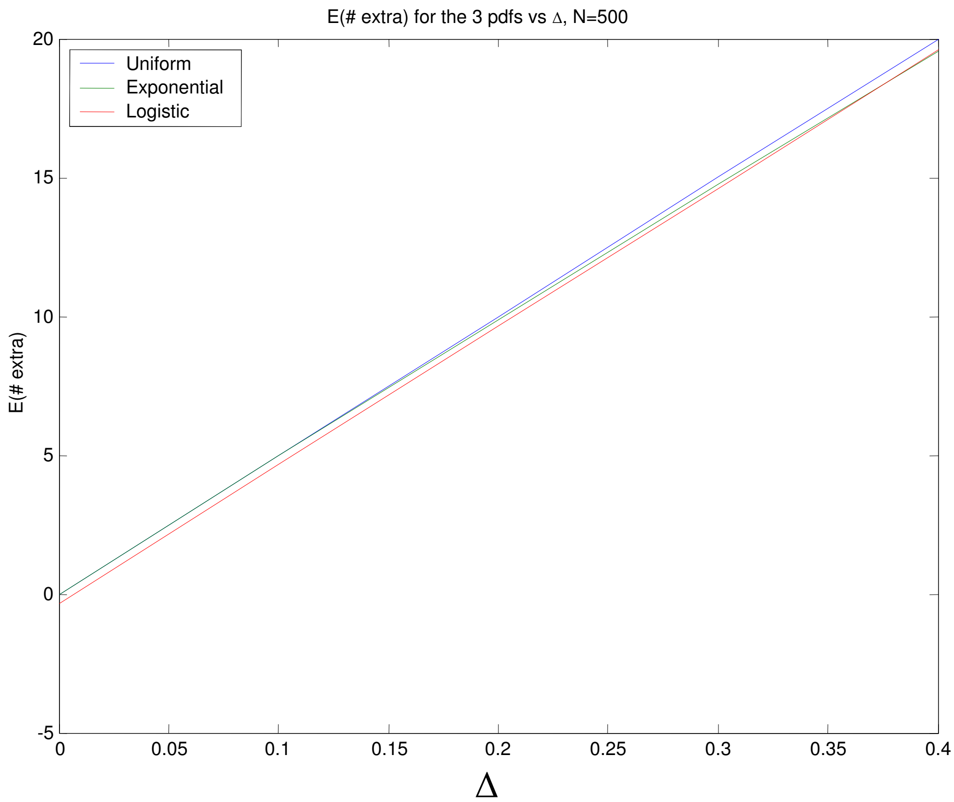


***Figure (2.11a)**: E(# extra) vs Δ*



***Figure(2.11b)**: E(#extra) vs Δ*

2.4.6.4 E(#extra) for the Uniform, Exponential and Logistic Distributions versus N

To compare the relationships between E(#extra) and N, for the three distributions, we draw the graphs relating E(#extra) versus N for different values of N, where N ranges from 1-1000, and Δ = 0.1, 0.3, 0.5, 0.9, as shown in Figures 2.12a,b,c,d,e,f .

The logistic distribution outperforms the uniform distribution for all values of N and Δ, it also outperforms the exponential distribution for values of N ranging from 1 to N = x, where x decreases with increase of the value of Δ.

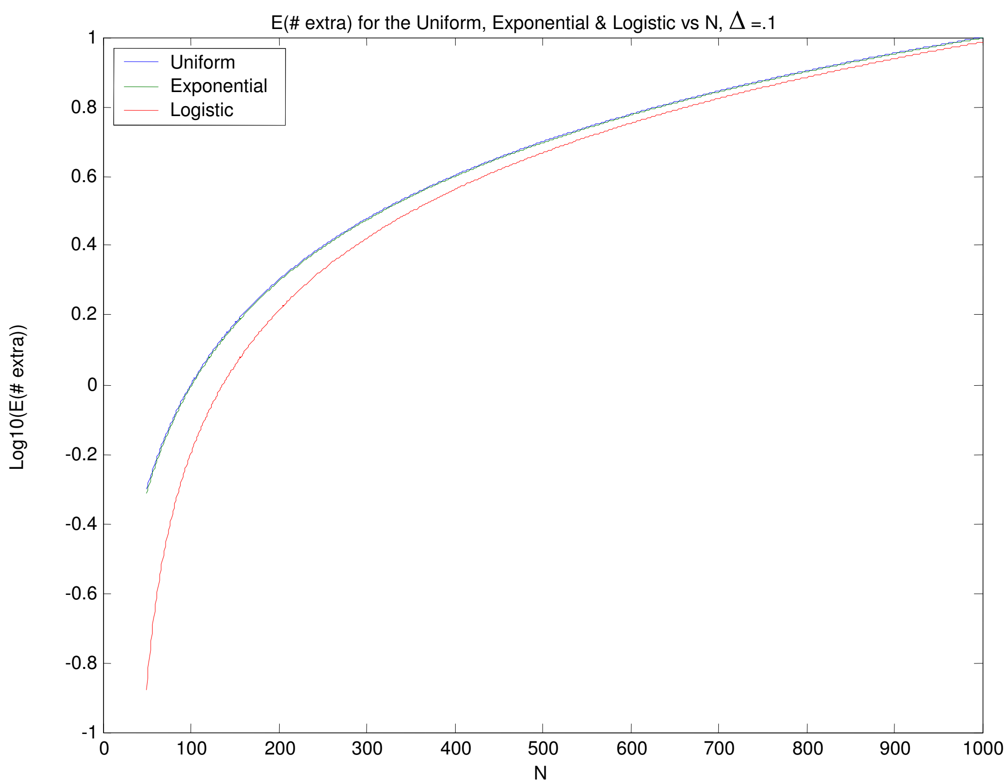


***Figure (2.12a)**: Comparison: E(# extra) vs N*

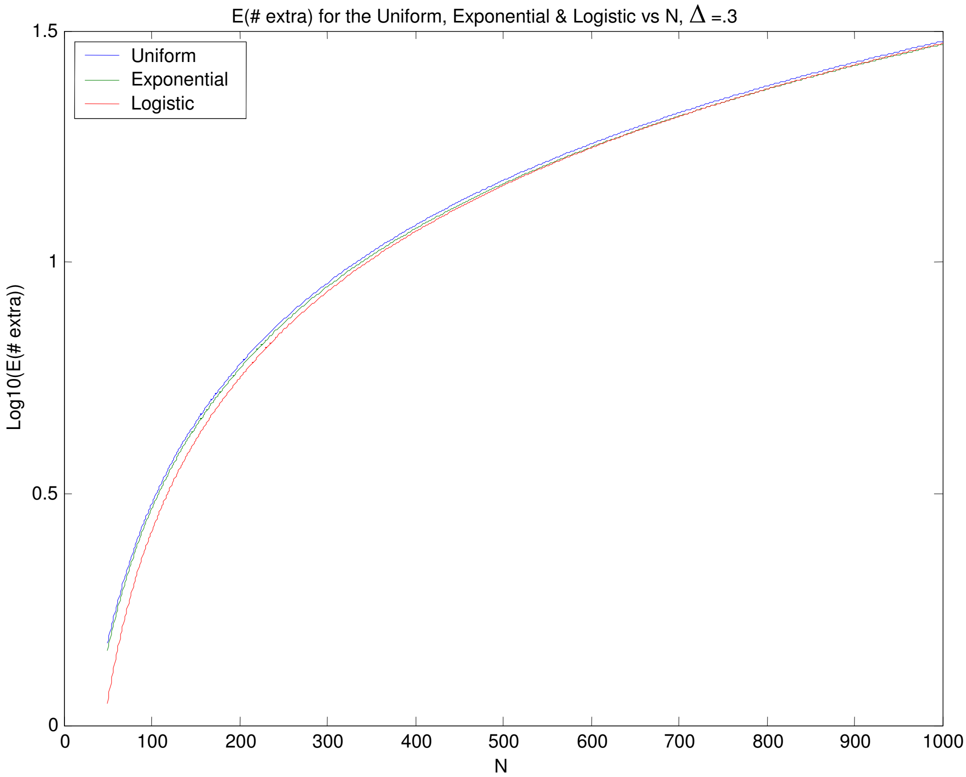


***Figure (2.12b)**: Comparison: E(# extra) vs N*

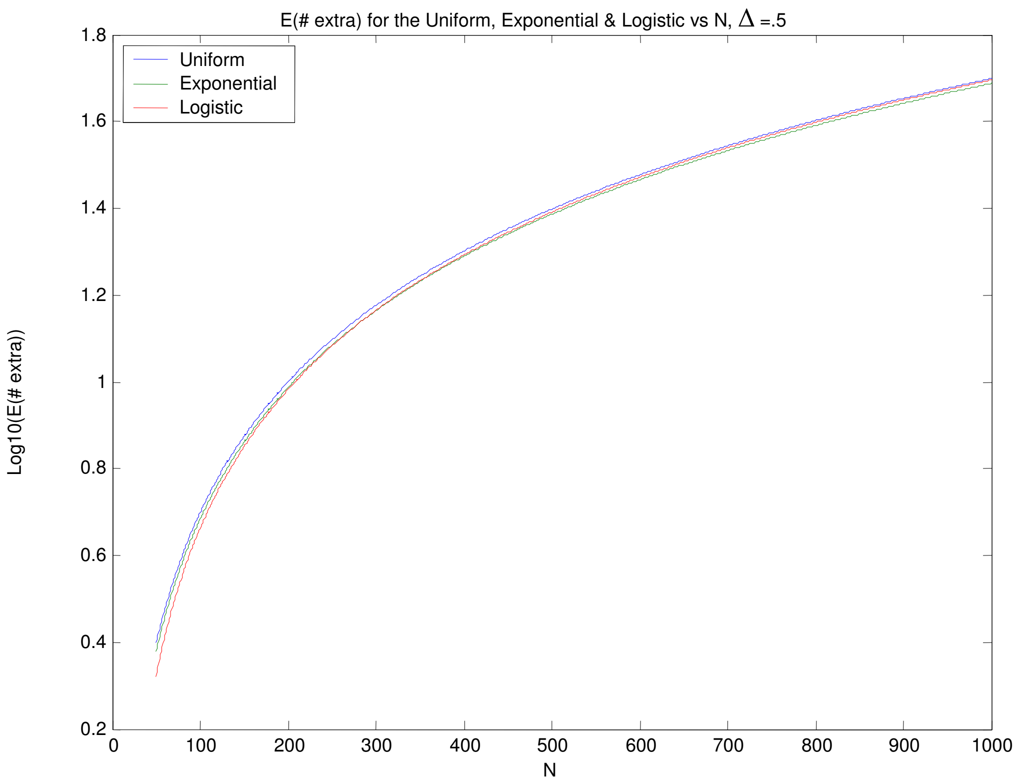


***Figure (2.12c)**: Comparison: E(# extra) vs N*

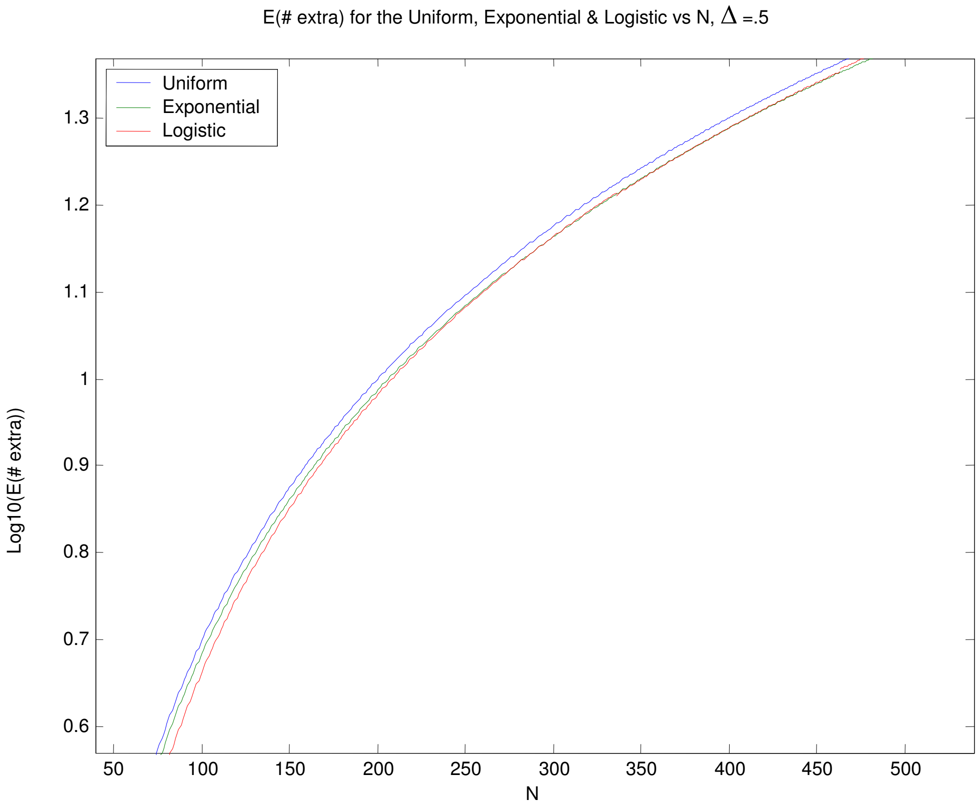


***Figure (2.12d)**: Comparison: E(# extra) vs N*

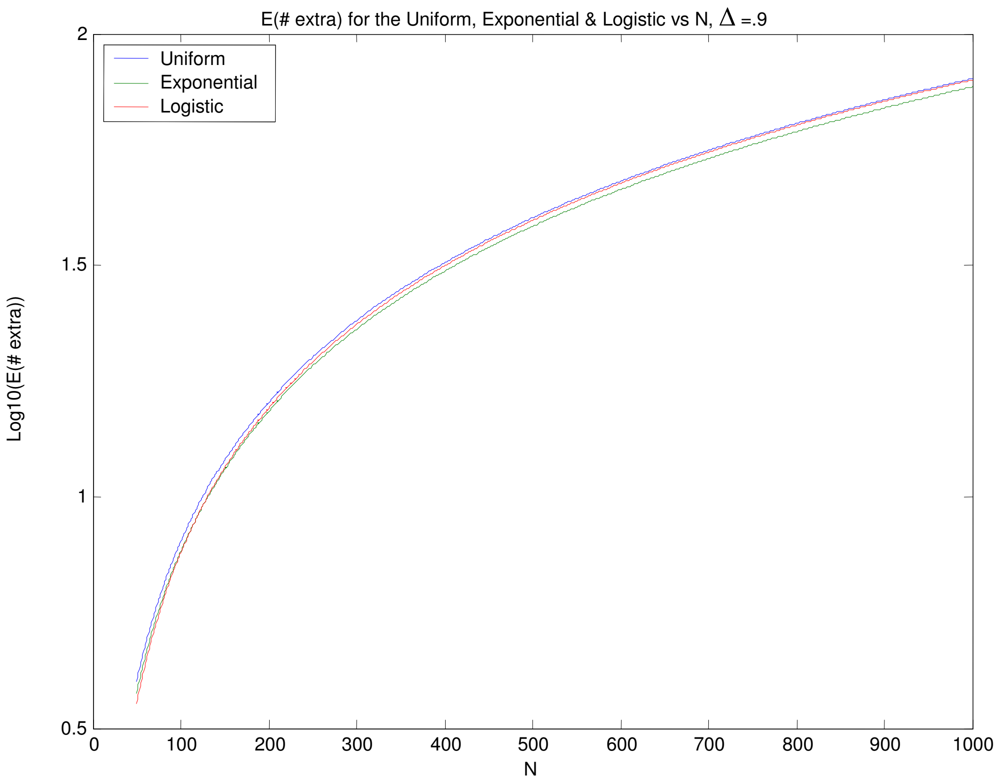


***Figure (2.12e)**: Comparison: E(# extra) vs N*

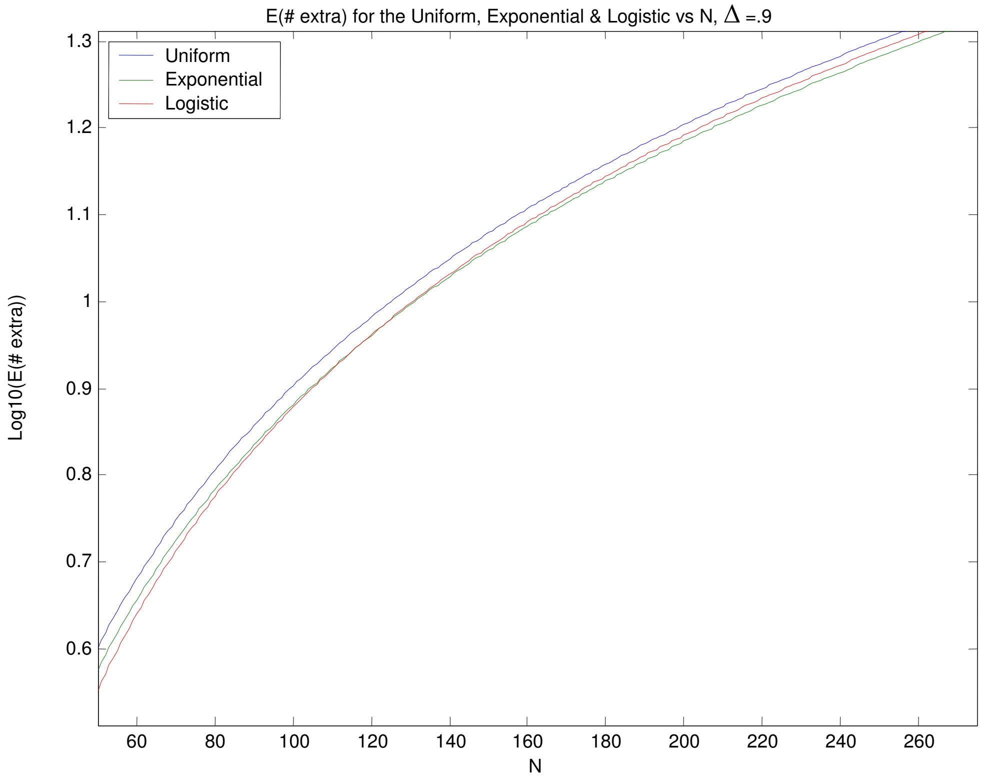


***Figure (2.12f)**: Comparison: E(# extra) vs N*

2.5 Conclusions

In this chapter we studied two performance metrics, time elapsed and extra messages, in the non lossy case which is the ideal case when no loss of messages occurs, and could be used when the probability of loss is small enough.

2.5.1 Time Elapsed

The metric used here to measure the time elapsed is $E(t_{min})$, which is the expected value of t_{min} which is the minimum elapsed time before one of the processes wakes up and sends a message.

It was found, from studying the relationship between $E(t_{min})$ and N(# of processes) for the logistic distribution, that $E(t_{min})$ decreases with increase of N, sharply at first, but slowly for large values of N, which means that when the number of participators of the group is small, the addition or the departure of a few number of participators greatly affects $E(t_{min})$; but it is of little significance if N is large.

$E(t_{min})$ was found to decrease with increase of N, for the three distributions, the uniform, exponential and the logistic distribution, with the logistic distribution outperforming the other two distributions in general, this out performance becomes markedly significant for large values of N.

2.5.2 Extra Messages

Extra messages are produced because of the existence of Δ (the time delay), i.e. the time taken by the earliest message to reach the other processes causing them to send unnecessary messages (Extra Messages). The extra messages metric is measured by E(# extra).

For the logistic distribution, E(# extra) was found to increase as Δ increases for a certain value of N; also for the same value of Δ, E(# extra) increases as N increases.

Number of extra messages, increases with increase of N, for all three distributions. Number of extra messages also increases with increase of Δ, for all three distributions.

In comparing the performance of the three pdfs regarding extra messages, the logistic distribution outperforms the uniform distribution function for all N and Δ, it also outperforms the exponential distribution for values of N in the interval [1, x], where x decreases with increase of the value of Δ.

Chapter Three

Suppression With Loss

3.1 Introduction

Loss in multicast networking denotes the case when a message sent by a sender, is not received by one or more of the potential destinations. It is worth noting that loss occurs in the majority of cases, though with differing degrees.

In lossy multicasting, we differentiate between two types of loss: correlated loss i.e. the loss occurs near the sender, which affects many receivers, and uncorrelated loss which occurs close to the receivers, whose effect differs with the location of the concerned receivers in the network. (see Figure 3.1). In fact, we limit our study to the two extreme cases: fully correlated loss (closest to the sender) and fully uncorrelated loss (closest to the receivers), though in real – life situation, most losses lie in between the two extreme cases (Schooler, 2001, p.40).

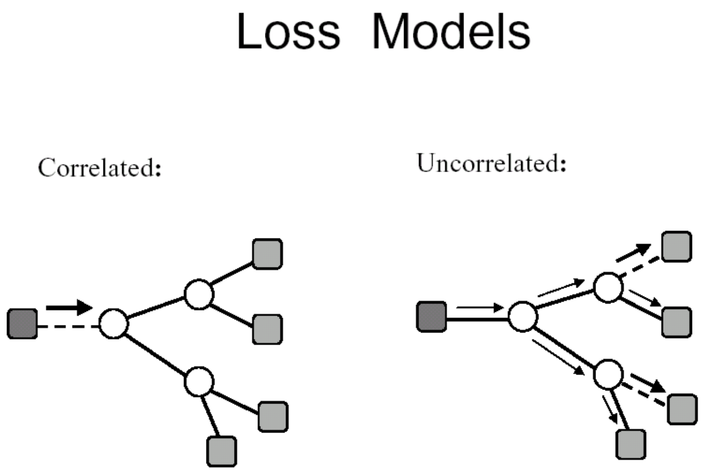


***Figure (3.1)**: Correlated & Uncorrelated Loss*

In assessing the suppression technique, in the lossy case, new performance metrics are introduced, in addition to those discussed in the non-lossy case:

1) Time Elapsed:

   a) $E(t_{min\ k})$ the expected value of the k^{th} smallest time.

   b) $E(t_{min\ e})$ or effective $E(t_{min})$.

   c) $E(t_{max})$ the maximum time elapsed.

2) Extra Messages:

   a) E(# messages), expected number of messages generated.

   b) E(# required), expected number of messages required.

   c) E(# extra), expected number of extra messages with loss.

These performance metrics will be introduced to answer questions like:- How much delay is incurred when N processes participate? How much messaging overhead is generated when the loss probability is l? How long before all processes have completed the algorithm?

These performance metrics will be studied, and compared for different pdfs in the following sections.

3.2 Time Elapsed with Loss

In the non-lossy case it was sufficient to use the expected time when the first message is sent, namely $E(t_{min})$; but in the lossy case the first message may be lost, and hence we may depend on two new time performance metrics:

a- Effective $E(t_{min}) = E(t_{min\ e})$

b- $E(t_{max})$.

In studying these two time metrics, we have to study first the metric $E(t_{min\ k})$.

3.2.1 $E(t_{min\ k})$ Time Metric

$E(t_{min\ k})$, the expected value of the k^{th} smallest time, is particularly important in calculating the values of $E(t_{min\ e})$ and $E(t_{max})$, the two main time performance metrics in the lossy case.

For time t_i to be the k^{th} smallest time, there must be k processes of shorter times and (N-1-k) processes of longer time than it. This event has the probability

$$\binom{N-1}{k}(P(t_i))^k(1-P(t_i))^{N-1-k}$$

where N = # of processes

i = index # of process

k= # of elapsed times less than elapsed time of i'th process

For calculating the probability for a particular time t_i to be the k^{th} smallest time, we notice that the number of groups each of which has k elements, taken from (N-1) elements = $\binom{N-1}{k}$; since the i^{th} element is not included.

For a specific group of k elements with time less than t_i, we have also a specific group of (N-k-1) elements with time greater than t_i.

These two specific groups have probability of happening

$$= (P(t_i))^k(1-P(t_i))^{N-1-k}$$

Therefore the probability for all the groups

$$= \binom{N-1}{k} (P(t_i))^k (1 - P(t_i))^{N-1-k} .$$

In general, expectation will have the formula

$$\mathrm{E(t)} = \int tp(t)dt$$

Noting that for $E(t_{\min k})$ depending on process (i), t will be

$$\mathrm{t} = t_i \binom{N-1}{k} (P(t_i))^k (1 - P(t_i))^{N-1-k}$$

since we have N processes we multiply by N

$$E(t_{\min k}) = \int_0^T Ntp(t) \binom{N-1}{k} (P(t))^k (1 - P(t))^{N-1-k} dt \qquad (3.1)$$

3.2.1.1 $E(t_{\min k})$ with the Distributions

$E(t_{\min k})$, the expected value of the k^{th} smallest time, depends on N (# of processes) and k, which takes the values from 1 to N-1, $E(t_{\min k})$ is calculated by formula (3.1) .

3.2.1.1.1 $E(t_{\min k})$ for the Uniform Distribution

$$E(t_{\min k}) = \int_0^T Ntp(t) \binom{N-1}{k} (P(t))^k (1 - P(t))^{N-1-k} dt$$

$$p(t) = \frac{1}{T}, \; P(t) = \frac{t}{T}$$

$$E(t_{\min k}) = \int_0^T N \binom{N-1}{k} t.\frac{1}{T} (\frac{t}{T})^k (1 - (\frac{t}{T}))^{N-1-k} dt$$

$$= \frac{(N-1)!N}{k!(N-1-k)!}\int_0^T (\frac{t}{T})^{k+1}(1-\frac{t}{T})^{N-1-k}\,dt$$

$$= \frac{(N)!}{k!(N-1-k)!}\int_0^T (\frac{t}{T})^{k+1}(1-\frac{t}{T})^{N-1-k}\,dt \qquad (3.2)$$

Solving for the integration

$$\mathrm{I} = \int_0^T (\frac{t}{T})^{k+1}(1-\frac{t}{T})^{N-1-k}\,dt$$

Let $x = \frac{t}{T} \Rightarrow \frac{dx}{dt} = \frac{1}{T} \Rightarrow dt = Tdx$

$$\mathrm{I} = T\int x^{k+1}(1-x)^{N-1-k}\,dx$$

Putting

$$x = \sin^2 y \Rightarrow 1 - x = \cos^2 y$$

$$\frac{dx}{dy} = 2\sin y\cos y \Rightarrow dx = 2.\sin y.\cos y.dy$$

$$\mathrm{I} \quad = T\int (\sin^2 y)^{k+1}(\cos^2 y)^{N-1-k}\,2\sin y\cos y.dy$$

$$= 2T\int (\sin y)^{2k+3}(\cos y)^{2N-1-2k}\,dy$$

calculating the new integration boundaries

$\mathrm{x} = \frac{t}{T} \qquad \Rightarrow \ \mathrm{t} = \mathrm{Tx}$

$\mathrm{t} = 0, \mathrm{T} \qquad \Rightarrow \ \mathrm{x} = 0, 1$

x = $\sin^2 y$

x = 0 => y = 0

x = 1 => y = $\frac{\pi}{2}$

$$I = 2T\int_0^{\frac{\pi}{2}} (\sin y)^{2k+3} (\cos y)^{2N-1-2k} dy$$

Using the formula followed to compute the integration

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x.dx = \frac{(m-1)(m-3)...(2)(n-1)(n-3)...(2)}{(m+n)(m+n-2)...(2)}$$ (Gillespie, (1959), p. 22)

m and n should be odd numbers.

Using this method we have the value of the integration to be

$$=(2T)\frac{(2k+3-1)(2k+3-3)...(2)(2N-2k-1-1)(2N-2k-1-3)...(2)}{((2k+3)+(2N-2k-1))((2k+3)+(2N-2k-1)-2)...(2)}$$

$$=(2T)\frac{(2k+3-1)(2k+3-3)...(2)(2N-2k-1-1)(2N-2k-1-3)...(2)}{((2N+2)(2N)(2N-2)...(2)}$$

$$=(2T)\frac{(2^{k+1})(k+1)!(2^{N-k-1})(N-k-1)!}{(2^{N+1})(N+1)!}$$

$$=(2T)\frac{(2^{N-k-1)+(k+1)-(N+1)})(k+1)!(N-k-1)!}{(N+1)!}$$

$$=(2T)\frac{2^{-1}(k+1)!(N-k-1)!}{(N+1)!} \qquad (3.3)$$

from (3.2), (3.3)

$$E(t_{\min k}) = \frac{(N)!}{k!(N-1-k)!}.(T)\frac{(k+1)!(N-k-1)!}{(N+1)!}$$

$$= \frac{(N)!}{k!(N-1-k)!}.(T)\frac{(k+1)(k)!(N-1-k)!}{(N+1)(N)!}$$

$$= \frac{(T)(k+1)}{(N+1)} \qquad (3.4)$$

3.2.1.1.2 E($t_{\min k}$) for the Exponential Distribution

For the exponential distribution

$$p(t) = \frac{1}{\alpha}e^{\frac{-t}{\alpha}},$$

$$P(t) = 1-e^{\frac{-t}{\alpha}},$$

Substituting these values of p(t), and P(t) into the general formula of E($t_{\min k}$):

$$E(t_{\min k}) = \int_0^\infty Ntp(t)\binom{N-1}{k}(P(t))^k(1-P(t))^{N-1-k}\,dt$$

we have

$$E(t_{\min k}) = \int_0^\infty Nt(\frac{1}{\alpha}e^{-\frac{t}{\alpha}})\binom{N-1}{k}(1-e^{-\frac{t}{\alpha}})^k(1-(1-e^{-\frac{t}{\alpha}}))^{N-1-k}\,dt$$

$$= \int_0^\infty (\frac{Nt}{\alpha})\binom{N-1}{k}(1-e^{-\frac{t}{\alpha}})^k e^{-\frac{t(N-k)}{\alpha}}\,dt \qquad (3.5)$$

3.2.1.1.3 E($t_{\min k}$) for the Logistic Distribution

For the logistic distribution

$$p(t) = \frac{2ae^{-at}}{(1+e^{-at})^2},$$

$$P(t) = \frac{2}{1+e^{-at}} - 1,$$

$$(1 - P(t)) = \frac{2e^{-at}}{1+e^{-at}},$$

Substituting these values of p(t), and P(t) into the general formula of $E(t_{\min k})$,

$$E(t_{\min k}) = \int_0^\infty Ntp(t)\binom{N-1}{k}(P(t))^k (1-P(t))^{N-1-k}\,dt$$

we have

$$E(t_{\min k}) = \int_0^\infty Nt\frac{2ae^{-at}}{(1+e^{-at})^2}\binom{N-1}{k}\left(\frac{2}{1+e^{-at}}-1\right)^k \left(1-\left(\frac{2}{1+e^{-at}}-1\right)\right)^{N-1-k} dt$$

$$= \int_0^\infty \frac{aNt}{1+e^{-at}}\binom{N-1}{k}\left(\frac{2}{1+e^{-at}}-1\right)^k \left(\frac{2e^{-at}}{1+e^{-at}}\right)^{N-k} dt \qquad (3.6)$$

Figure 3.2 shows the relationship between $E(t_{\min k})$ and k for N=10 (# of processes), k takes the values (1, 2… N-1 = 9).

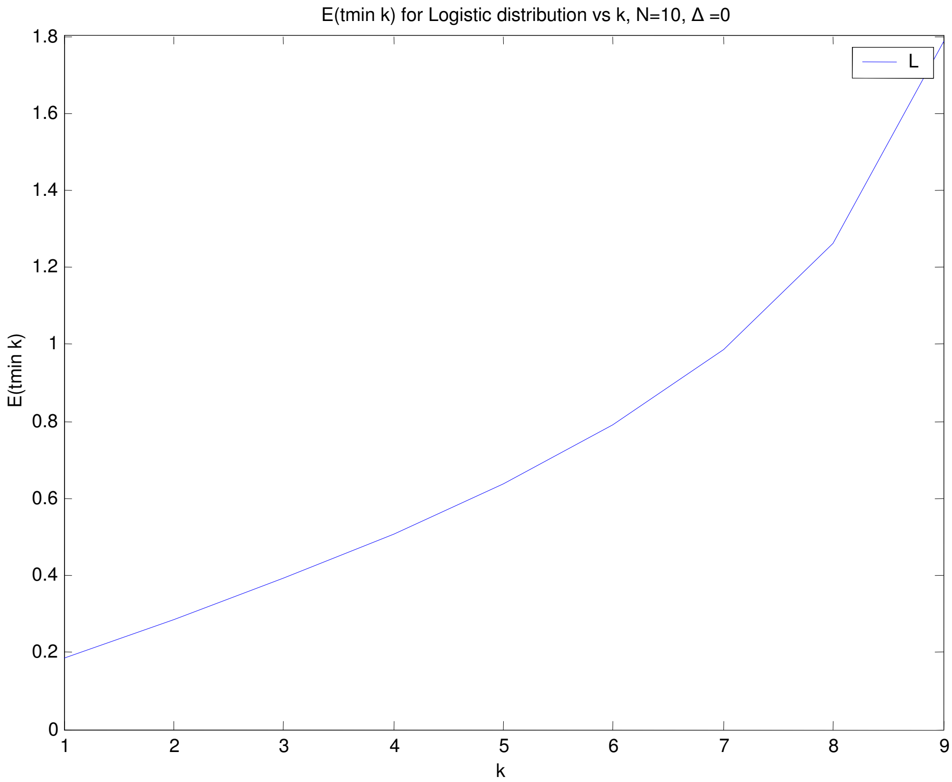


Figure (3.2): *$E(t_{min\ k})$ vs k*

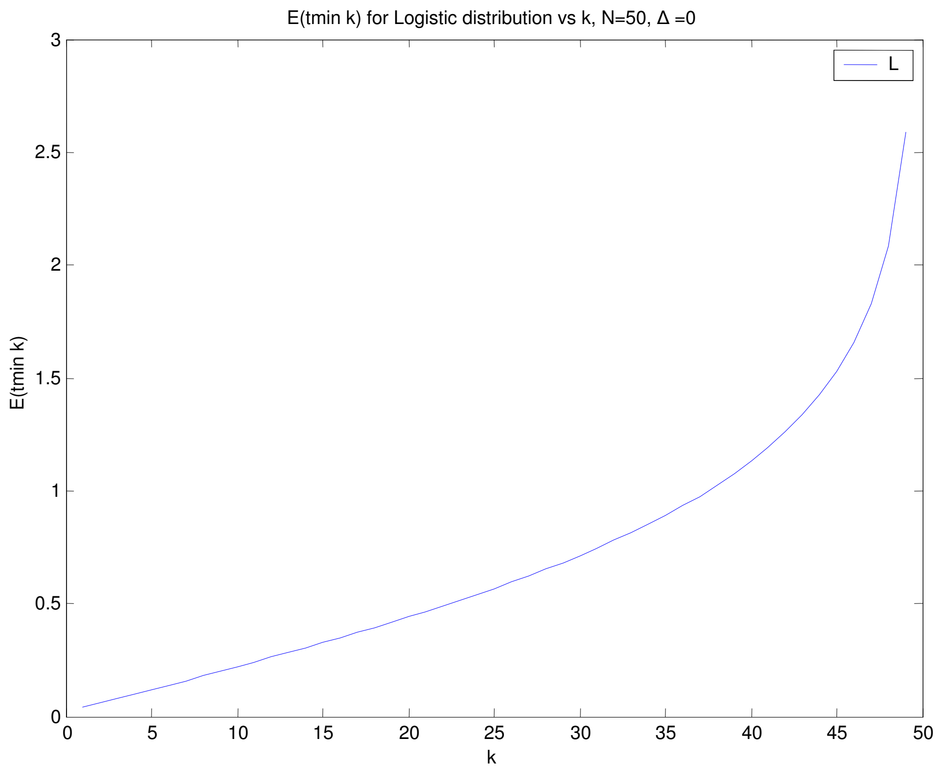


Figure (3.3): *$E(t_{min\ k})$ vs k*

From the Figure 3.2 (when N = 10) we notice that the values of $E(t_{min\ k})$ increase with k from ≈ 0.2 (when k = 1) to ≈ 1.8 (when k = 9).

Figure 3.3 shows the relationship between $E(t_{min\ k})$ and k, for N=50 ; the values of $E(t_{min\ k})$ increase when k increases, similar to Figure 3.2, but corresponding values of $E(t_{min\ k})$ for the same k, differ in the two figures such that those in Figure 3.3 are less than the corresponding ones in Figure 3.2.

This result is expected: as N increases the probability of t_{min} having smaller values becomes higher, e.g. in Figure 3.2 in which N=10, the value of $E(t_{min\ 5})$ was about 0.6, while in Figure 3.3 in which N=50, the value of $E(t_{min\ 5})$ was about 0.1 .

3.2.1.1.4 $E(t_{min\ k})$ vs k for the Uniform, Exponential, and Logistic pdf's

Figure 3.4 shows the relationship between $E(t_{min\ k})$ and k for the three pdf distributions, where N = 10.

From Figure 3.4 we notice the following:-

1- The curves of the graphs (3 pdf's) show that $E(t_{min\ k})$ increases with the increase of the values of k.

2- The least value of $E(t_{min\ k})$ for the three pdf's is nearly 0.25, but with increase of k, the difference between values of $E(t_{min\ k})$ increases.

3- The uniform distribution outperforms the other two pdf's in the time metric $E(t_{min\ k})$, with the logistic lying in between the other two.

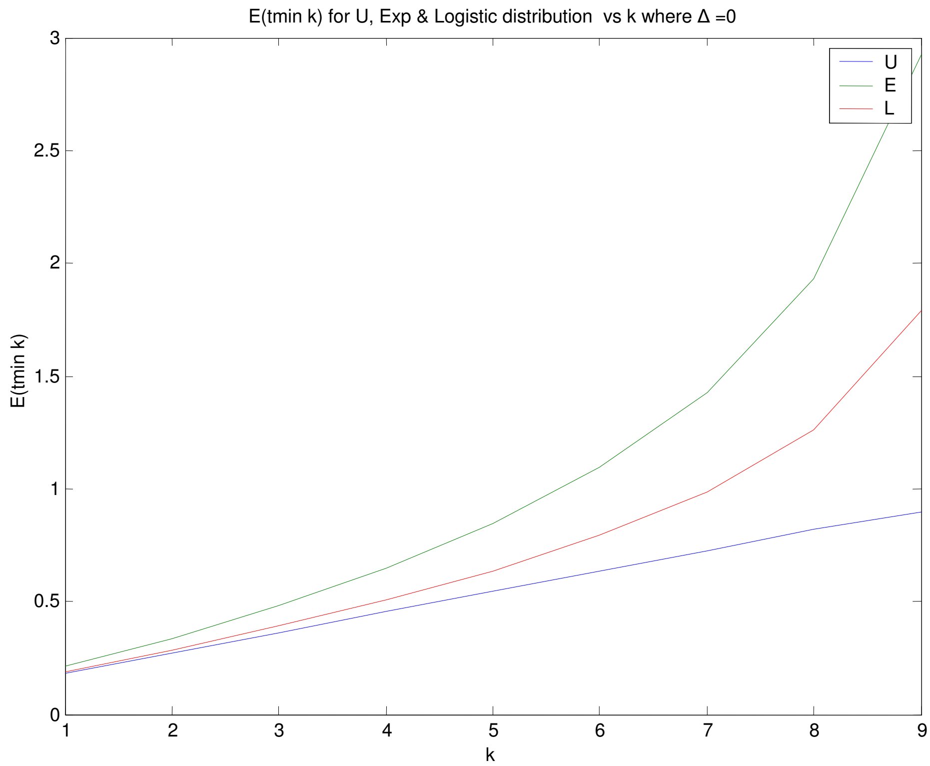


***Figure (3.4)**: Comparison $E(t_{min\ k})$ vs k*

Figure 3.5 where N=50, and k takes the values from 1 to 49, shows many similarities to Figure 3.4 .

The value of $E(t_{min\ k})$ increases with increase of k for the three pdf's, the differences between the $E(t_{min\ k})$'s also increase with increase of k, and each value $E(t_{min\ k})$ of the logistic lies between the two corresponding values of the other two pdf's. (for explanation see sec. 3.2.1.1.3).

On the other hand, comparing the two figures: Figure 3.4, Figure 3.5, we notice that for a given k and a given pdf, $E(t_{min\ k})$ in the second figure is less than the corresponding value in the first figure, i.e. $E(t_{min\ k})$ for a certain k decreases with increase of N.

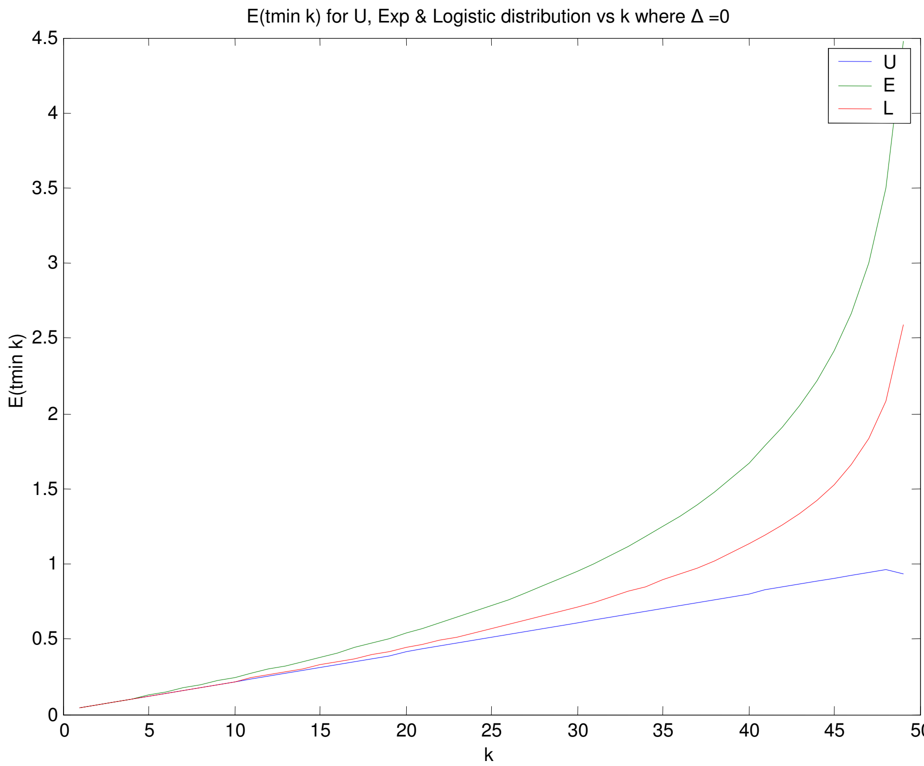


***Figure (3.5)**: $E(t_{min\ k})$ vs k*

3.2.2 Effective Minimum Time Elapsed $E(t_{min\ e})$

$E(t_{min\ e})$ or effective minimum time elapsed, is defined as the expected time of the earliest message sent but not completely dropped in the network, i.e. the time at which the first message is sent that could suppress some other process in the group.

In the lossy case, since a message may be lost for some processes in the network and received by others, the time order of $(t_{min\ e})$ of the processes has to be given a new order other than t_{min0}, $t_{min1}, \ldots, t_{minN-1}$ for the N processes network. Depending on this new ordering we apply the new performance metric $E(t_{min\ e})$.

Since more messages are lost in correlated loss than in uncorrelated loss, it follows that $E(t_{min\ e})$ will be higher in the correlated case; being an upper bound for $E(t_{min\ e})$, the correlated loss $E(t_{min\ e})$ will be adopted as a time performance metric in this study.

To compute $E(t_{min\ e})$ we designate first the probability of message loss by (l), the probability of receiving the message successfully will be (1- l).

Supposing that t_{min0}, t_{min1},… t_{minN-1}, are the earliest suppression times of the processes given in ascending order, it is evident that the probability that

$t_{min\ k}$ is the smallest time equals (1-l) l^k , where (1-l) represents the success of sending a message by the k^th^-process, and l^k represents the failure of k-processes (0,1,2,….,k-1) in sending successful messages

$E(t_{min\ e})$ =

$$E\left[\frac{(1-l)t_{\min 0} + (1-l)lt_{\min 1} + (1-l)l^2 t_{\min 2} + ... + (1-l)l^k t_{\min k} + ... + (1-l)l^{N-1} t_{\min N-1}}{(1-l^N)}\right] \quad (3.8)$$

where (1-l^N) is the normalization factor expressing the fact that one message at least was successful.

$$E(t_{min\ e}) = \frac{1-l}{1-l^N} E\left[\sum_{0 \le k < N} l^k . t_{\min k}\right] \quad (3.9)$$

Substituting for $t_{min\ k}$ we have

$$E(t_{min\ e}) = \frac{1-l}{1-l^N} \sum_{0 \le k < N} l^k \int_0^T Ntp(t) \binom{N-1}{k} (P(t))^k (1-P(t))^{N-1-k} dt \quad (3.10)$$

To find a simplified form of the formula, we first prove that

$$(lP(t) + (1-P(t)))^{N-1} = \binom{N-1}{k} (P(t))^k (1-P(t))^{N-1-k}$$

using the binomial theorem

$$(a+b)^N = \sum_{k=0}^{N} \binom{N}{k} a^{N-k} b^k$$

let a = 1 - P(t)

b = l.P(t)

substituting in the right hand side of the identity, we have

$$(a+b)^{N-1} = \sum_{0 \le k < N-1} \binom{N-1}{k} (1-P(t))^{N-1-k} (lP(t))^k$$

$$E(t_{\min e}) = \frac{1-l}{1-l^N} \sum_{0 \le k < N} l^k \int_0^T Ntp(t) \binom{N-1}{k} (P(t))^k (1-P(t))^{N-1-k} dt$$

Hence

$$E(t_{\min e}) = \frac{1-l}{1-l^N} \int_0^T Ntp(t)(lP(t)+(1-P(t)))^{N-1} dt \qquad (3.11)$$

Putting Q(t) = (1-l)(P(t)), we find that

lP(t)+1-P(t) = P(t)(l-1)+1 = 1 – P(t)(1-l) ≡ 1-Q(t)

Substituting in (3.11) we get

$$E(t_{\min e}) = \frac{-1}{1-l^N} \int_0^T N \underbrace{(1-l)\,p(t)}_{q(t)} t (1-Q(t))^{N-1} dt$$

$$E(t_{\min e}) = \frac{-1}{1-l^N} \int_0^T Nq(t)(1-Q(t))^{N-1} dt$$

$$E(t_{\min e}) = \frac{-1}{1-l^N} \int_0^T td[1-Q(t)]^N \qquad (3.12)$$

Integrating by parts $\int u dv = uv - \int v du$

u = t

v = $(1\text{-}Q(t))^N$

$$\int u dv = t(1-Q(t))^N - \int (1-Q(t))^N dt$$

(3.12) becomes

$$= \frac{-1}{1-l^N}\left[t(1-Q(t))^N \Big|_0^T - \int_0^T (1-Q(t))^N dt \right]$$

$$= \frac{-1}{1-l^N}\left[t(1-(1-l)P(t))^N \Big|_0^T - \int_0^T (1-Q(t))^N dt \right]$$

$$= \frac{-1}{1-l^N}\left[T(1-(1-l)\underbrace{P(T)}_{1})^N - 0(1-(1-l)P(0)) - \int_0^T (1-Q(t))^N dt \right]$$

$$= \frac{-1}{1-l^N}\left[T(1-1+l)^N - \int_0^T (1-Q(t))^N dt \right]$$

$$= \frac{-1}{1-l^N}\left[Tl^N - \int_0^T (1-Q(t))^N dt \right]$$

$$= \frac{-Tl^N + \int_0^T (1-Q(t))^N dt}{1-l^N} \tag{3.13}$$

This formula, being easy to manipulate, is sometimes used for simplifying the calculations related to $E(t_{\min e})$

3.2.2.1 E($t_{\min e}$) and the Probability Distributions

From the general formula of $E(t_{\min e})$ (3.11), we notice that the formula contains both p(t) and P(t), which means that its value varies with the used pdf in the suppression technique.

3.2.2.1.1 The Uniform Distribution:-

$$p(t) = \frac{1}{T}, \qquad 0 \le t < T,$$

$$P(t) = \frac{t}{T}, \qquad 0 \le t < T$$

let c = 1-l, Q(t) = cP(t), q(t) = cp(t)

$$E(t_{\min e}) = \frac{-Tl^N + \int_0^T (1-Q(t))^N dt}{1-l^N}$$

$$= \frac{-Tl^N + \int_0^T (1-cP(t))^N dt}{1-l^N}$$

$$= \frac{-Tl^N + \int_0^T \left(1-\frac{ct}{T})\right)^N dt}{1-l^N}$$

$$= \frac{-Tl^N + \left.\frac{\left(1-\frac{ct}{T}\right)^{N+1}}{(N+1)(\frac{-c}{T})}\right|_0^T}{1-l^N}$$

$$= \frac{-Tl^N + \left[\frac{(1-c)^{N+1}}{(N+1)\frac{-c}{T}} - \frac{(1)^{N+1}}{(N+1)\frac{-c}{T}}\right]}{1-l^N}$$

$$= \frac{-Tl^N + \left[\frac{T(l^{N+1}-1)}{(N+1)(l-1)}\right]}{1-l^N}$$

$$= \frac{-Tl^N(N+1) + \dfrac{Tl^{N+1} - T}{l-1}}{(1-l^N)(N+!)}$$

$$= \frac{T}{(N+!)(1-l^N)}\left[\frac{l^{N+1}-1}{l-1} - (N+1)l^N\right]$$

$$= \frac{T}{N+1}\frac{1}{1-l^N}\left[\frac{1-l^{N+1}}{1-l} - (N+1)l^N\right] \qquad (3.14)$$

3.2.2.1.2 The Exponential Distribution

$$\text{p(t)} = \frac{1}{\alpha}e^{-\frac{t}{\alpha}}, \qquad 0 \le t\,,$$

$$\text{P(t)} = 1 - e^{-\frac{t}{\alpha}}, \qquad 0 \le t$$

let c = 1 - *l*, Q(t) = cP(t), q(t) = cp(t)

$$\text{E}(t_{\text{min e}}) = \frac{-Tl^N + \int_0^\infty (1-Q(t))^N\,dt}{1-l^N} \qquad (3.15)$$

The formula of

$$\text{E}(t_{\text{min e}}) = \frac{\alpha}{N}\left[\frac{-Tl^N N}{(1-l^N)\alpha} + \frac{(1-l)^N}{(1-l^N)}\right] \text{ (Schooler, p.52)}$$

Was found incompatible with the integration formula using the trapezoidal rule which we used in our calculations.

3.2.2.1.3 The Logistic Distribution

$$\text{p(t)} \quad = \frac{2ae^{-at}}{(1+e^{-at})^2}, \qquad 0 \le t$$

$$P(t) = \frac{2}{(1+e^{-at})} - 1, \qquad 0 \le t$$

$$Q(t) = (1-l)P(t) = (1-l)\left(\frac{2}{(1+e^{-at})} - 1\right) \tag{3.16}$$

$$E(t_{\min e}) = \frac{-Tl^N + \int_0^{\infty} (1 - Q(t))^N \, dt}{1 - l^N}$$

3.2.2.2 $E(t_{\min e})$: Graphical Representation

The following realistic values of the variables and parameters T, N, a and *l* were adopted in the following graphs:

T = 1

α = T = 1

N = ranges from 1 to 1000

a $= \frac{2}{T} = 2$

$l \in (0,1)$

The formulas used are those relating to correlated loss, see section (3.1)

3.2.2.2.1 Variation of $E(t_{\min e})$ for the Logistic Distribution vs N

To illustrate the relationship between $E(t_{\min e})$ and N, we draw the graph of $E(t_{\min e})$ vs N for different values of N, where N ranges from 1 to 1000 processes, taking $l = 0.2, 0.5, 0.9$.

Figure 3.6a, which describes the above relationship shows that $E(t_{\min e})$ decreases as N increases, which is intuitively true since when more processes participate, any value of t_{min} including a small one becomes more probable.

Figure 3.6a also shows that the values of $E(t_{min\ e})$ increase with the increase of the loss parameter l which is also expected since small values of $(t_{min\ k})$ may not be effective due to loss

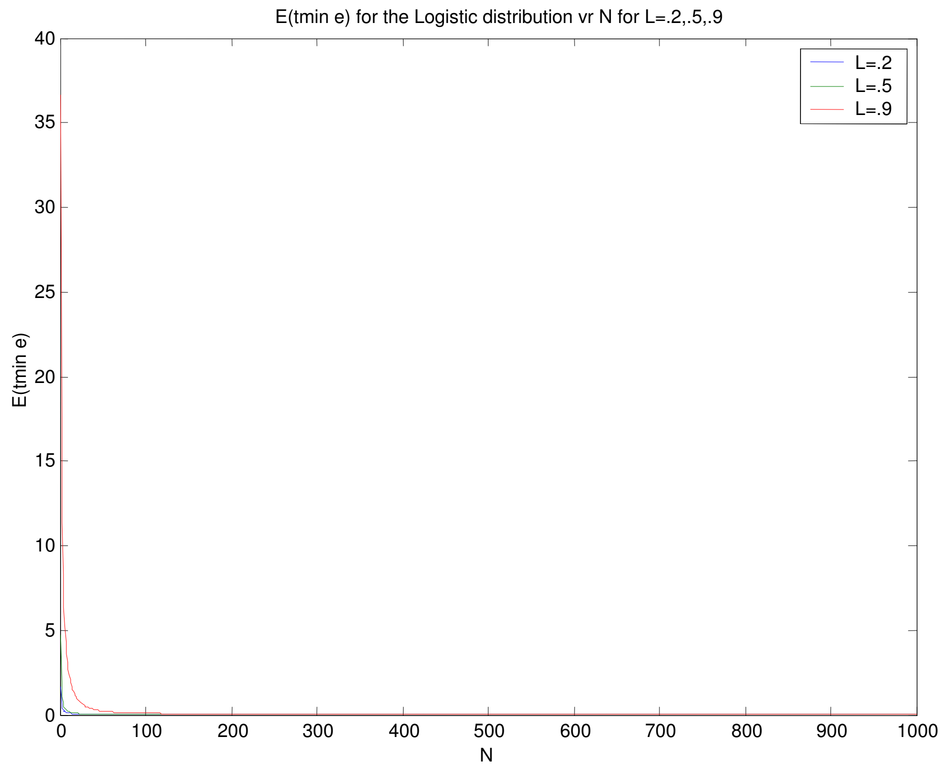


***Figure (3.6a)**: $E(t_{min\ e})$ vs N*

To show clearly the effect of l on $E(t_{min\ e})$ we magnify Figure 3.6a for the values 1-60 for N, as shown in Figure 3.6b.

3.2.2.2.2 $E(t_{min\ e})$ for Uniform, Exponential and Logistic Distributions

To compare the relationship between $E(t_{min\ e})$ and N (correlated case) for the 3 pdfs, we draw the graphs of $E(t_{min\ e})$ vs N Figures 3.7a, b, c, d, e, f where N ranges from (1–100), and l takes the values 0.1, 0.4, 0.8 .

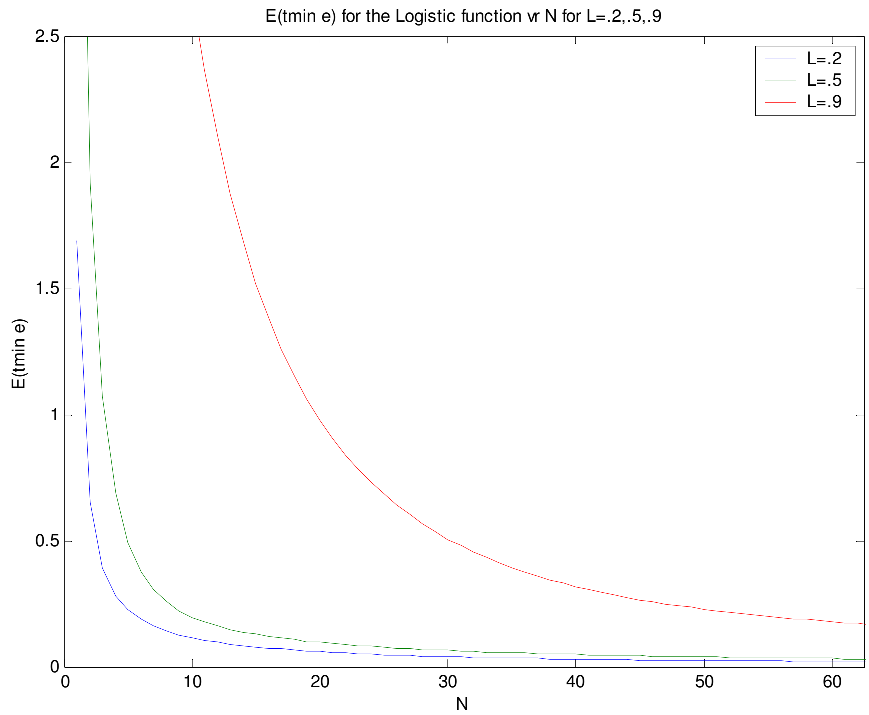


***Figure (3.6b)**: $E(t_{min\ e})$ vs N (different scale)*

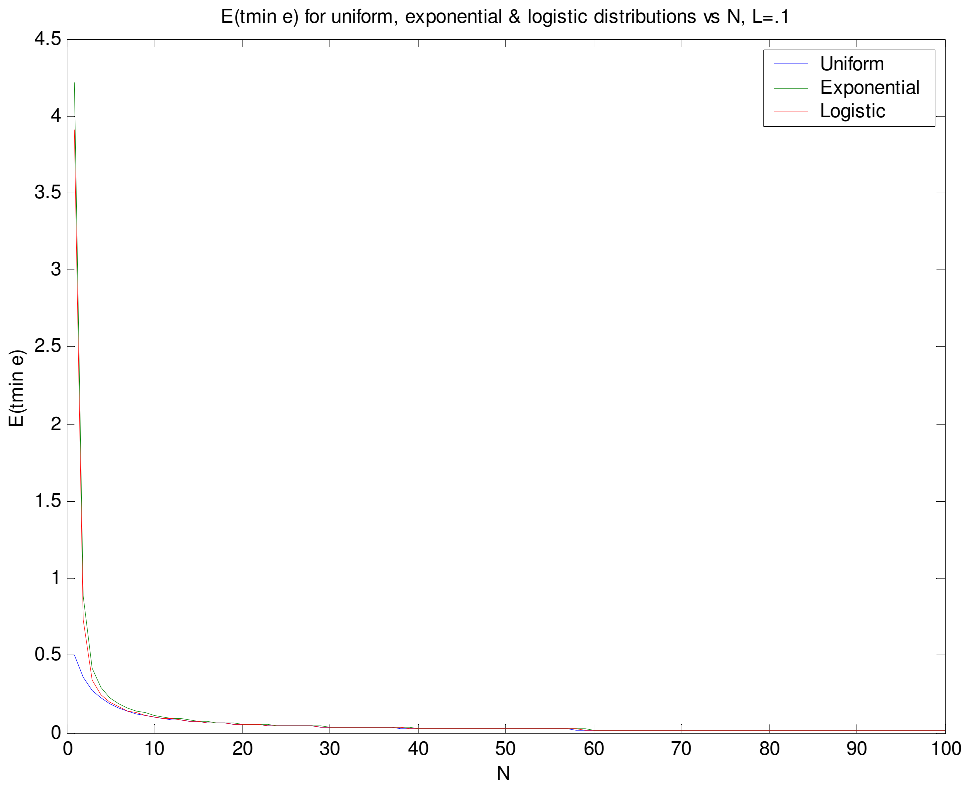


***Figure (3.7a)**: Comparison: $E(t_{min\ e})$ vs N*

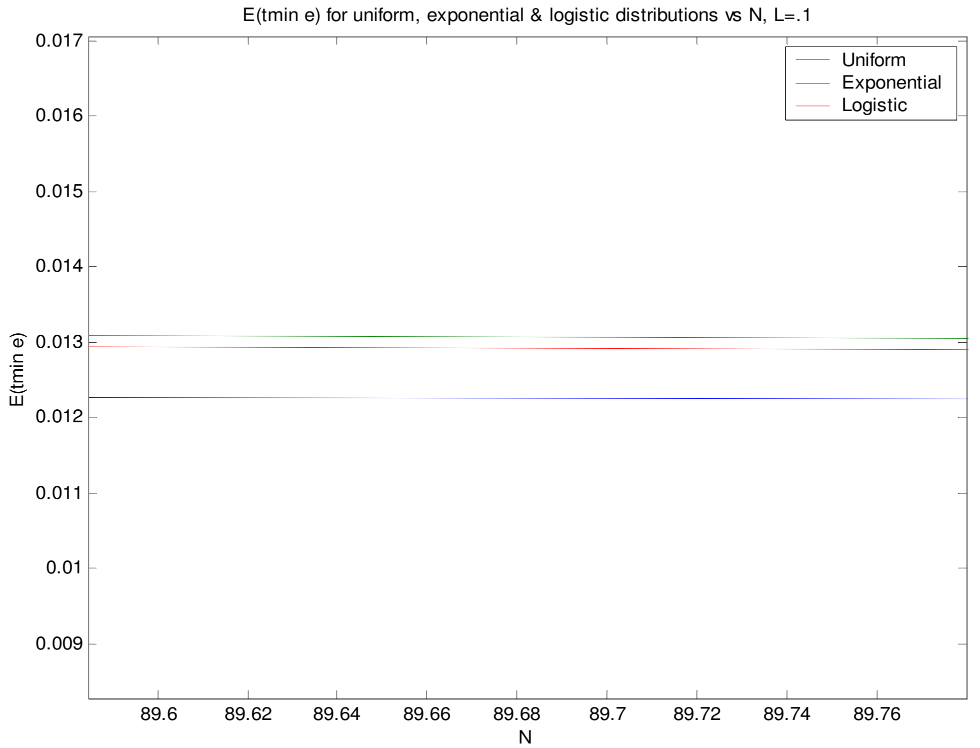


Figure (3.7b)*: Comparison: $E(t_{min\ e})$ vs N (different scale)*

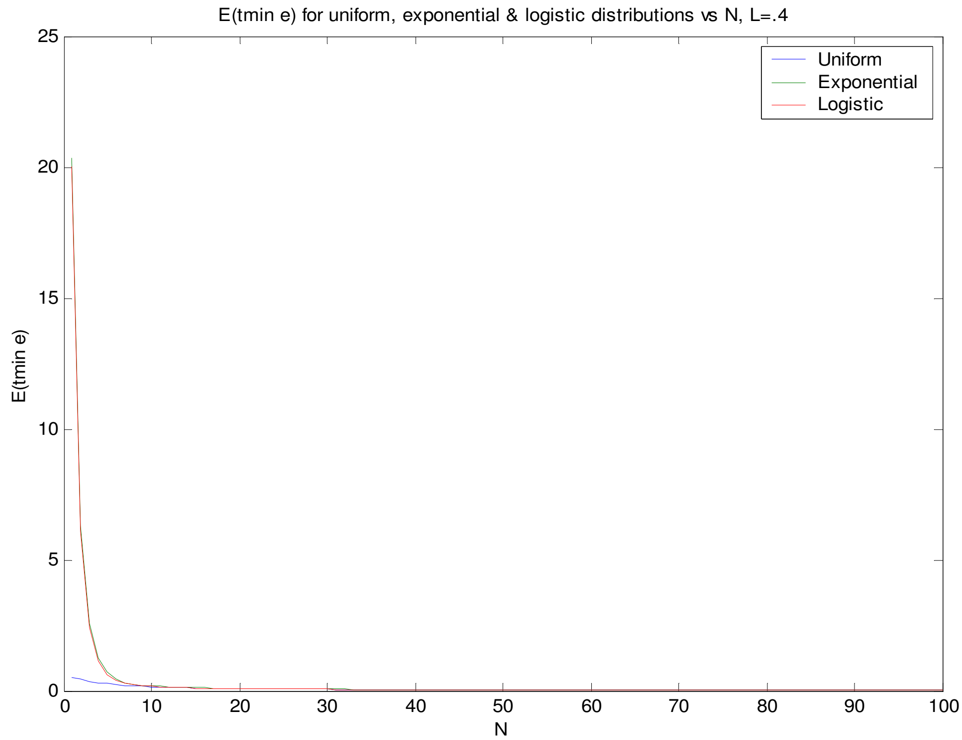


Figure (3.7c)*: Comparison: $E(t_{min\ e})$ vs N*

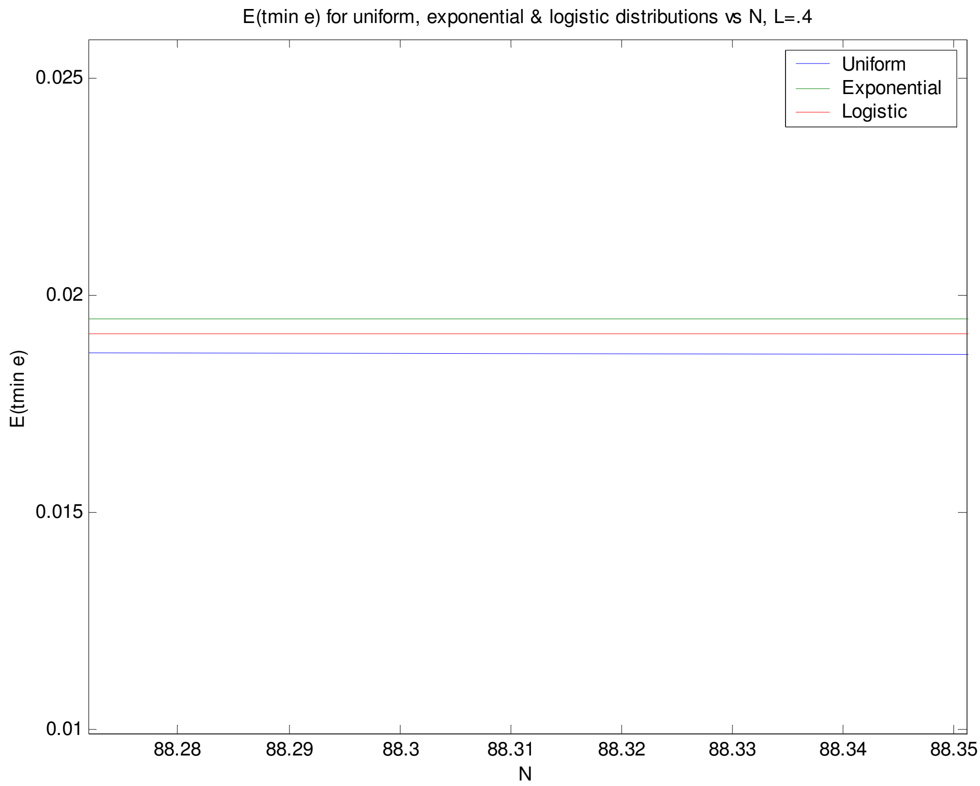


Figure (3.7d)*: Comparison: $E(t_{min\ e})$ vs N (different scale)*

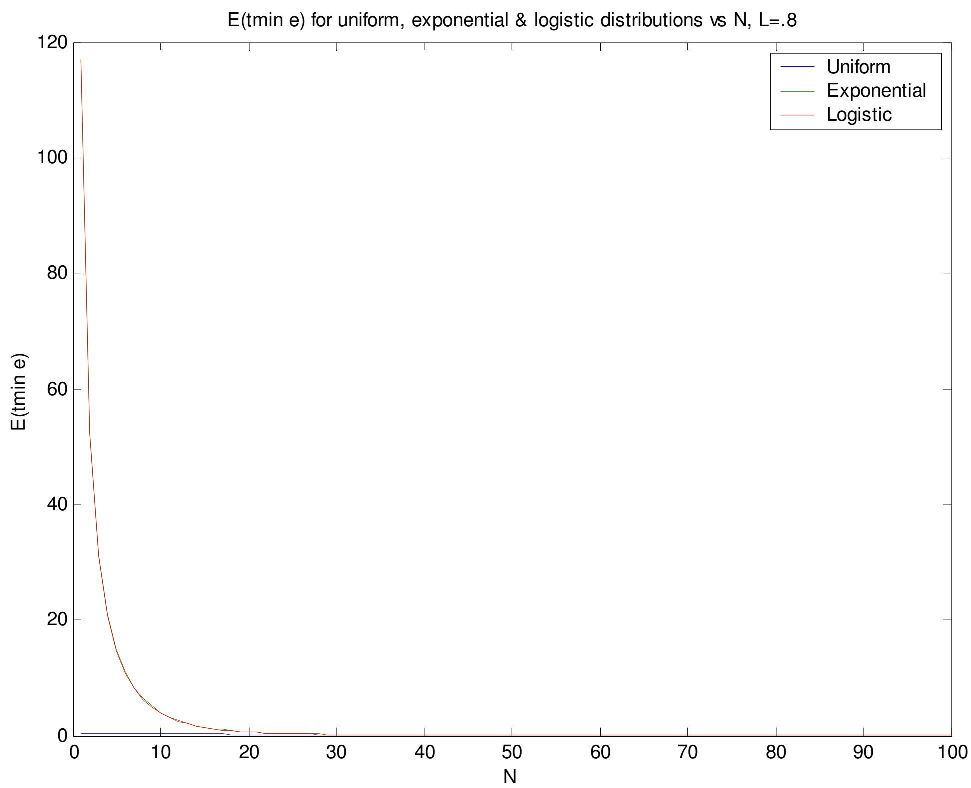


Figure (3.7e)*: Comparison: $E(t_{min\ e})$ vs N*

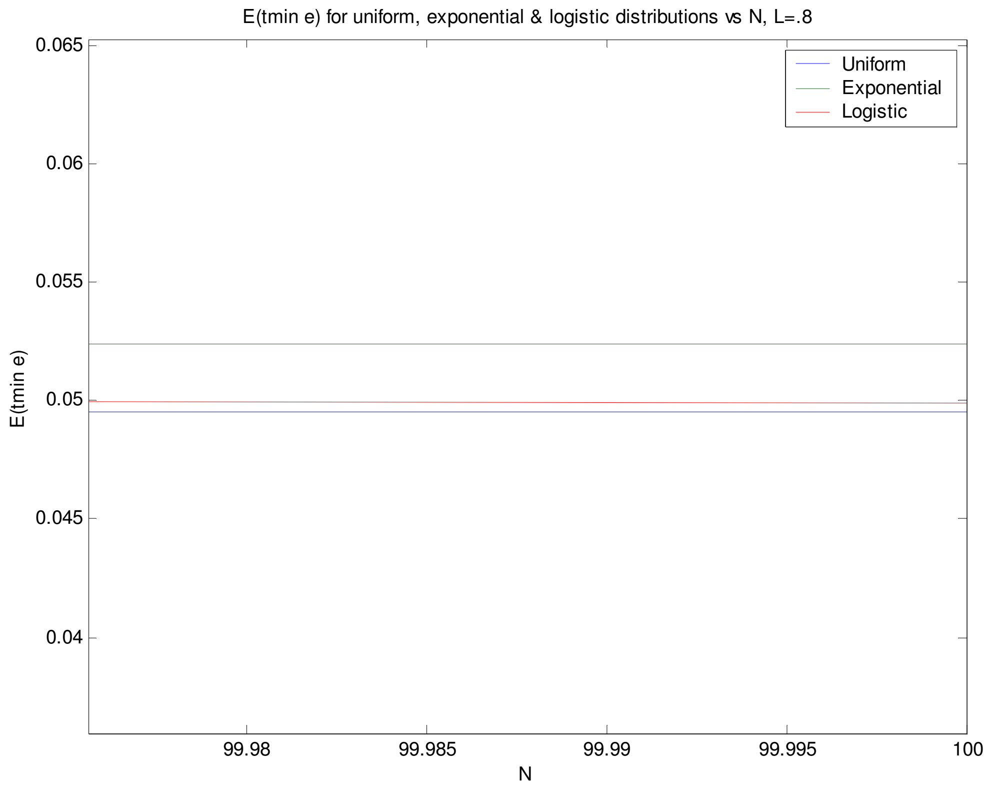


***Figure (3.7f)**: Comparison: $E(t_{min\ e})$ vs N (different scale)*

The relationship between $E(t_{min\ e})$ vs N for the three distributions depends on the value of *l*. The following summarizes this relationship:-

a. As N increases $E(t_{min\ e})$ decreases for the three pdfs.

b. For small values of N, (1 – 15) the uniform distribution function outperforms the exponential and the logistic distributions irrespective of the value of *l*.

c. For values of N greater than (15), The logistic distribution outperforms the exponential for all values of *l* and N; the uniform distribution outperforms the logistic generally but the greater the value of *l*, the performance of the logistic becomes nearly equal to that of the uniform.

3.2.3 Maximum Time Elapsed, $E(t_{max})$

$E(t_{max})$: the expected time selected by the last process that actually generates a message; this time is generally much less than $E(t_{min\ N-1})$. It is useful, particularly with message loss in knowing how long it takes the algorithm to complete. It also denotes completion time that is important when suppression by an algorithm is followed by another algorithm.

In lossless case $E(t_{max}) \leq E(t_{min})+\Delta$, which means that each message is sent within Δ of the earliest message. In the lossy case $E(t_{max})$ is not defined in terms of $E(t_{min})$, In both cases lossy and nonlossy $E(t_{max})+\Delta$ means the expected time after which all nodes are in agreement to halt the algorithm.

3.2.3.1 Calculating $E(t_{max})$

We calculate $E(t_{max})$ by using the probability distribution of $t_{min\ k}$

$$E(t_{max})_{\Delta=0} = \sum_{0 \leq k < N} t_{\min k} * \Pr(t_{\max} = t_{\min k})_{\Delta=0} \tag{3.17}$$

Supposing we have a very small value of delay i.e. near zero, and very small relative to T, and noting that $E(t_{max})$ in uncorrelated loss is higher than the correlated case, we calculate $E(t_{max})$ in these conditions:-

($\Delta = 0$) and uncorrelated loss.

We first find a formula $\Pr(t_{max} = t_{min\ k})$; $t_{max} = t_{min\ k}$ occurs if the two conditions are satisfied:-

1- when process k loses all the messages sent from processes j, $0 \leq j < k$

2- processes $k+1 \leq j < N-1$ receive at least one message from processes $0 \leq j \leq k$ that generate messages. (see Figure 3.8)

Suppose that the processes $0 \le j < k$ generate i messages where i ranges from 1 to k.

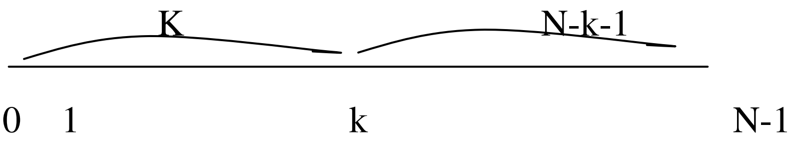


Figure (3.8): *Messages in E(t_{max})*

If *i* messages were sent, k must have lost them all while the processes from $k+1 \le j < N-1$ each must have received 1 message at least, the probability of this event = $l^i(1-l^{i+1})^{N-K-1}$.

In the nonlossy case: $t_{max} = t_{min\ k} = t_{min}$ happens when all subsequent N-1 processes receive the first message

$$\Pr(t_{max} = t_{min\ 0})_{\Delta=0} = (1-l)^{N-1}$$

$$\Pr(t_{\max} = t_{\min k})_{\Delta=0} = \sum_{0 \le i \le k} \Pr(exactly \quad i \quad messages \quad sent) * l^i (1-l^{i+1})^{N-K-1} \quad (3.18)$$

Equation (3.17) takes the form

$$E(t_{\max})_{\Delta=0} = \sum_{0 \le k < N} t_{\min k} * \sum_{0 \le i \le k} \Pr(exactly \quad i \quad messages \quad sent) * l^i (1-l^{i+1})^{N-K-1} \quad (3.19)$$

Let P(i,n): represent the probability that exactly *i* messages are sent by n processes, this event can be decomposed into two disjoint parts:-

1- Process n-1 sends a message and processes $0 \le j < n-1$ send (i-1) messages; in this case process (n-1) must lose the (i-1) messages sent, which happens with probability (l^{i-1}),

2- Process n-1 does not send a message, and processes $0 \le j < n-1$ send (*i*) messages; in this case process (n-1) must receive at least one of the (i) messages sent, which happens with probability $(1-l^i)$.

$$P(i,n) = P(i-1,n-1) * l^{i-1} + P(i,n-1) * (1-l^i)$$

so $P(0,n) = 0$

$$P(i,i) = l^1.l^2.l^3....l^{i-1}$$

To find the values of P(i,n) the following procedure was used noting that no points lie under the diagonal in Figure 3.9 because the number of processes n will always exceed or equal number of messages:

a- Diagonal points:

The rule $P(i,i) = l^{(i-1)*\frac{i}{2}}$ was used, where i ranges between the values 1 to n.

b- For points P(1,n):

using induction we have

$P(1,1) = 1$, (from P(i,i) rule)

$P(1,2) = P(1,1) * (1-l^1) = (1-l)^1$, (from the P(i,n) rule)

$P(1,3) = P(1,2) * (1-l^1) = (1-l)^2$

…

$P(1,n) = P(1,n-1) * (1-l^1) = (1-l)^{n-1}$

c- For points other than those of previous parts (a,b):

we use the general rule

$$P(i,n) = P(i-1,n-1) * l^{i-1} + P(i,n-1) * (1-l^{i})$$

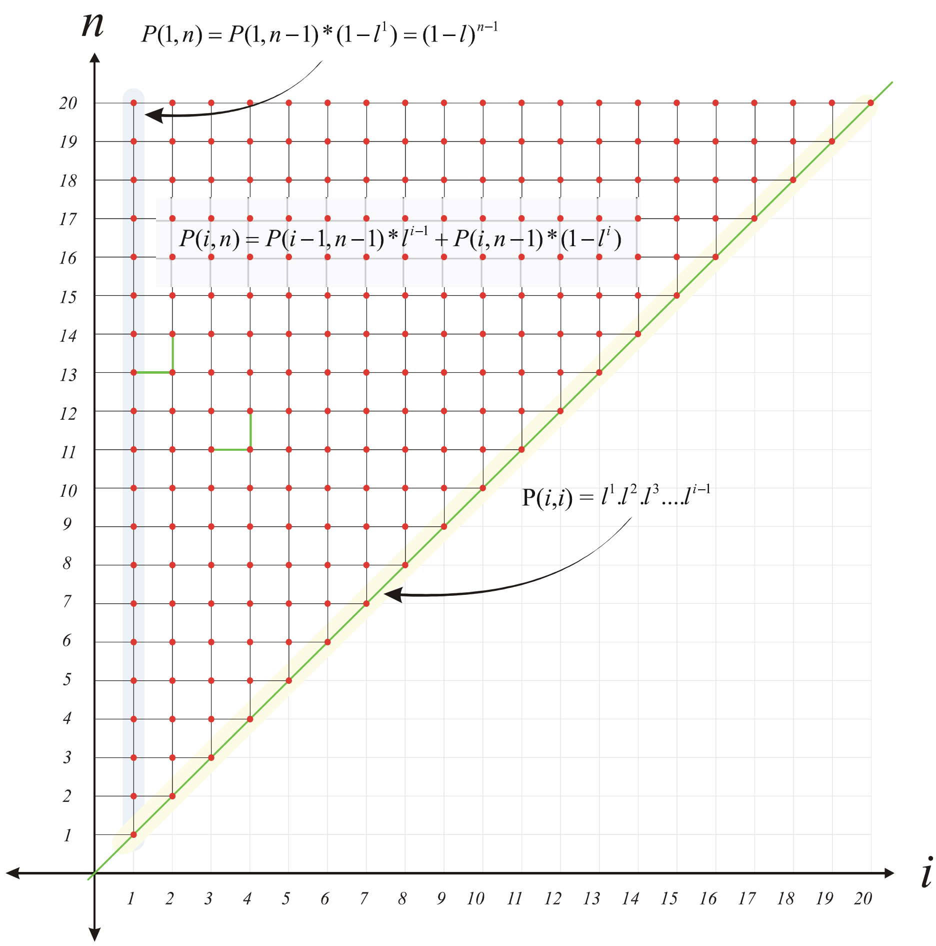


***Figure (3.9)**: Representing P(i,n), (i ≤ n).*

The above procedure was used in a Matlab program for determining the values of P(i,n) for calculating the time metric E(t_{max}) and E(# required) messages in the lossy case.

Summing up, the formula for E(t_{max}) is derived as follows:-

$$E(t_{\max} = t_{\min k})_{\Delta=0} = \sum_{1 \le i < k} \Pr(exactly \quad i \quad messages \quad sent) * l^{i} (1-l^{i+1})^{N-K-1}$$

$$E(t_{\max})_{\Delta=0} = \sum_{0 \le k < N} t_{\min k} * \sum_{0 \le i < k} \Pr(exactly \quad i \quad messages \quad sent) * l^{i} (1-l^{i+1})^{N-K-1}$$

$$E(t_{\min k}) = \int_0^T Ntp(t)\binom{N-1}{k}P(t)^k(1-P(t))^{N-1-k}\,dt$$

$$E(t_{\max}) = \sum_{0\le k<N}\left[\int_0^T Ntp(t)\binom{N-1}{k}P(t)^k(1-P(t))^{N-1-k}\,dt * \sum_{0\le i\le k}P(i,k)*l^i(1-l^{i+1})^{N-1-k}\right]$$

3.2.3.2 E(t_{max}): Graphical Representation

Figure 3.10 and Figure 3.11 show the relationship between E(t_{max}) for the logistic distribution function and a small number of processes N (N=10) for various values of the loss factor (l = 0.2, 0.4, 0.8) and also for (N=50) with the same values of (l).

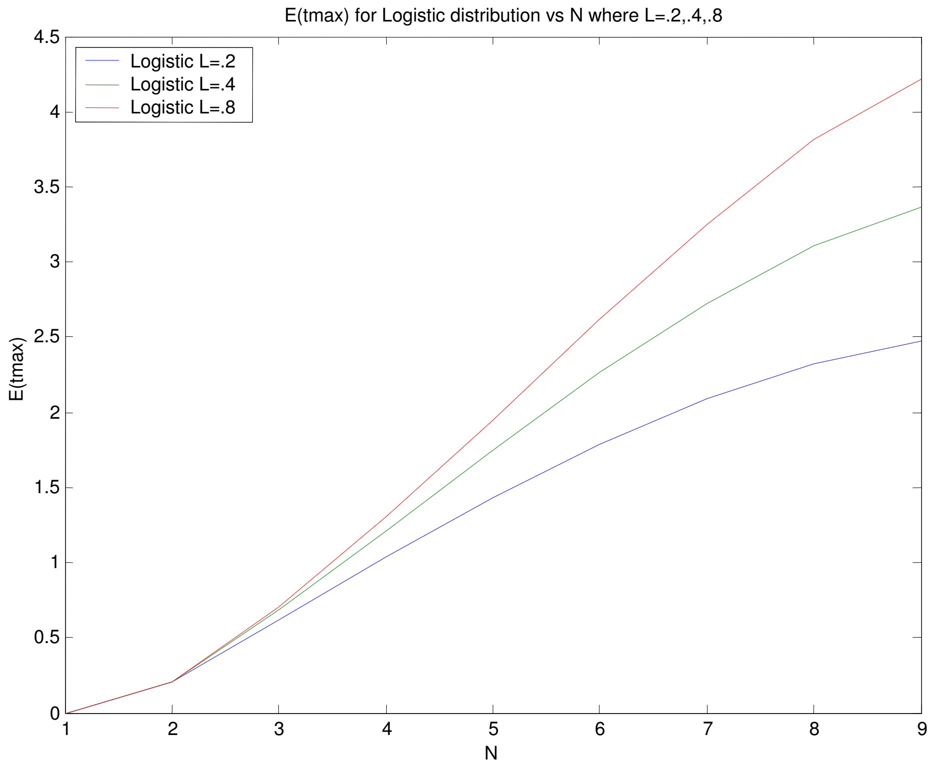


***Figure (3.10)**: E(t_{max}) vs N*

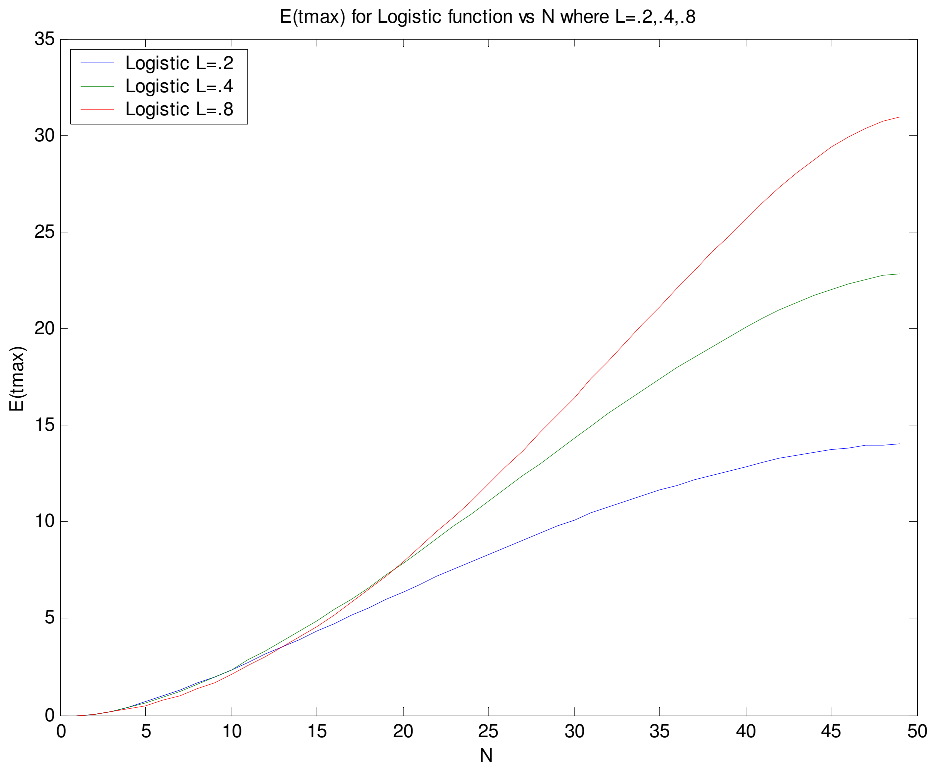


Figure (3.11): *$E(t_{max})$ vs N*

From the figures we notice the following:-

1- $E(t_{max})$ increases with increase of N.

2- From nearly equal values at N=2, the 3 curves for different l become wide apart for values near 10 for the first figure and similarly in the second, with the largest value of $E(t_{max})$ corresponding to the largest loss factor ($l = 0.8$).

3- There is some indication that for low values of (l) the curves tend to become steady before other values of (l).

The value of $E(t_{max})$ depends on the number of processes N, and the value of k (formula (3.17)). Hence in comparing $E(t_{max})$ for the same value of k, in the two figures, we notice that different relationships might exist between the three curves representing the different values of l.

3.2.3.3 $E(t_{max})$ vs N for the Uniform, Exponential and Logistic Distributions

The Figures 3.12a,b,c, show the relationship between $E(t_{max})$ for the three pdf's: uniform, exponential and logistic with the # of processes N at $l = 0.2, 0.5, 0.9$.

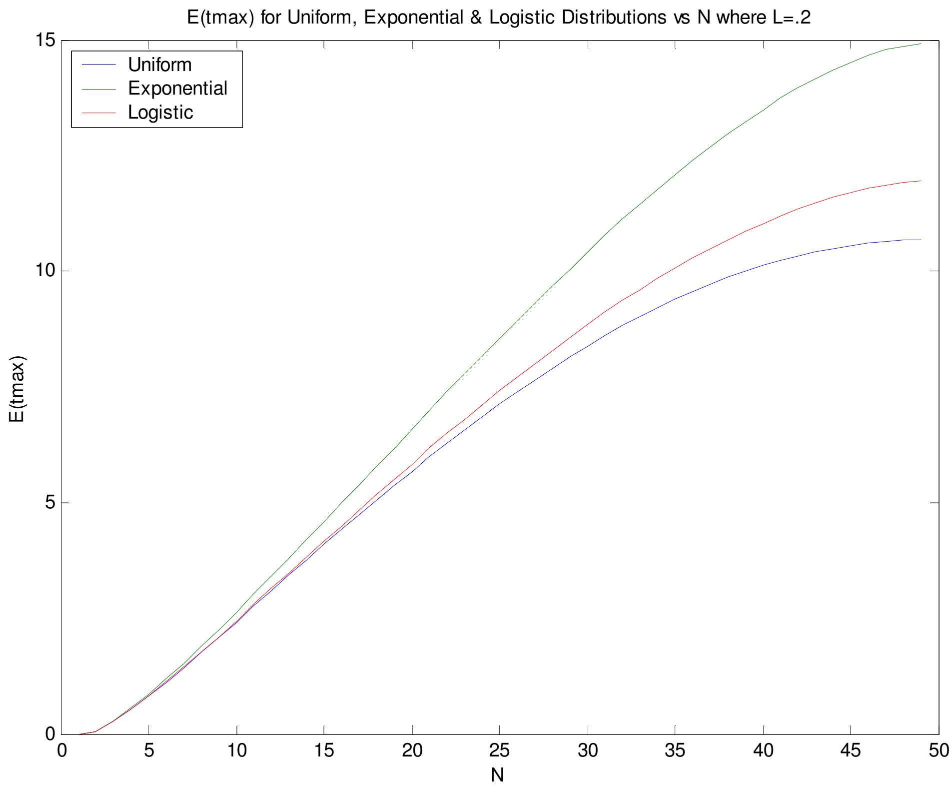


***Figure (3-12a)**: Comparison: $E(t_{max})$ vs N*

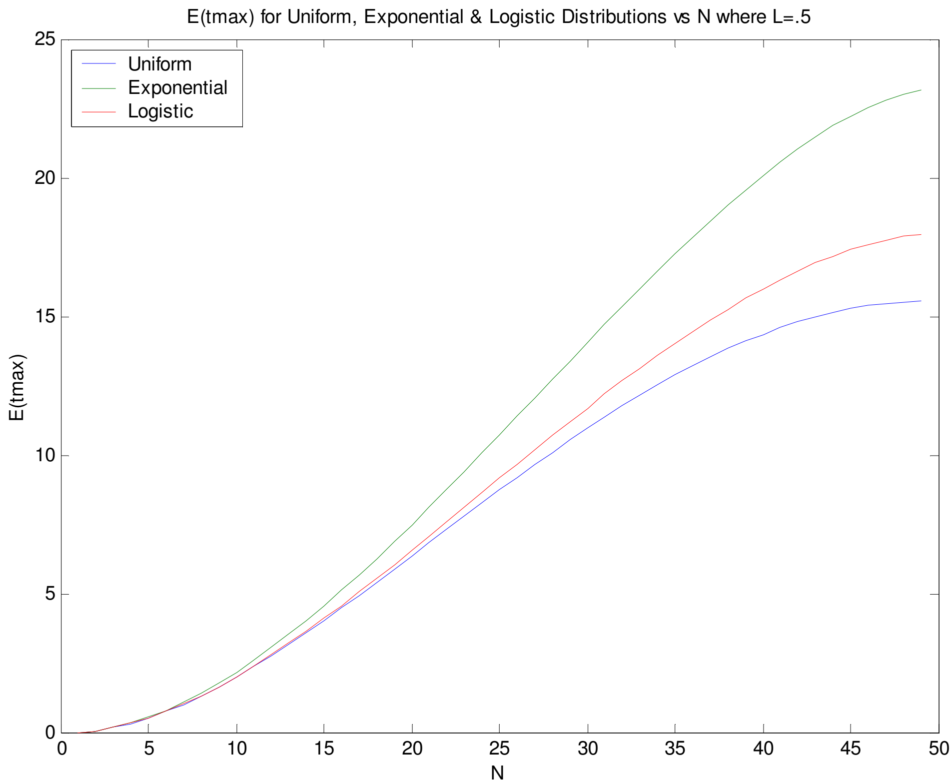


Figure (3-12b): *Comparison: $E(t_{max})$ vs N*

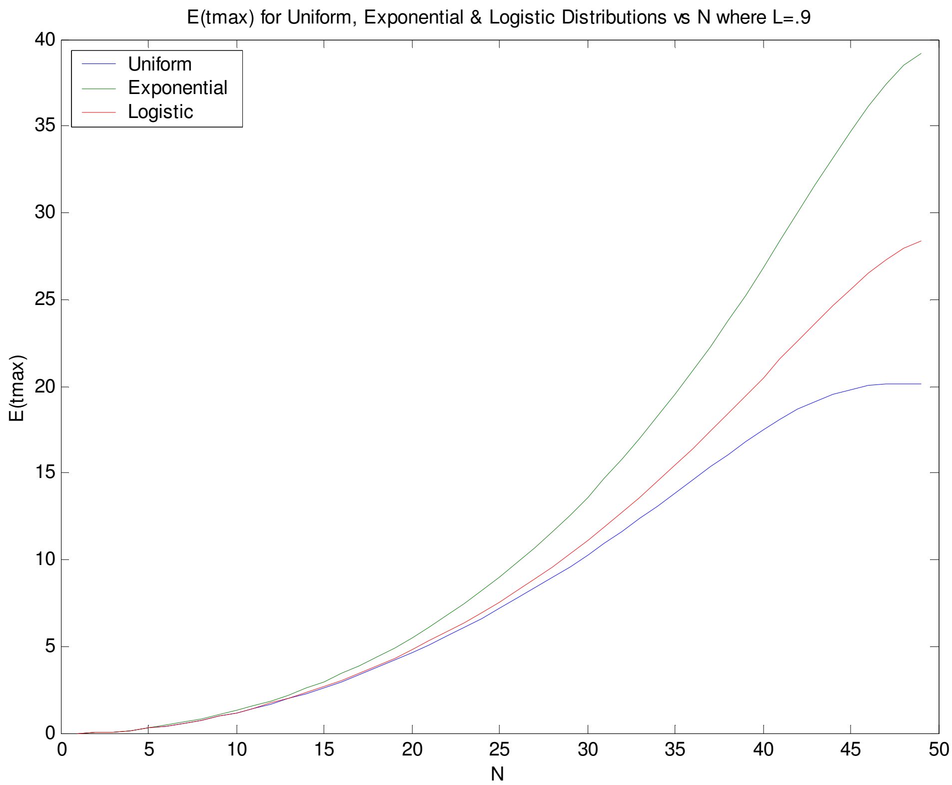


Figure (3-12c): *Comparison: $E(t_{max})$ vs N*

From the Figures 3.12a,b,c , we notice the following:-

1- $E(t_{max})$ increases with increase of N for all the three pdfs.

2- The lowest values of $E(t_{max})$ are for the uniform distribution and the highest are for the exponential, while those of the logistic lie in between the two.

3.3 Extra and Required Messages in Lossy Conditions

3.3.1 Introduction

In the non- lossy case, it was the first sent message that had to be taken into consideration in the suppression of other messages. In fact, the first message is the only required message (that affects suppression); all other issued messages are regarded extra messages.

In the lossy case, the first sent message, like other messages, may be lost; the loss factor (l) and the existence of transmission delay (Δ) may complicate the calculation of required and extra messages.

Taking into consideration the possible values of (l) and(Δ), we differentiate between four cases in calculating the number of required messages and extra messages:

1) $\boldsymbol{l = 0 , \Delta = 0}$

In this case, the first sent message suppresses all other messages i.e. we have one required message, no extra messages, and the total number of messages = 1.

2) $\boldsymbol{l = 0, \ \Delta \neq 0}$

This is the non- lossy case which we studied in chapter 2, the first sent message is the only required message, E(t_{max}) = $N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)(1 - P(t-\Delta))^{N-1}\,dt$,

The total number of messages = $N.P(\Delta) - 1 + N\int_{\Delta}^{T} p(t)(1 - P(t-\Delta))^{N-1}\,dt + 1$

$$= N.P(\Delta) + N\int_{\Delta}^{T} p(t)(1 - P(t-\Delta))^{N-1}\,dt$$

3) $l \neq 0, \Delta = 0$

In this case no extra messages are produced, all the produced messages are required messages; this case will be treated in detail in the following sections.

4) $l \neq 0, \Delta \neq 0$

This case is difficult to treat analytically; it is outside the scope of our study, and it suffices here to use the results of case 3 as an approximation to this case.

3.3.2 Uncorrelated Loss

In the lossy case many messages may contribute to suppressing different processes, so we may think of different numbers of messages (i) used in the process of suppression: hence we may use the following equation to determine the number of messages sent

$$\text{E(\#messages)} = \sum_{1 \leq i \leq N} i * \Pr(\mathit{exactly} \quad i \quad \mathit{messages} \quad \mathit{sent})$$

In the uncorrelated case, which is accompanied by a higher # of generated messages, the last equation will be modified according to the fact that the probability that (i) messages sent in the zero delay case can be calculated. The last equation will be of the following form

$$\mathrm{E}(\#\text{messages})_{\Delta=0} = \sum_{1\le i\le N} i * \Pr(\textit{exactly} \quad i \quad \textit{messages} \quad \textit{sent})$$

$$= \sum_{1\le i\le N} i * P(i,N)$$

To calculate the number of required messages, we notice that this number is equivalent to the number of messages generated when there is no transmission delay, in fact if all messages are received in the same time there is no chance of sending extra messages and hence we may consider:

$$\mathrm{E}(\#\text{required})_{l=\mathrm{p},\ \Delta=\mathrm{d}} = \mathrm{E}(\#\text{messages})_{l=\mathrm{p},\ \Delta=0}$$

i.e. the number of required messages is independent of the value of Δ.

In the uncorrelated case simulation demonstrates that higher number of required messages are produced (see also sec. 3.3.4). From the above, it follows that:

$$\begin{aligned}\mathrm{E}(\#\text{required})_{l=\mathrm{p},\ \Delta=\mathrm{d}} &= \mathrm{E}(\#\text{messages})_{l=\mathrm{p},\ \Delta=0} \\ &= \sum_{1\le i\le N} i * \Pr(\textit{exactly} \quad i \quad \textit{messages} \quad \textit{sent}) \\ &= \sum_{1\le i\le N} i * P(i,N)\end{aligned}$$

for evaluating *P(i,N)*, a special algorithm was formulated by using the two recurrence formulas in section (3.2.3.1) using the MatLab software.

3.3.3 Correlated Loss

In the last section, we illustrated the fact that E[#messages] and E[#required] are higher in the case of uncorrelated loss. It follows from this fact that E[#extra] are comparatively low with the uncorrelated loss; this may lead us to suggest that extra messages E[#extra] will be high with correlated loss, which is certified by the results of simulation (Schooler, p: 49).

To calculate E[#extra] we find E[#required] first:

$$\text{E[\#required]} \quad = \sum_{1\le k\le N} k * \Pr[k \quad messages \quad sent]$$

But k messages sent, implies the loss of k-1 messages, and the success of reception of the kth message, hence probability of this event = $(1-l)l^{k-1}$

$$\therefore E[\#\,required] = \sum_{1\le k\le N} k(1-l) * l^{k-1}$$

$$= (1-l)\sum_{1\le k\le N} k * l^{k-1}$$

$$S = \sum_{1\le k\le N} k * l^{k-1}$$

$$S = 1 + 2l^1 + 3l^2 + 4l^3 + \ldots + (N-1)l^{N-2} + Nl^{N-1}$$
$$l.S = 0 + l^1 + 2l^2 + 3l^3 + \ldots + (N-2)l^{N-2} + (N-1)l^{N-1} + Nl^N$$

$$S(1-l) = 1 + l^1 + l^2 + l^3 + \ldots + l^{N-1} - Nl^N$$

$$= \frac{1-l^N}{1-l} - Nl^N$$

$$E[\#\,required] = (1-l)S = \frac{1-l^N}{1-l} - Nl^N$$

$$\text{E[\#extra]}_{l = \text{p}, \Delta = \text{d}} = \text{E[\# messages]}_{l = \text{p}, \Delta = \text{d}} - \text{E [\# required]}_{l = \text{p}, \Delta = \text{d}}$$

$$= \text{E[\# messages]}_{l = \text{p}, \Delta = \text{d}} - \text{E [\# required]}_{l = \text{p}, \Delta = 0}$$

3.3.4 Comparisons of Required Messages

Figure 3.13 shows E(#required) vs N in the correlated case for different values of Loss l =0.9, 0.5, 0.2 and N takes the values from 1 to 100.

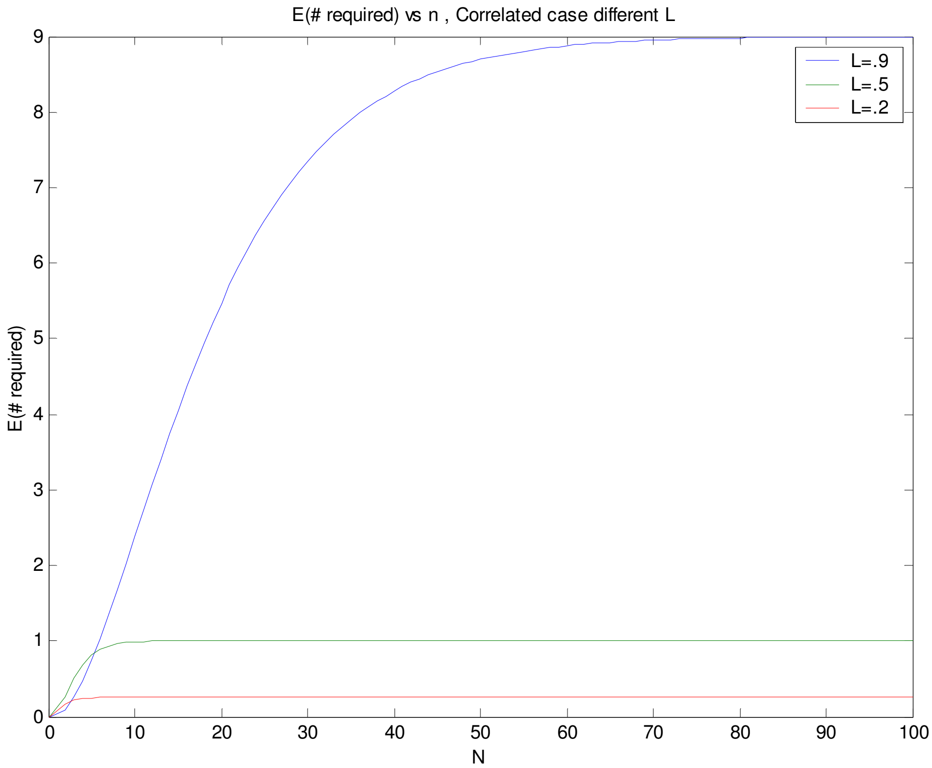


Figure (3.13): *E(# required) vs N*

It's clear from the figure that the required messages increase with increase of the value of l, also for small values of l, the # of required messages tends to be steady for relatively small values of N,

Figure 3.14 shows E(# required) vs N in the uncorrelated case for different values of loss l = 0.9, 0.5, 0.2 and N takes the values from 1 to 100.

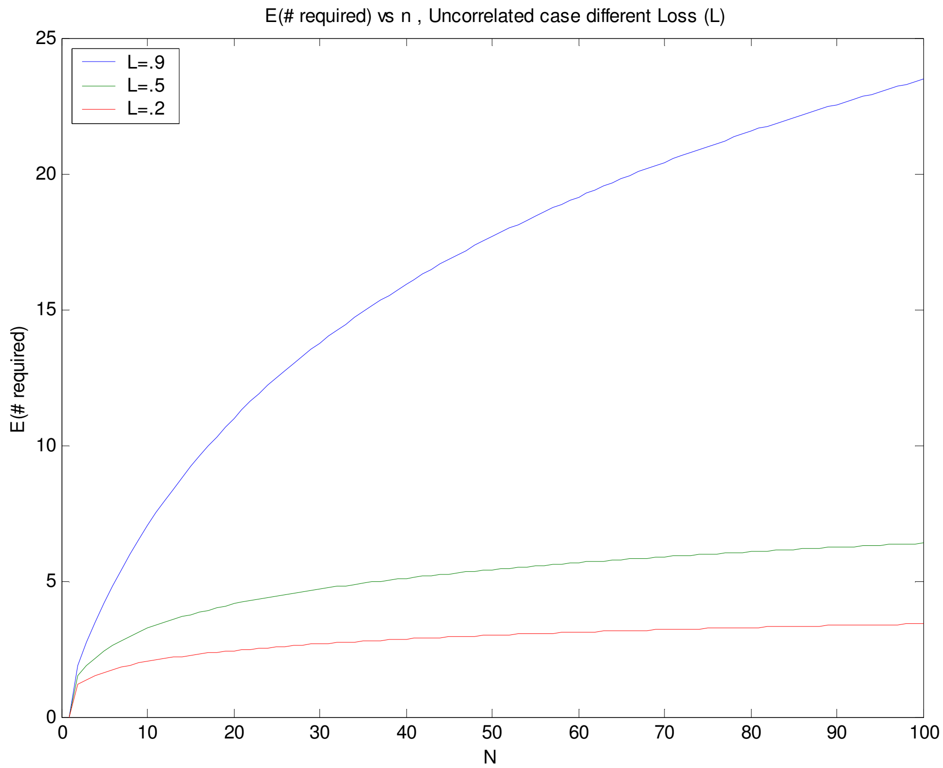


Figure (3.14)*: E(# required) vs N*

It's clear from the figure that the required messages increase with increase of the value of *l*, we may conclude from the figure that E(# required) becomes quickly almost constant for small values of *l*.

Figure 3.15a compares E(# required) for the correlated and uncorrelated case for different values of N, where N ranges between 1 and 200, and *l* = 0.9, it is clear from the Figure 3.15a, that the values of E(# required) in the uncorrelated case are greater than those of the correlated case for all values of N.

The same result holds in Figure 3.15b, but the ratio between the two values increases as *l* decreases.

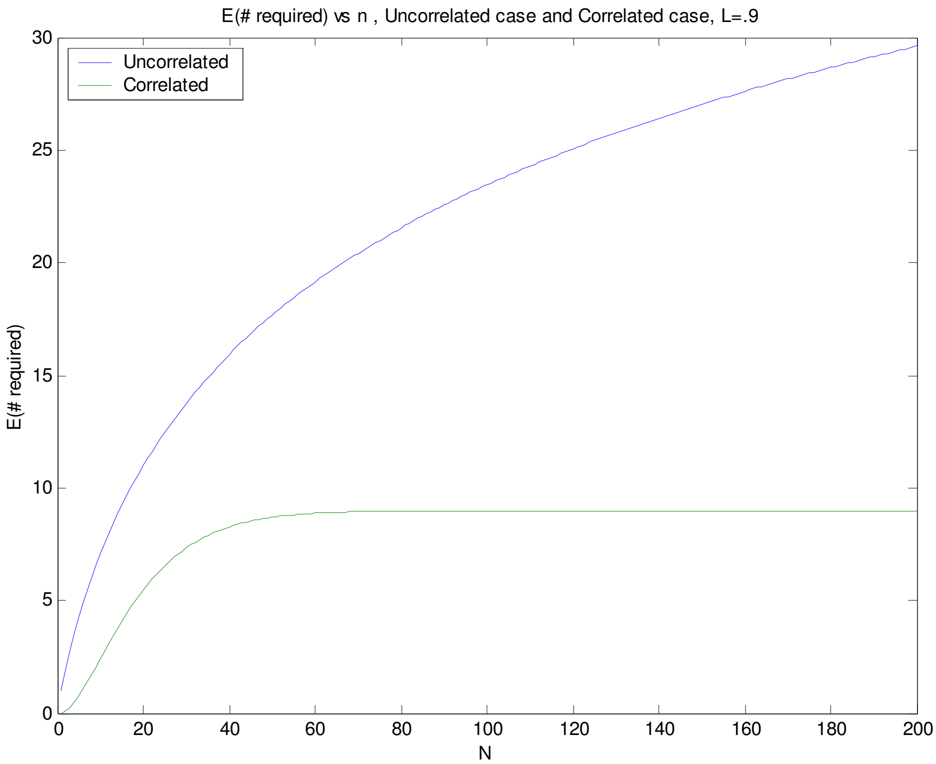


Figure (3.15a): *E(# required) vs N*

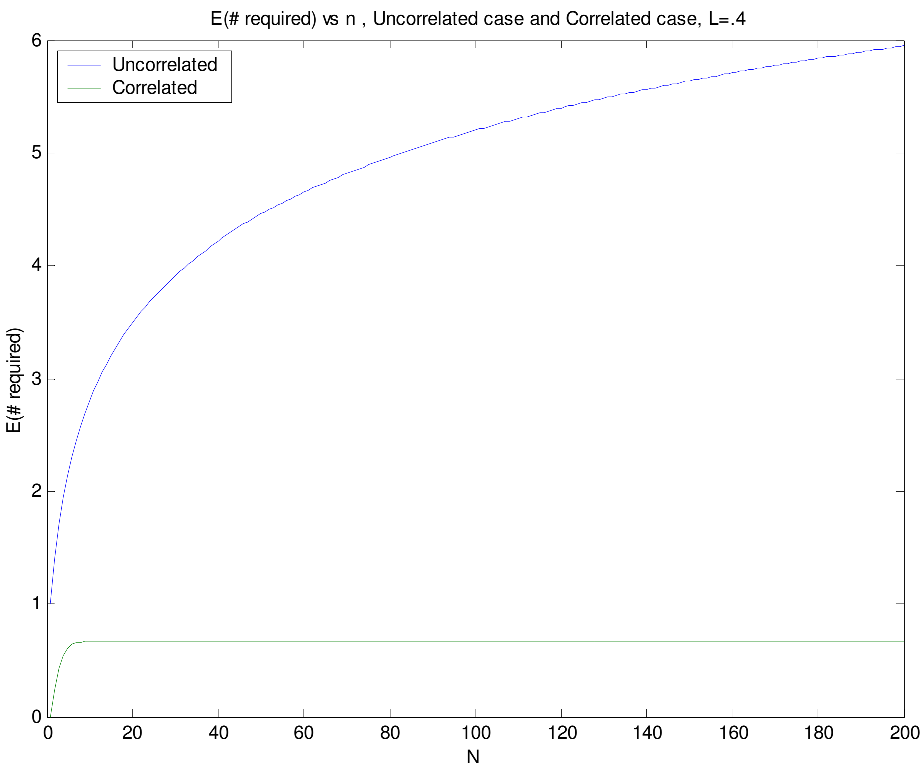


Figure (3.15b): *E(# required) vs N*

3.4 Conclusions

There are certain similarities between the results we found in this chapter and those of the previous chapter regarding the relationship between the pdfs of the study and the performance metrics of both time and extra messages, but we had to add new more performance metrics due to the effect of probability of loss (*l*) in receiving the messages in the network.

3.4.1 Time Performance Metrics

a- $E(t_{min\ e})$ or effective $E(t_{min})$: the expected time of the earliest messages sent but not completely dropped in the network – which corresponds to $E(t_{min})$ in non lossy case – was found similar to the nonlossy case, in that:

$E(t_{min\ e})$ decreases as N (# of processes) increases, and that the uniform distribution outperforms – though slightly – the logistic pdf, and the logistic distribution outperforms the decaying exponential distribution.

For the logistic distribution, regarding the probability of loss (*l*), it was found that $E(t_{min\ e})$ increases with increase of (*l*) in the correlated case which is the case studied owing to the fact that the loss in this case is greater than in the uncorrelated case.

b- $E(t_{max})$: expected time selected by the last process that actually generates messages, which is a characteristic metric of the loss case, was found to increase with increase of N for the three pdfs; and also as (*l*) increases, $E(t_{max})$ increases.

Comparing $E(t_{max})$ for the three pdfs in the uncorrelated case where it is higher than that of the correlated case, it was found that the uniform distribution was the best pdf in this case, with the logistic coming next to it.

3.4.2 Extra and Required Messages

Owing to the possibility of loss of messages, we introduced a new metric in calculating messages, i.e. the required messages, which means the # of messages required to fully suppress a group of processes of size N, this is equivalent to the number of messages generated when there exists no transmission delay.

For the uncorrelated case, E(# required) was found to be higher than that of the correlated case, and also that the E(# required) increases with the increase of (*l*), while E(# extra) is higher in the correlated than in the uncorrelated case.

Chapter Four

Optimization

of the Performance Metrics

4.1 Introduction

So far, we have studied the two performance metrics i.e. time elapsed and extra messages separately, and found that the logistic distribution outperforms generally both the uniform and the exponential distributions with regard to time elapsed, and it was also comparable in the extra messages produced, to the exponential distribution, which is generally used in the suppression technique.

The question arises as to the minimization of both performance metrics (time elapsed and extra messages). The difficulty of solving this problem lies in the fact that trying to minimize one of them leads to increase of the values of the other.

In this chapter we will try to examine the relationship between the two performance metrics for the three pdf's in both lossy and nonlossy cases, with the aim of improving the contribution of the pdf's in the suppression technique to enhance the scalability of the communication in the networks.

4.2 Optimization in the Nonlossy Case

The presence of simple formulas for the time elapsed metric for the three pdfs makes it comparatively easy to build the relation between extra messages and time elapsed in the nonlossy case; generally, we substitute time elapsed represented by $E(t_{min})$ for the parameters (T, α, a) in the formulas of the extra messages represented by E(#extra). Then we illustrate the relationship by drawing the corresponding figures depending on:

1- Selected values of N (# of processes) 3, 10, 50 and 100 for a fixed value of Δ.

2- Selected values of Δ (delay) 0.01, 0.1, 1 and 10 for a fixed value of N.

4.2.1 Uniform Distribution

To show the relationship between the two performance metrics for various values of N and Δ we eliminate the common parameter (T) from the two formulas of the E(#extra) & E(t_{min}), we get an equation relating the two performance metrics where, extra messages is given as a function of time elapsed.

$$E(t_{min}) = \frac{T}{N+1}, \text{ and}$$

$$E(\#\text{ extra}) = N(\frac{\Delta}{T}) - (\frac{\Delta}{T})^N$$

Let Y = E(#extra) and x = E(t_{min}), then we have:

$$Y = \frac{N.\Delta}{x(N+1)} - \left(\frac{\Delta}{x(N+1)}\right)^N \qquad (4.1)$$

For Varying N:-

Figures 4.1a,b,c,d represents the graph of formula (4.1) for x = 1 to 10, and N having values 3,10, 50 and 100.

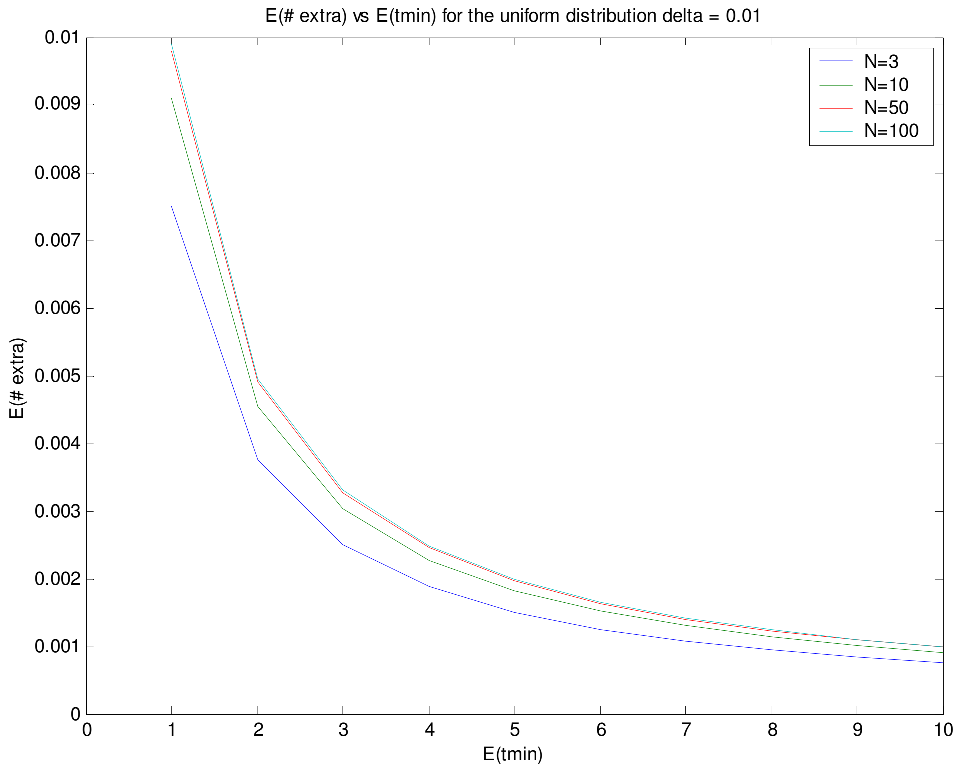


Figure (4.1a): *E(#extra) vs E(t_{min})*

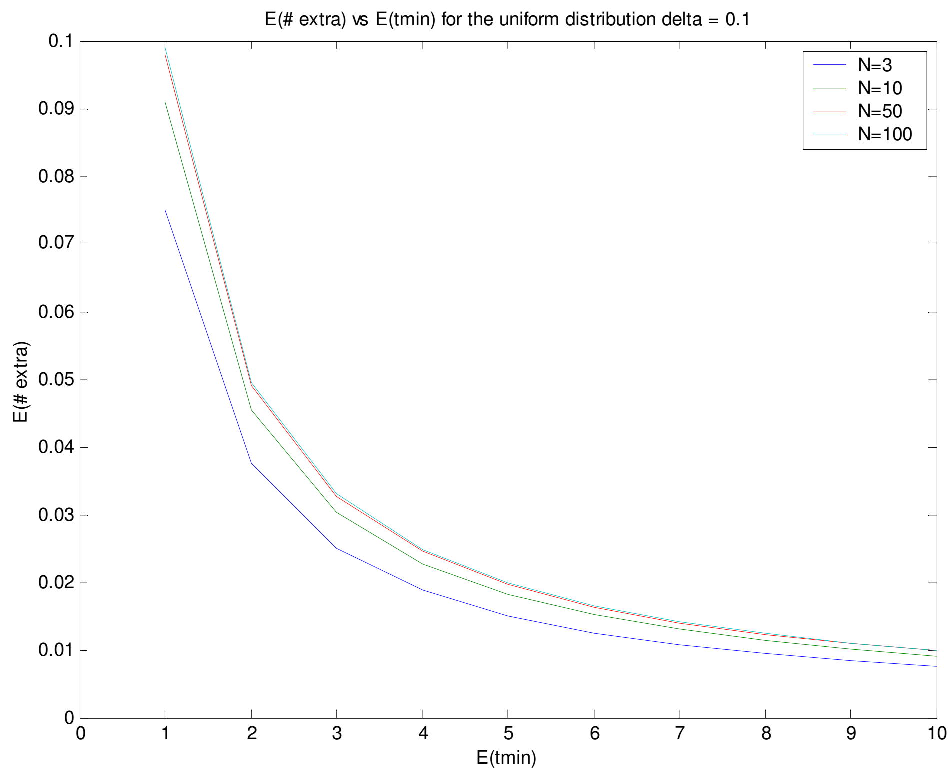


Figure (4.1b): *E(#extra) vs E(t_{min})*

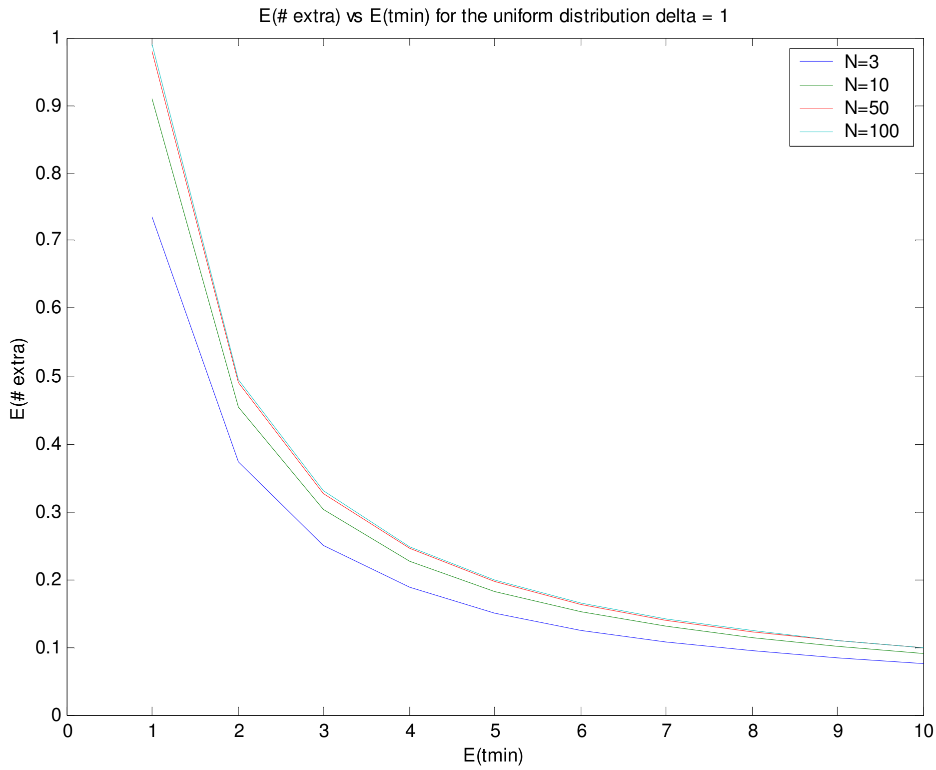


***Figure (4.1c)**: E(#extra) vs E(t_{min})*

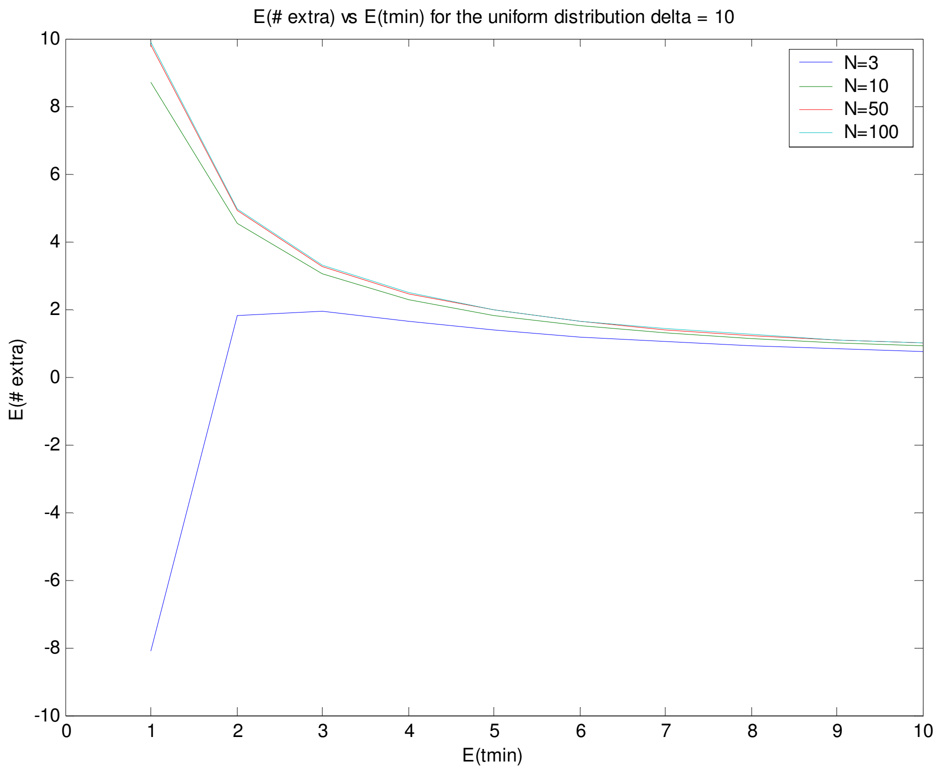


***Figure (4.1d)**: E(#extra) vs E(t_{min})*

From the above figures we notice the following:-

1- For a fixed N, the relationship between the number of extra messages which measures the overhead and time elapsed which measures the responsiveness of the system is an inverse one, i.e. as time elapsed increases, extra messages decrease and vice versa.

2- For a fixed $E(t_{min})$, increase of N leads to increase in E(#extra) also for a fixed value of E(#extra) increase of N leads to increase in $E(t_{min})$.

3- For large values of N, As $N \rightarrow \infty$ the relation E(#extra) vs $E(t_{min})$ is given by:

$$y \approx \frac{\Delta}{x}$$

4- The largest value of extra messages differs with value of Δ: it does not exceed the value of Δ in each case.

For Varying Δ:-

Figures 4.2a,b represent the graph of this formula (4.1) for x = 1 to 10, and Δ having values 0.01, 0.1, 1 and 10.

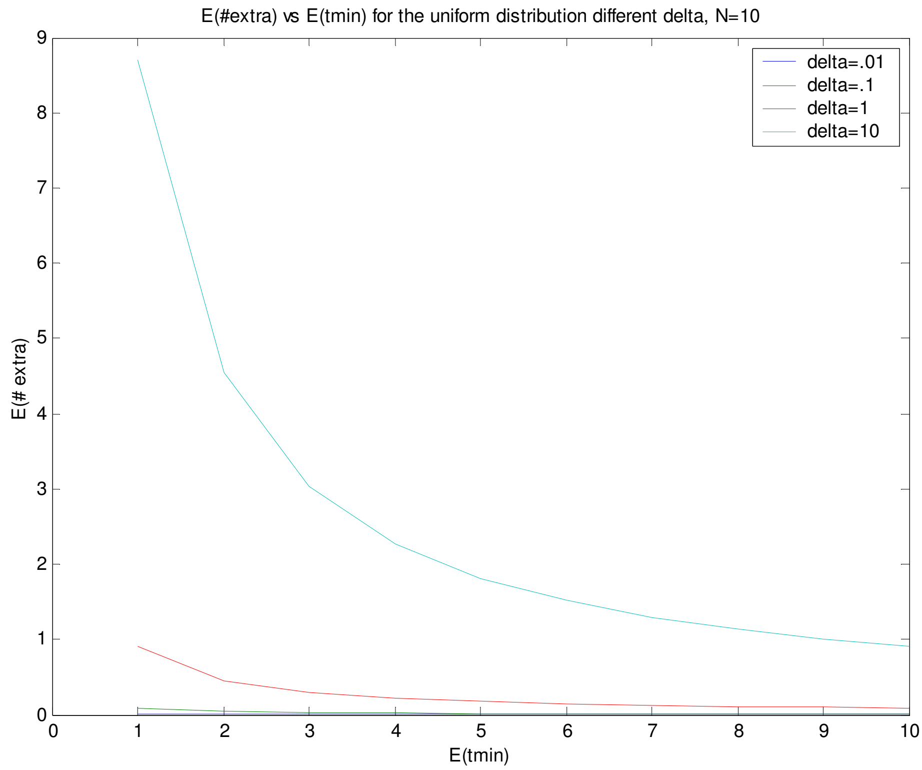


***Figure (4.2a)**: E(#extra) vs E(t_{min})*

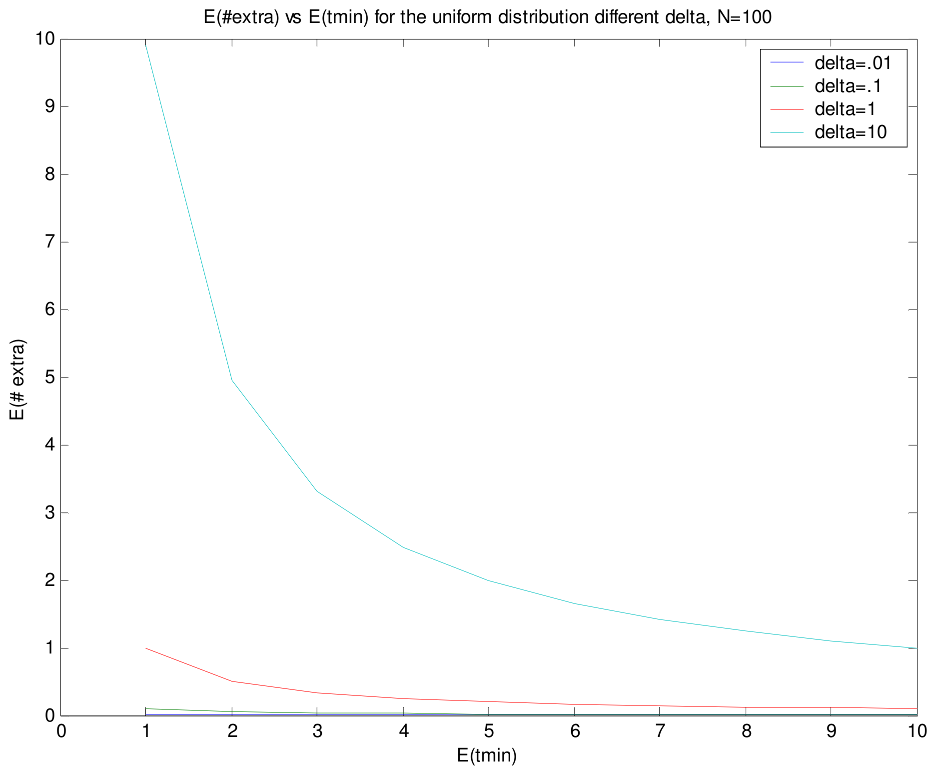


***Figure (4.2b)**: E(#extra) vs E(t_{min})*

From the above figures we notice the following:-

1- For a fixed Δ, the relationship between the number of extra messages which measures the overhead and the time elapse which measures the responsiveness of the system is an inverse one, i.e. as time elapse increases, extra messages decreases and vice versa.

2- For a fixed $E(t_{min})$, increase of Δ leads to increase in E(#extra) also for a fixed value of E(#extra) increase of Δ leads to increase in $E(t_{min})$

3- E(#extra) ranges between the two values 0 (for small values of Δ e.g. 0.01, 0.1) up to $\approx$10. (for $\Delta = 10$).

4- The largest value of extra messages differs with value of Δ: it does not exceed the value of Δ in each case.

4.2.2 Exponential Distribution

To show the relationship between the two performance metrics for various values of N and Δ, we eliminate the common parameter (α) from the two formulas of the E(#extra) and $E(t_{min})$, we get an equation relating the two performance metrics where, extra messages is given as a function of time elapse.

$$E(t_{min}) = \frac{\alpha}{N}, \text{ and}$$

$$E(\#extra) = (N-1)(1-e^{-\frac{\Delta}{\alpha}})$$

Let Y= E(#extra) and $x = E(t_{min})$,

x = $\frac{\alpha}{N} \rightarrow \alpha = Nx$ then we have:

$$Y = (N-1)(1-e^{-\frac{\Delta}{Nx}}) \tag{4.2}$$

For Varying N:-

Figures 4.3a,b,c,d represents the graph of this formula for x = 1 to 10, and N having values 3,10, 50 and 100.

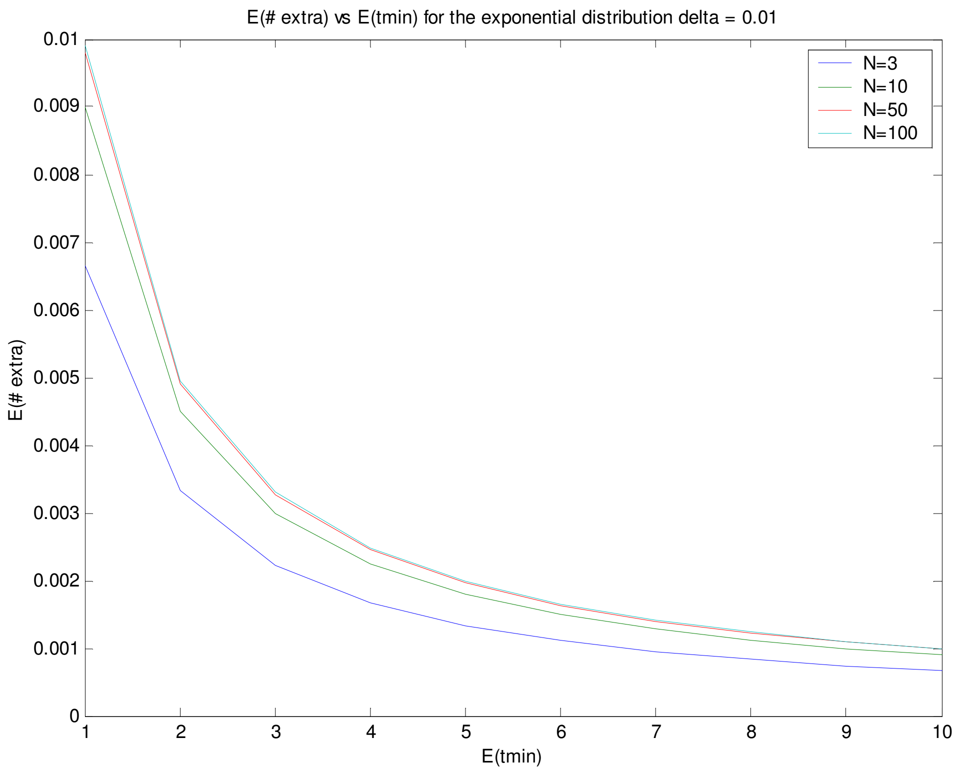


***Figure (4.3a)**: E(#extra) vs E(t_{min})*

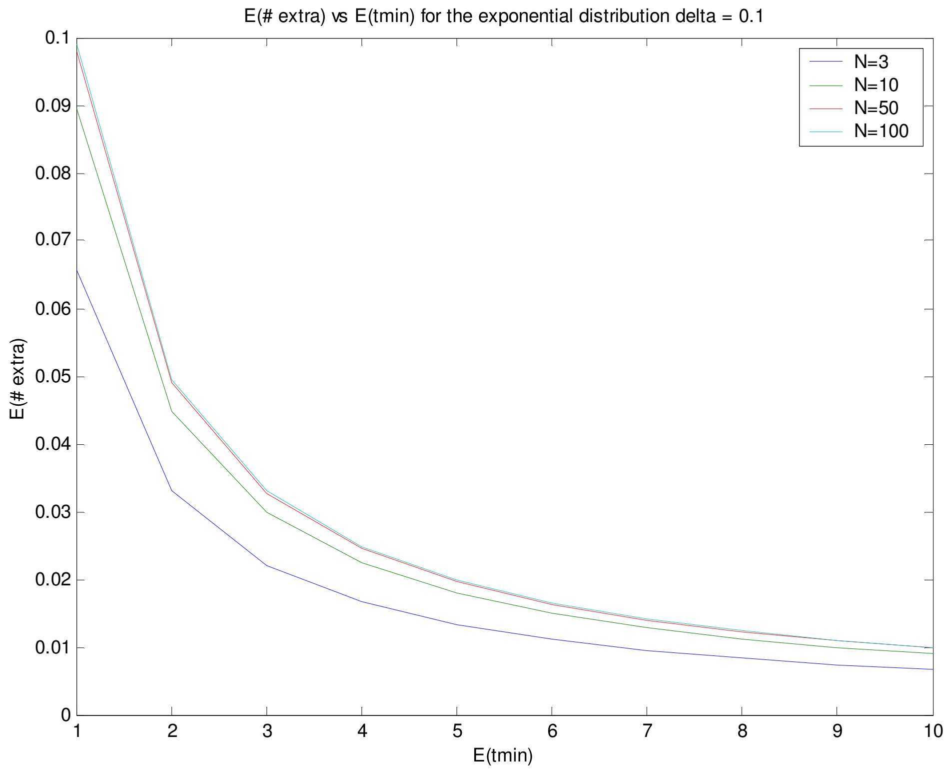


***Figure (4.3b)**: E(#extra) vs E(t_{min})*

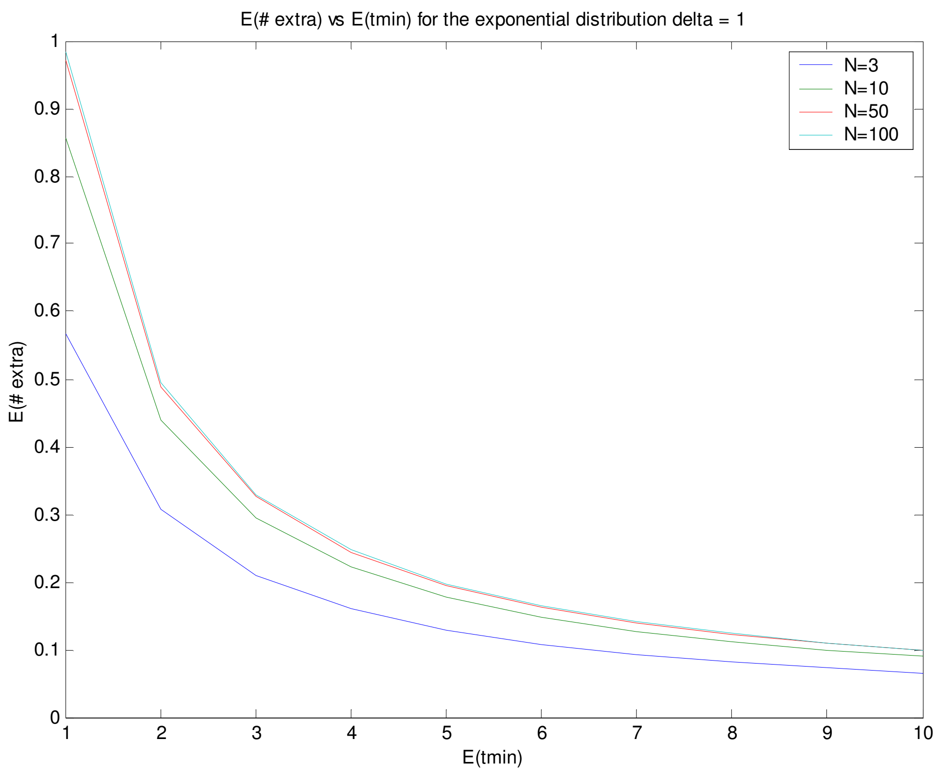


***Figure (4.3c)**: E(#extra) vs E(t_{min})*

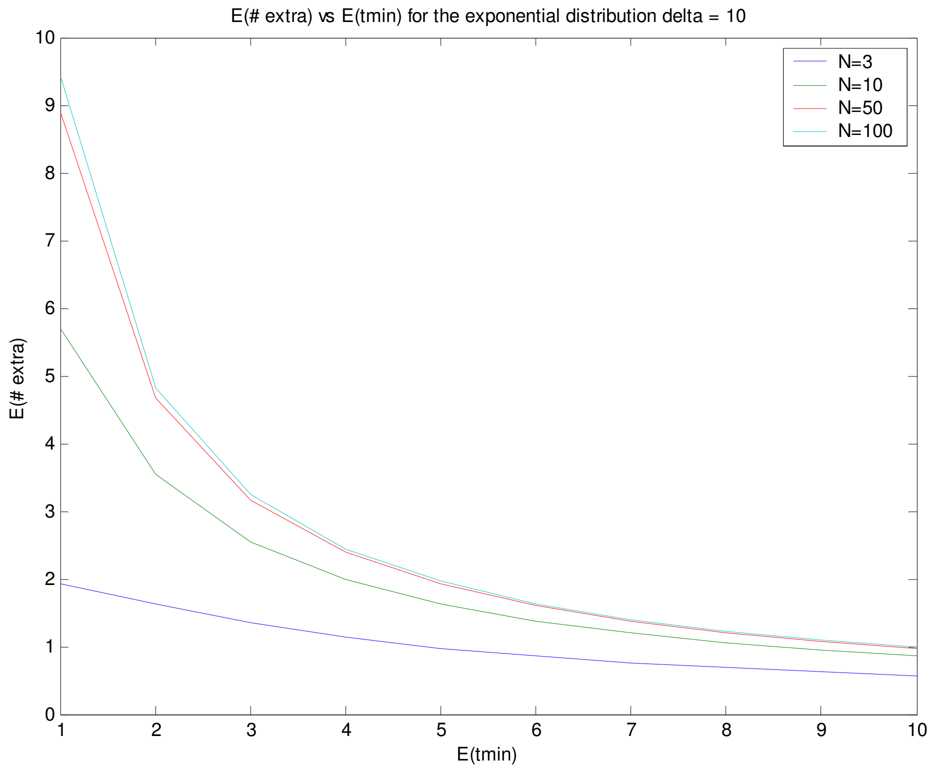


Figure (4.3d)*: E(#extra) vs E(t_{min})*

From the above figures we notice the following:-

1- The relationship between the number of extra messages which measures the overhead and the time elapse which measures the responsiveness of the system is an inverse one, i.e. as time elapse increases, extra messages decreases and vice versa.

2- For a fixed E(t_{min}), increase of N leads to increase in E(#extra) also for a fixed value of E(#extra) increase of N leads to increase in E(t_{min})

3- For determining the relationship between E(#extra) and E(t_{min}) for large values of N:

using the Taylor expansion for (e^x):

$$e^x = 1 + x + \frac{x^2}{2!} + ...$$

$$e^x \geq 1 + x \Rightarrow 1 - e^x \leq -x, x > 0$$

$$1 - e^{-\frac{\Delta}{Nx}} \leq \frac{\Delta}{Nx}$$

we have

$$E(\#_{extra}) = (N-1)(1 - e^{-\frac{\Delta}{Nx}}),$$

for large values of N,

$$E(\#_{extra}) \leq N.\frac{\Delta}{Nx} = \frac{\Delta}{x}$$

For Varying Δ:-

Figures 4.4a,b represents the graph of formula (4.2) for x = 1 to 10, and Δ having values 0.01, 0.1, 1 and 10.

From the above figures we notice the following:-

1- For a fixed Δ, the relationship between the number of extra messages which measures the overhead and the time elapse which measures the responsiveness of the system is an inverse one, i.e. as time elapse increases, extra messages decreases and vice versa.

2- For a fixed $E(t_{min})$, increase of Δ leads to increase in E(#extra) also for a fixed value of E(#extra) increase of Δ leads to increase in $E(t_{min})$

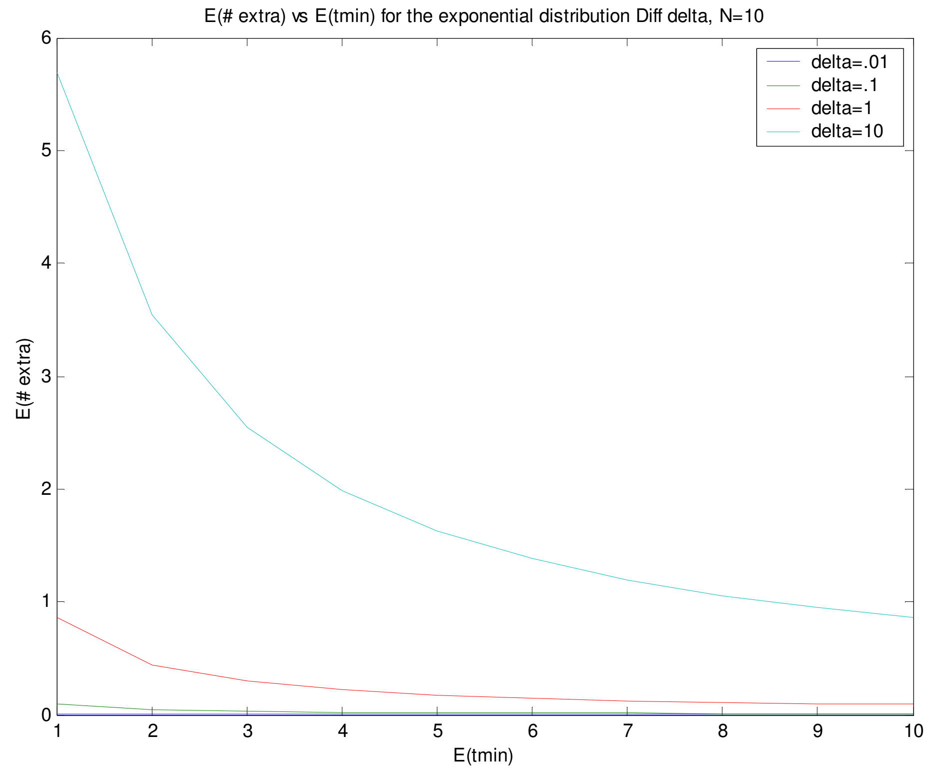


***Figure (4.4a)**: E(#extra) vs E(t_{min})*

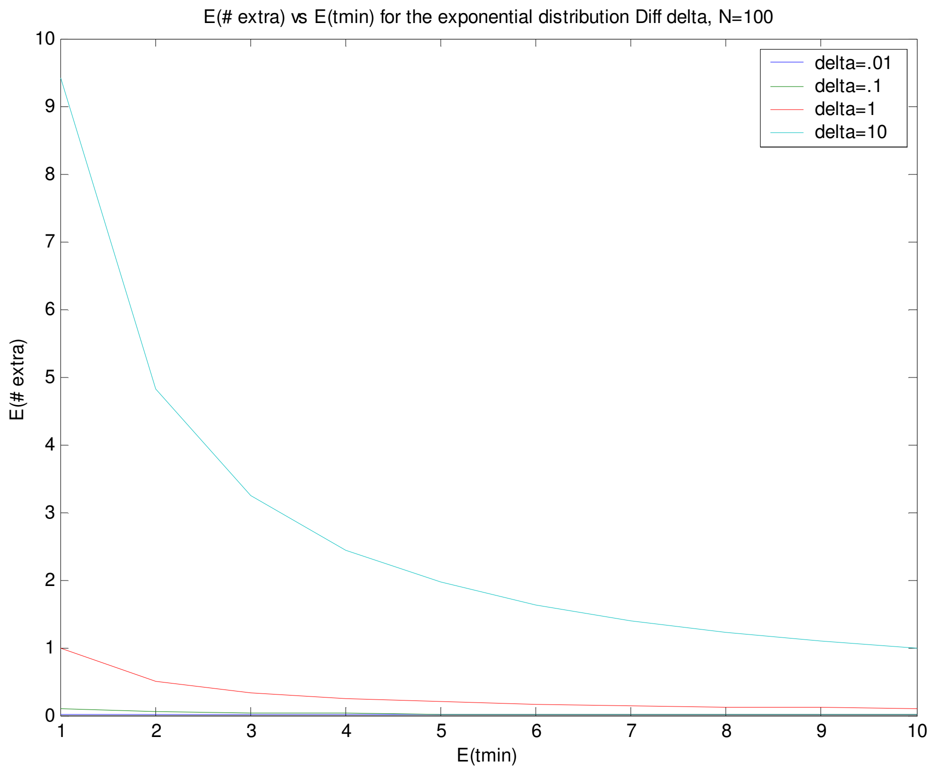


***Figure (4.4b)**: E(#extra) vs E(t_{min})*

3- E(#extra) ranges between the two values 0 (for small values of Δ e.g. 0.01, 0.1) up to ≈ 9 (for Δ = 10).

4- The largest value of extra messages differs with value of Δ: it does not exceed the value of Δ in each case.

4.2.3 Logistic Distribution

To show the relationship between the two performance metrics for various values of N and Δ, we eliminate the common parameter (a) from the two formulas of the E(#extra) and E(t_{min}), we get an equation relating the two performance metrics, where, extra messages is given as a function of time elapse.

E(t_{min}) $\approx \frac{2}{aN}$, (the approximation value of the logistic) and

$$E(\#extra) = NP(\Delta) - 1 + N e^{-\Delta a} \int_{\Delta}^{\infty} \frac{2ae^{-a(t-\Delta)}}{\left(1+e^{-at}\right)^2} \cdot \left[2 - \left(\frac{2}{1+e^{-a(t-\Delta)}}\right)\right]^{N-1} dt$$

Let y = E(#extra) and x = E(t_{min}), $a = \frac{2}{Nx}$ then we have:

$$y = NP(\Delta) - 1 + N e^{-\frac{2\Delta}{Nx}} \int_{\Delta}^{\infty} \frac{2 * 2e^{-\frac{2(t-\Delta)}{Nx}}}{Nx\left(1+e^{-\frac{2t}{Nx}}\right)^2} \cdot \left[2 - \left(\frac{2}{1+e^{-\frac{2(t-\Delta)}{Nx}}}\right)\right]^{N-1} dt \quad (4.3)$$

Varying N:-

Figures 4.5a,b,c,d represents the graph of this formula for x=1 to 10, and n having values 3,10, 50 and 100.

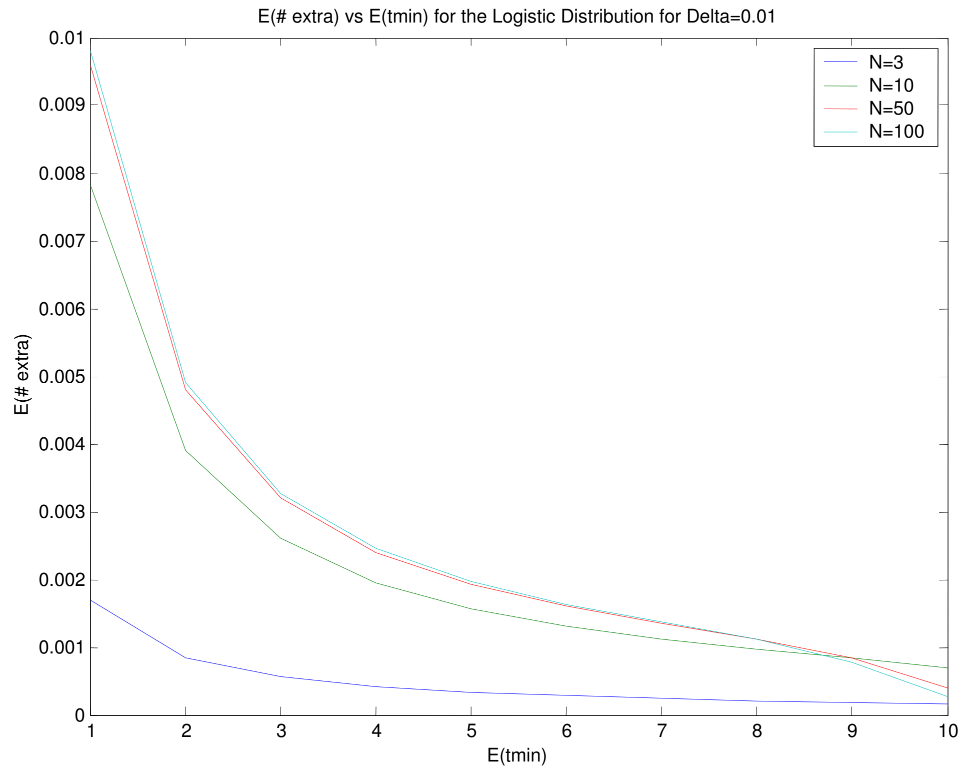


Figure (4.5a): *E(#extra) vs E(t_{min})*

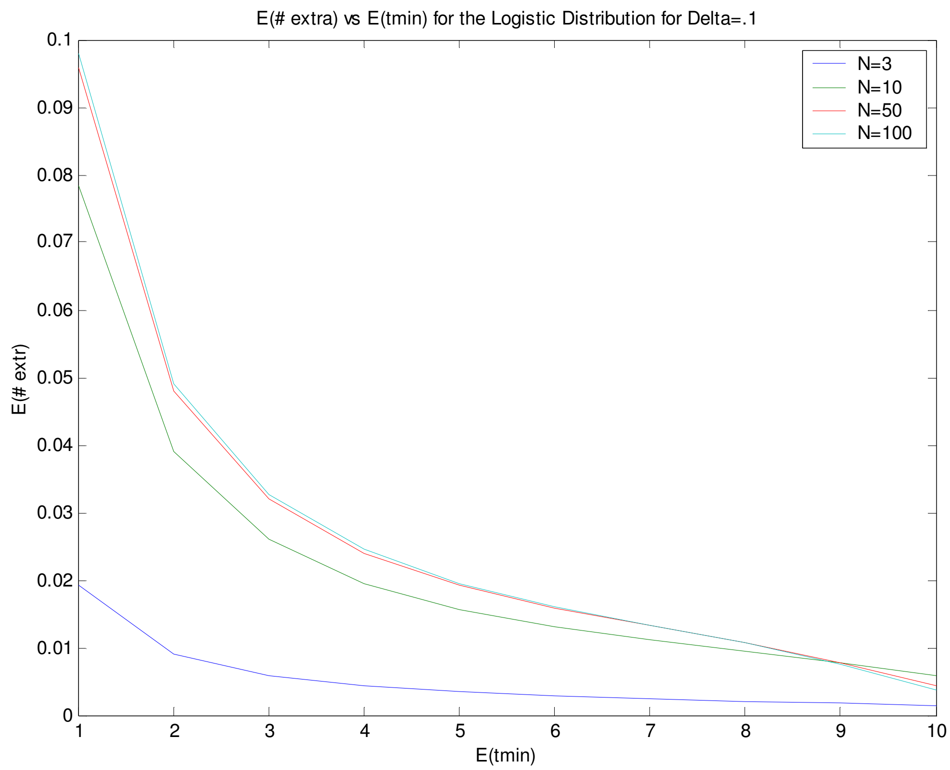


Figure (4.5b): *E(#extra) vs E(t_{min})*

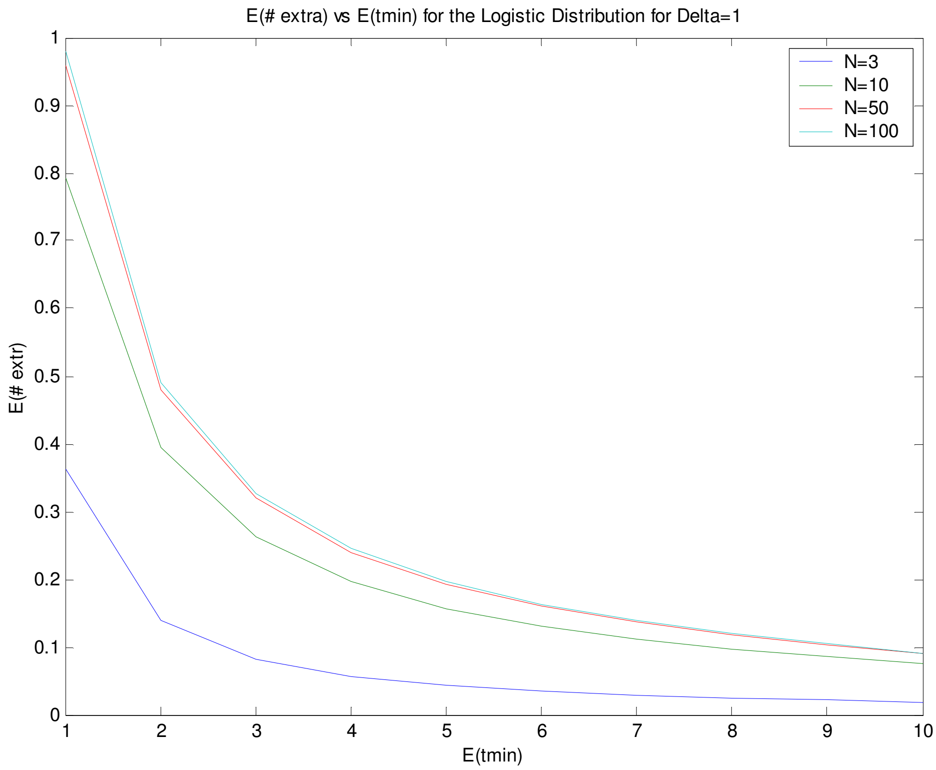


Figure (4.5c): *E(#extra) vs E(t_{min})*

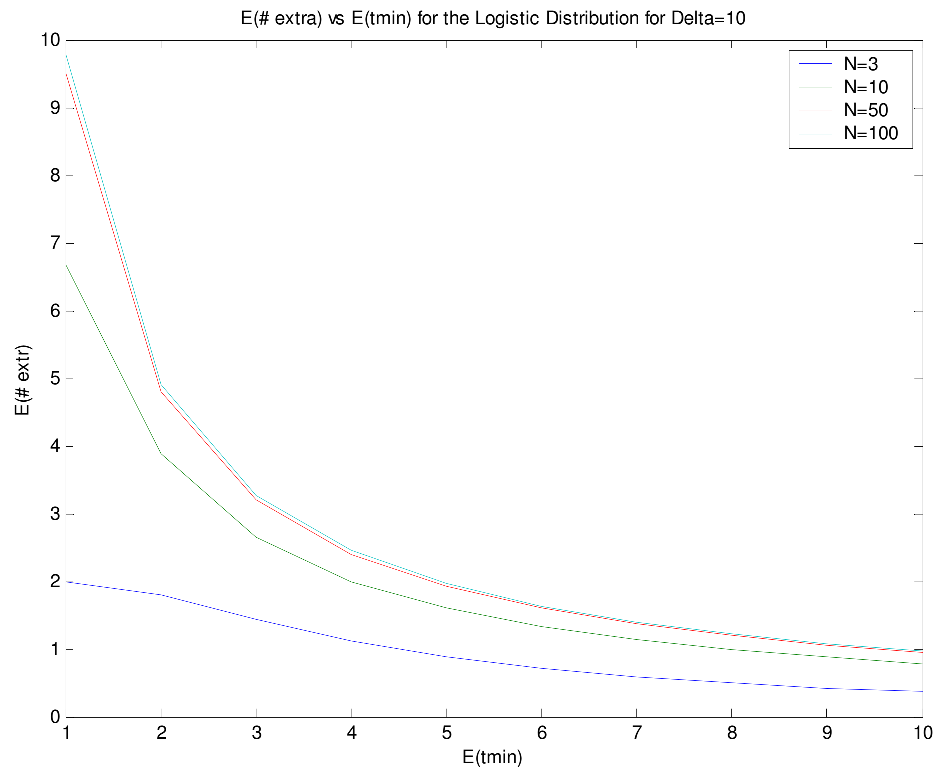


Figure (4.5d): *E(#extra) vs E(t_{min})*

From the above figures we notice the following:-

1- The relationship between the number of extra messages which measures the overhead and the time elapse which measures the responsiveness of the system is an inverse one, i.e. as time elapse increases, extra messages decreases and vice versa.

2- For a fixed $E(t_{min})$, increase of N leads to increase in E(#extra) also for a fixed value of E(#extra) increase of N leads to increase in $E(t_{min})$

For Varying Δ:-

Figures 4.6a,b represents the graph of this formula for x = 1 to 10, and Δ having values 0.01, 0.1, 1 and 10.

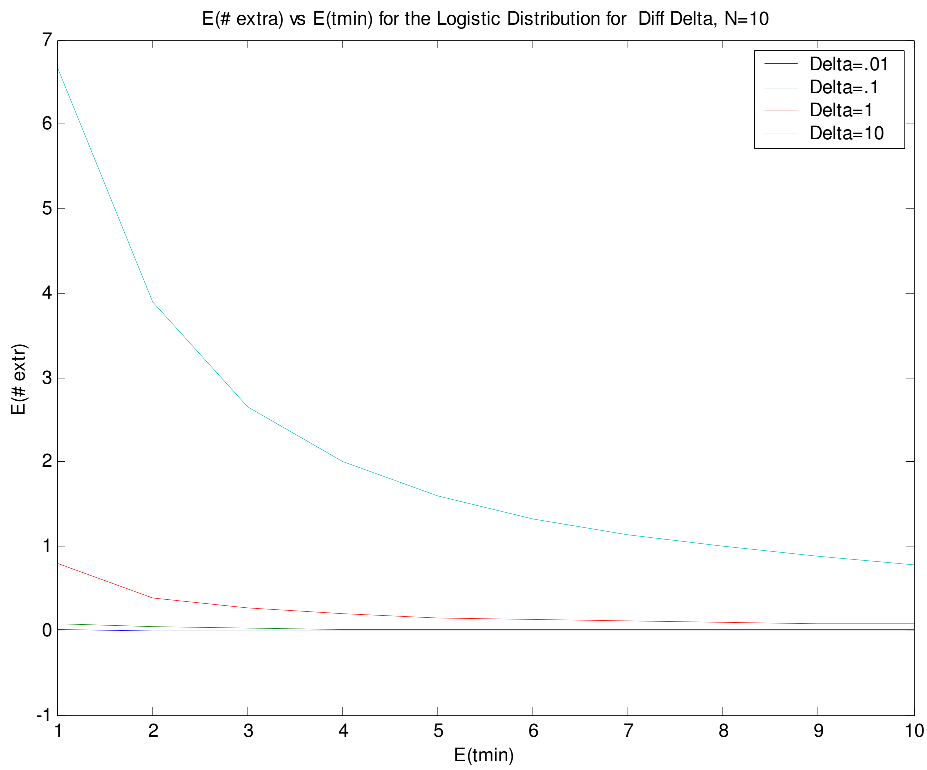


Figure (4.6a)*: E(*#extra*) vs E(t_{min})*

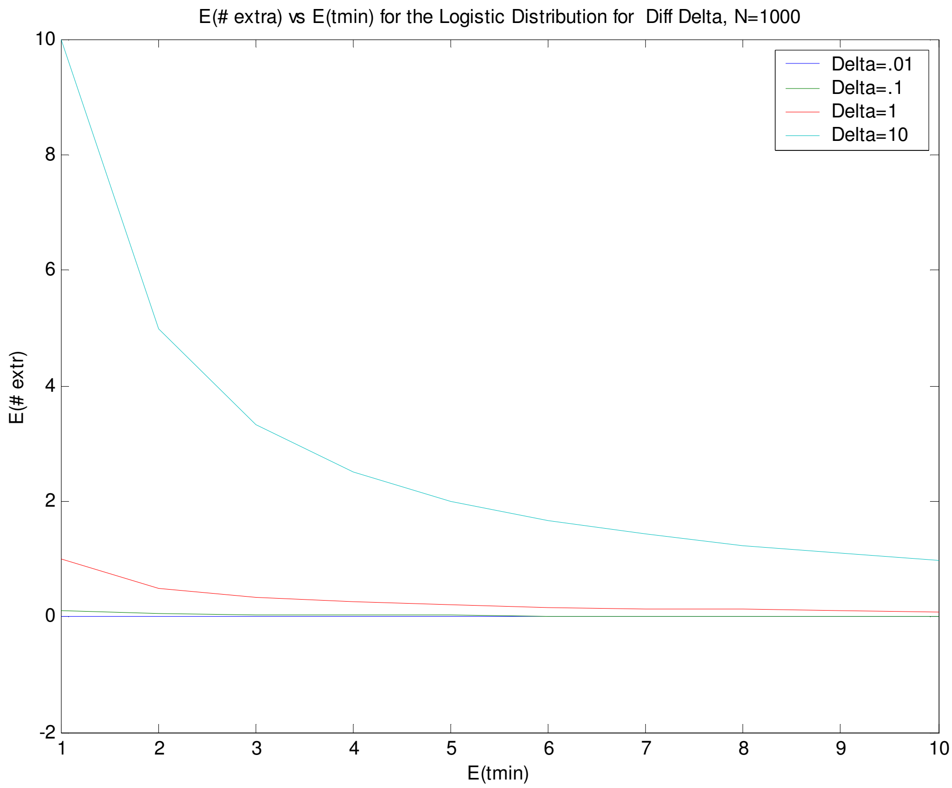


Figure (4.6b)*: E(#extra) vs E(t_{min})*

From the above figures we notice the following:-

1- For a fixed Δ, the relationship between the number of extra messages which measures the overhead and the time elapse which measures the responsiveness of the system is an inverse one, i.e. as time elapse increases, extra messages decreases and vice versa.

2- For a fixed E(t_{min}), increase of Δ leads to increase in E(#extra) also for a fixed value of E(#extra) increase of Δ leads to increase in E(t_{min})

3- E(#extra) ranges between the two values 0 (for small values of Δ e.g. 0.01, 0.1) up to ≈ 10 (for Δ = 10).

4- The largest value of extra messages differs with value of Δ: it does not exceed the value of Δ in each case.

4.2.4 Comparison of the Uniform, Exponential, and Logistic pdfs

Figures 4.7a, b give graphical representations of the relationship between E(#extra) and E(t_{min}) for the three pdfs, the first for N = 10 and the second for N = 50.

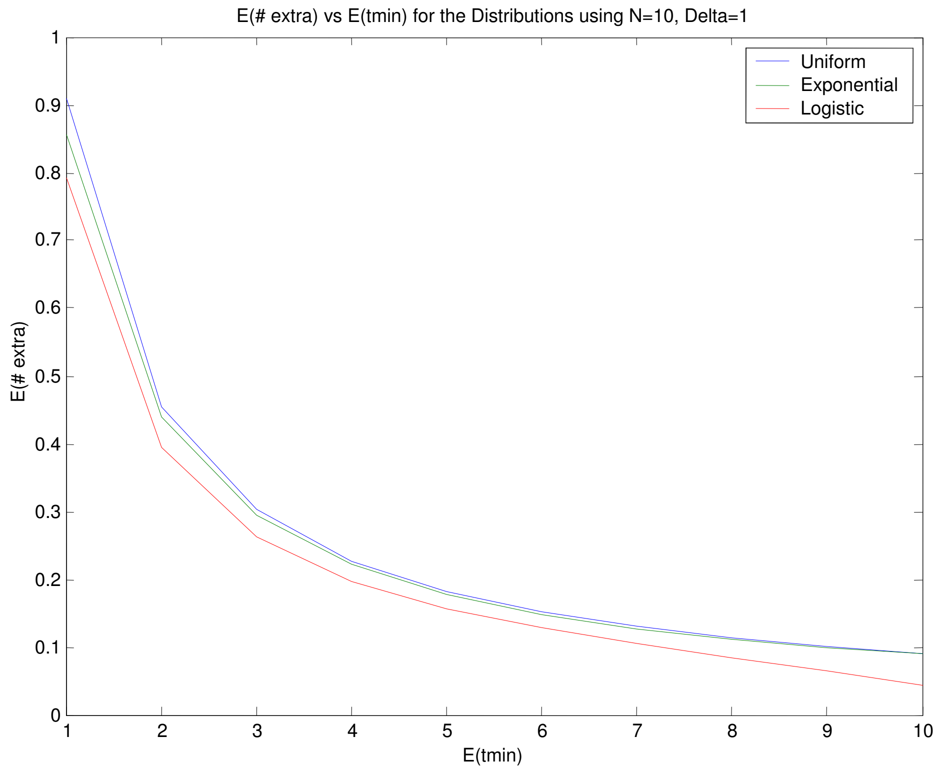


***Figure (4.7a)**: Comparison: E(#extra) vs E(t_{min})*

Both figures show that the logistic distribution function is the least sensitive one in the relationship between E(#extra) and E(t_{min}), and this is clear from the fact that its graph lies below the graph of the other two distribution functions in both Figures 4.7a,b.

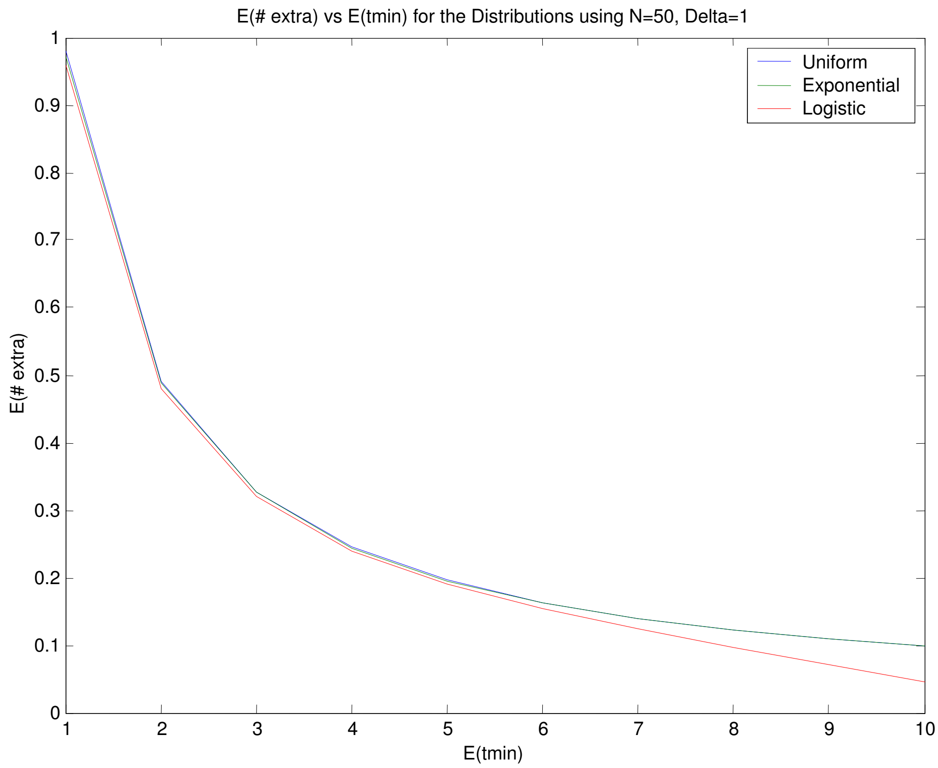


***Figure (4.7b)**: Comparison: E(#extra) vs E(t_{min})*

With increase of E(t_{min}) we notice that the uniform and exponential distributions become almost identical while the logistic becomes more distinct and still below both of them.

For large values of N and small values of E(t_{min}) the graphs corresponding to the three pdfs are very close to each other, but for large values of N and large values of E(t_{min}) the graph of the logistic becomes clearly distinct from the other two graphs and still lying below them.

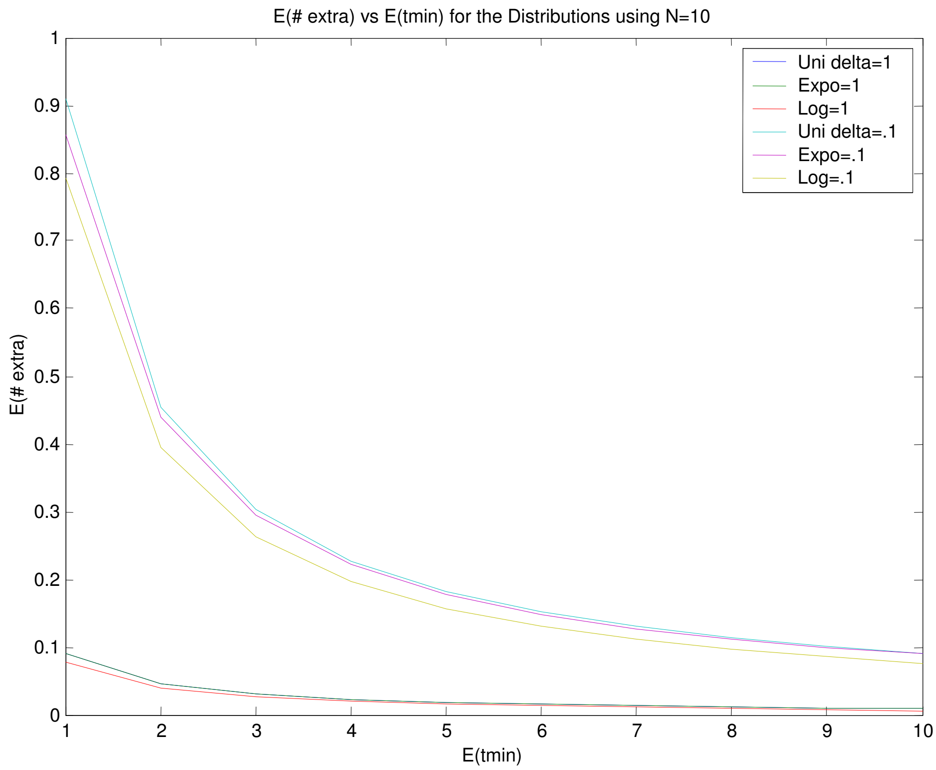


***Figure (4.8a)**: Comparison: E(#extra) vs E(t_{min})*

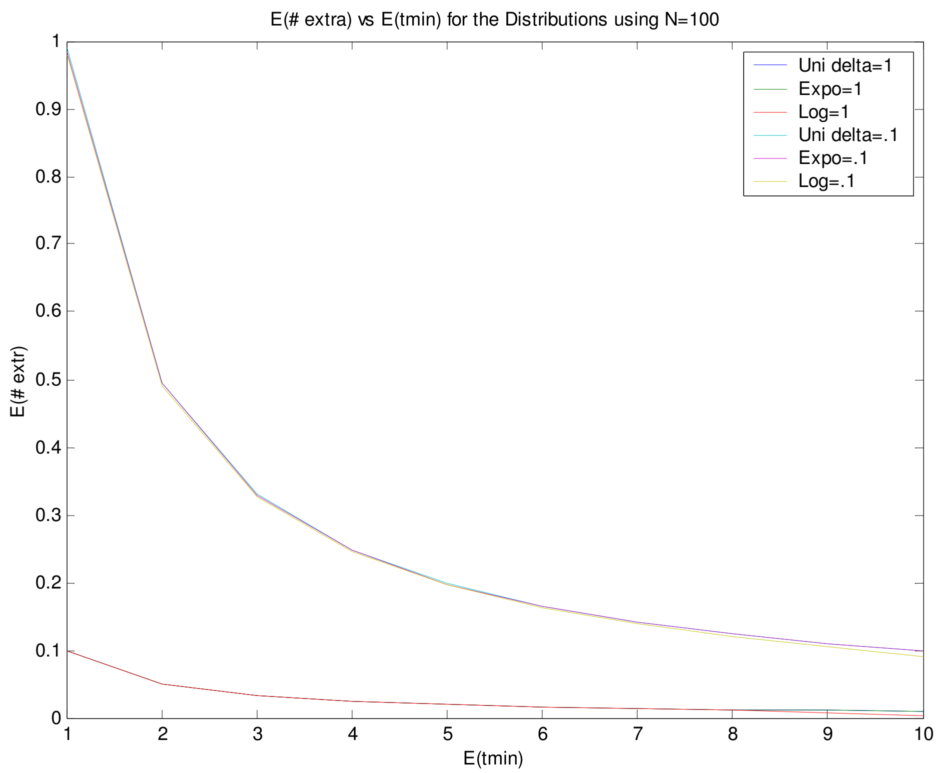


***Figure (4.8b)**: Comparison: E(#extra) vs E(t_{min})*

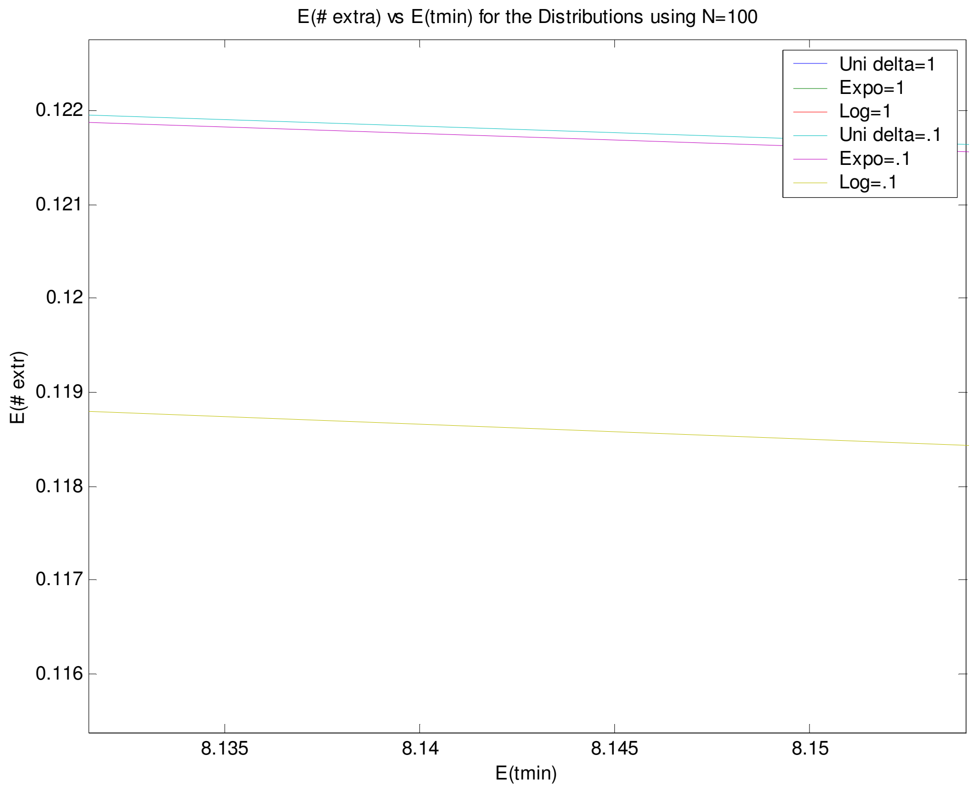


Figure (4.8c): *Comparison: E(#extra) vs E(t_{min})*

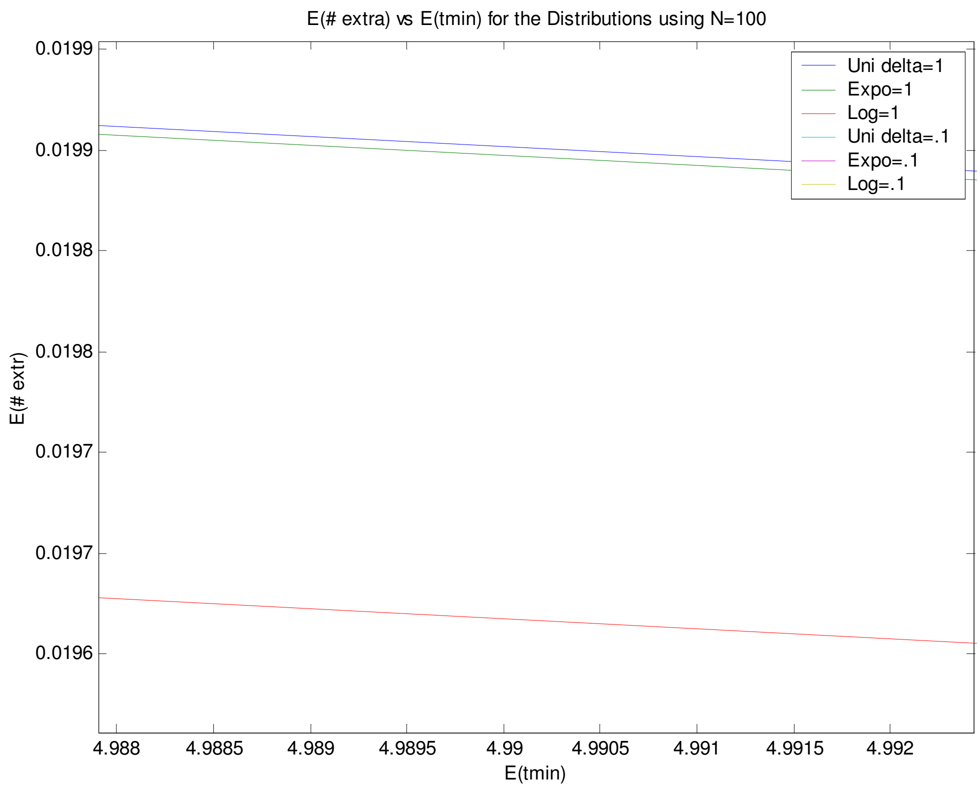


Figure (4.8d): *Comparison: E(#extra) vs E(t_{min})*

From Figures 4.8a,b,c,d we notice that:

* For large values of N, for the three pdfs, we can say that extra messages become inversely proportional to N.

* For large values of N, for the three pdfs, we can say that extra messages become directly proportional to the value of Δ.

* For large values of N, for the three pdfs, we can say that extra messages become inversely proportional to $E(t_{min})$.

Concluding, the logistic distribution is the least sensitive distribution in the relationship between E(#extra) and $E(t_{min})$, i.e. for the same value of $E(t_{min})$, the logistic distribution function produces less extra messages than the other two distributions, which means that it is a better candidate for solving the problem posed above: optimizing the performance metrics.

4.3 Optimization in the Lossy case

Our study of the lossy case in chapter three was limited to the state where the time delay ($\Delta = 0$); this choice of Δ excludes by definition the generation of extra messages and limits the messages generated to required messages, the number of which is independent of the pdf chosen; hence we will be left with measures of time elapsed in our attempt for optimization of the performance metrics. This was actually studied in chapter three, where we found that:

1. $E(t_{min\ e})$ decreases as N (# of processes) increases, and that the uniform distribution outperforms – though slightly – the logistic pdf, and the logistic distribution outperforms the decaying exponential distribution.

2. Comparing $E(t_{max})$ for the three pdfs in the uncorrelated case where it is higher than that of the correlated case, it was found that the uniform distribution was the best pdf in this case, with the logistic coming next to it, while the decaying exponential ranked third in this metric.

These results, though limited in scope to the performance metric of time, taken together with the results of the nonlossy case, encourage the application of the logistic distribution function in communication networks, and making further studies on the subject.

Chapter Five

Conclusion and Suggestions

for

Future Work

5.1 Conclusion

This thesis addresses the problem of scalability in multicast routing depending on the suppression technique. The suppression technique employs probabilistic distribution functions in its execution, of which the most commonly used are the uniform and the exponential distributions.

The need for the use of other pdfs is apparent, owing to the increase of multicast routing and increasing sizes of the participant processes in real-time applications like video conferencing, whiteboards, games, etc. Since it was found that while the uniform distribution is more efficient regarding one performance metric, namely time elapsed which measures response time, it is less efficient than the exponential distribution for the other performance metric, namely extra messages which measures messaging overhead, this is generally true in both types of networks: the lossy, and the nonlossy ones.

A new pdf was proposed to enhance the suppression technique, which is a modified form of the logistic distribution. This distribution was tested regarding the performance metrics of the suppression technique in the lossy and the nonlossy networks and it proved to excel the two other pdfs in the time elapsed in the nonlossy case; In the lossy case it comes next to the uniform distribution excelling the exponential in the time metric $E(t_{max})$ which measures the expected completion time of the algorithm, while in the time metric $E(t_{min\ e})$ which measures the expected minimum time for suppression of the algorithm, the performance of the three distributions where almost alike with the uniform distribution coming first followed by the logistic distribution, the curves of the two distributions become closer, with increase of N or Δ.

As regards the other performance metric in the nonlossy case, i.e. extra messages, the proposed distribution outperformed the uniform pdf while it competes with the exponential excelling it for values of Δ less than 0.9 for small values of N, and for values of Δ less than 0.4 for large values of N, In the lossy case (Δ = 0) required messages in the uncorrelated case were proved to be greater than in the correlated case for all values of N and Δ.

In studying the optimization of both metrics, the logistic distribution is the least sensitive distribution in the relationship between E(#extra) and $E(t_{min})$, i.e. for the same value of $E(t_{min})$, the logistic distribution produces less extra messages than the other two distributions, which means that it is a better candidate for solving the problem posed above: optimizing the performance metrics.

In trying to implement analytical methods of study, we deduced some approximate formulas, proved given formulas using calculus methods and rules, and in other cases Matlab software was used to compare the values of the infinite integrals using the Trapezoidal rule to calculate the values of the performance metrics for the three distributions: uniform, exponential and logistic.

While trying to prove the formulas used in related work to this study (Schooler, 2001; Schooler, et al, 2001),

$$E[\#required] = \frac{l(1-l^N)}{1-l} - Nl^N$$

the formula which we arrived at for calculating the average value of required messages in the correlated case was found to be

$$E[\#required] = \frac{1-l^N}{1-l} - Nl^N$$

in fact, the original formula leads to much smaller values compared with the proved formula.

5.2 Suggestions for Future Work

The work of this thesis can be extended in the following areas:

- Simulation may be used to check the results of the study practically for various values of number of processes and different values of transmission delay.
- Simulation may also be used to compare the performance of the three pdfs in lossy networks in the case where ($l \neq 0$) and ($\Delta \neq 0$).
- The proposed distribution should be implemented in suppression algorithms to be used in real networks.
- Studying the performance metrics for the 3 pdfs in the case of delay variation.
- We would also like to compare and evaluate other scalability techniques with the one used in this thesis.

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Appendices

Appendix (I)

The Logistic Differential Equation

$$\frac{dP}{dt} = rP(K-P)$$

The solution of separating the variables, we have:

$$rdt = \frac{dP}{P(K-P)}$$

by integrating both sides and using the integration formula

$$\int \frac{dP}{P(K-P)} = \frac{1}{K}\ln(\frac{P}{K-P})$$

for the right side, it results to the following

$$rt + c = \frac{1}{K}\ln(\frac{P}{K-P})$$

taking the exponential for both sides

$$e^{K(rt+c)} = \frac{P}{K-P}$$

$$\frac{K}{P} = \left[\frac{1}{e^{K(rt+c)}} + 1\right]$$

$$P = \left[\frac{K}{e^{-K(rt+c)} + 1}\right]$$

let the constant part in the integration be $e^{-Kc} = C$ and substitute it in the formula

$$P = \left[\frac{K}{Ce^{-K(rt)} + 1} \right]$$

The solution of this logistic differential equation is the logistic distribution

$$P = \left[\frac{K}{1 + Ce^{-Krt}} \right]$$

Appendix (II)

The Mean of the Logistic Distribution (used in the study)

The mean of a continuous random variable x is calculated from the formula

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx$$

In our study we use the time random variable (t) and solve for the mean

$$E(t) = \int_{0}^{\infty} tp(t)dt$$

$$p(t) \quad = \frac{2ae^{-at}}{(1+e^{-at})^2},$$

$$P(t) \quad = \frac{2}{1+e^{-at}} - 1,$$

$$E(t) = \int_{0}^{\infty} t\frac{2ae^{-at}}{(1+e^{-at})^2}dt$$

$$\int_{0}^{\infty} td(\frac{2}{1+e^{-at}} - 1)$$

but $\int udv = uv - \int vdu$

$$E(t) \ = t*\left[\frac{2}{1+e^{-at}} - 1\right] - \int (\frac{2}{1+e^{-at}} - 1)dt\Bigg|_{0}^{\infty}$$

$$= \frac{2t}{1+e^{-at}} - t + t - \int (\frac{2}{1+e^{-at}})dt\Bigg|_{0}^{\infty}$$

$$= \frac{2t}{1+e^{-at}} - \int \left(\frac{2}{1+e^{-at}}\right) dt \Big|_0^\infty$$

$$= \frac{2t}{1+e^{-at}} - 2\int \left(\frac{1+e^{-at} - e^{-at}}{1+e^{-at}}\right) dt \Big|_0^\infty$$

$$= \frac{2t}{1+e^{-at}} - 2\int \left(1 - \frac{e^{-at}}{1+e^{-at}}\right) dt \Big|_0^\infty$$

$$= \frac{2t}{1+e^{-at}} - 2t + 2\int \left(\frac{e^{-at}}{1+e^{-at}}\right) dt \Big|_0^\infty$$

$$= \frac{2t}{1+e^{-at}} - 2t + \frac{2}{-a}.\ln(1+e^{-at}) \Big|_0^\infty$$

$$= \frac{2}{a}\ln(2)$$

Appendix (III)

The Variance of the Logistic Distribution

The variance of a continuous random variable x is calculated from the formula:

$$\sigma^2 = E\left[(x-\mu)^2\right] = E\left[x^2\right] - \mu^2 \text{, where } \mu = \text{mean.}$$

For finding the variance of the logistic distribution; we use the second form:

$$\sigma^2 = E\left[x^2\right] - \mu^2$$

$$= \int_0^\infty t^2 f(t) dt - \mu^2$$

$$= \int_0^\infty t^2 \frac{2ae^{-at}}{(1+e^{-at})^2} dt - \mu^2$$

$$= \frac{1}{a}\int_0^\infty 2\frac{t^2 a^2 e^{-at}}{(1+e^{-at})^2} dt - \mu^2$$

putting T = a * t

$$= \frac{1}{a}\int_0^\infty 2\frac{t^2 a^2 e^{-at}}{(1+e^{-at})^2} dt - \mu^2$$

$$\sigma^2 = \frac{1}{a^2}\int_0^\infty 2\frac{T^2 e^{-at}}{(1+e^{-T})^2} dT - \mu^2$$

The value of the integral as computed by the trapezoidal rule =3.2787 hence

$$\sigma^2 = \frac{1}{a^2}(3.2787) - \left(\frac{2\ln(2)}{a}\right)^2$$

in case a =1 then $\sigma^2 = 1.3597$

بسم الله الرحمن الرحيم

جامعة النجاح الوطنية

كلية الدراسات العليا

دراسة استخدام التوزيع اللوجيستي في تكنيك الكبت من أجل توسيع البث المتعدد

إعداد

هادي علي خليل حمد

إشراف

د. محمد نجيب أسعد

قدمت هذه الأطروحة استكمالاً لمتطلبات درجة الماجستير في الرياضيات المحوسبة بكلية الدراسات العليا في جامعة النجاح الوطنية في نابلس، فلسطين.

2007

دراسة استخدام التوزيع اللوجيستي في تكنيك الكبت

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الملخص

إن النمو الكبير في أنظمة الاتصال التي يدعمها الحاسوب، وبخاصة الإنترنت، جعل من الملزم تصميم بروتوكولات تتميز بالجودة وقابلية التوسع لدعم أداء البنية التحتية للشبكات. ويقصد بقابلية التوسع هنا قدرة البروتوكول على مواكبة متطلبات مجموعات الأجهزة المتواصلة عندما يصبح عددها كبير جدا.

إن الطلب المتنامي باستمرار على الاتصالات، والقدرة الكبيرة للشبكات الحديثة يتطلبان باستمرار حلولا ناجعة لمشكلات الاتصال وقد كان من بين هذه الحلول إدخال " توجيه الإرسال

المتعـــدد" "Multicast Routing" وكذلك اســـــتعمال الإرســـــال الدوري غير المجاب "Un Acknowledged Periodic Messeging".

ويرتبط بهذين الحلين لمشكلة قابلية التوسع، أي قدرة المجموعات المتواصلة على التزايد الكبير في العدد، ضرورة استعمال تكنيكات معينة للتغلب على مشكلة قابلية التوسع، ومنها تكنيك الكبت "Suppression".

تستخدم هذه الدراسة اقترانات التوزيعات الاحتمالية في تكنيك الكبت بهدف تحسين قابلية التوسع لتوجيه الإرسال المتعدد في شبكات الاتصال. ويَعد التوزيعان الأكثر استخداما في تكنيك الكبت التوزيع المنتظم (Uniform) والتوزيع الأسي (Exponential) ويتفوق أولهما على الآخر في مقياس الأداء الزمني "Time Elapse" في حين يتفوق ثانيهما في مقياس أداء الرســـــــائل الزائدة "Extra Messages".

أدخلت هذه الرسالة شكلا معدلا للتوزيع الاحتمالي اللوجستي "Logistic" كمرشح للاستخدام في تكنيك الكبت كما أنها قامت بمقارنة التوزيع اللوجستي مع التوزيعين المذكورين آنفا. وقد استخدم برنامج MATLAB في حساب مقاييس الأداء ورسم الأشكال المناظرة في مقارنة النتائج.

برهنت الدراسة على تفوق التوزيع اللوجستي على التوزيعين الآخرين في مقاييس الأداء الزمني بوجه عام وعلى منافسته لهما في مقاييس أداء الرسائل الزائدة أيضا.