Coupled Parallel Flow through Porous Layers of Variable Thicknesses

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Abstract:

We consider the coupled, parallel flow of an incompressible, viscous fluid through two porous layers of variable thicknesses, shown in Fig. 1. The flow in each layer is governed by the dimensionless equations in Fig. 1, where in for each layer i = 1,2, v_i is the velocity, ϑ_i is the ratio of effective viscosity to base fluid viscosity, and Da is the Darcy number. Governing equations are to be solved subject to no slip on the solid walls, and velocity and shear stress continuity at the interface, namely: $v_1 = v_2$ and $\frac{dv_1}{dY} = \frac{dv_2}{dY}$ at $Y = \alpha$.

Letting $Y^* = \frac{Y}{\sqrt[3]{\vartheta_1 Da\alpha}}$ and $V_1 = V_1(Y^*) = v_1(Y)$, we express the equation of the lower layer as:

$\frac{d^2V_1}{dY^{*2}} - V_1Y^* + \sqrt[3]{\frac{(\alpha Da)^2}{\vartheta_1}} = 0$; $0 \le Y^* \le (\sqrt[3]{\vartheta_1 Da\alpha})\alpha$, which is Airy's inhomogeneous differential equation. General solutions to the governing equations are:

$$v_1 = aA_i(\gamma_1 Y) + bB_i(\gamma_1 Y) + \gamma_4 N_i(\frac{\gamma_3 Y}{\alpha})$$

and $v_2 = cExp(\frac{Y}{\gamma_2}) + dExp(\frac{-Y}{\gamma_2}) + Da$, where:

$\gamma_1 = \sqrt{\vartheta_1 Da}$, $\gamma_2 = \sqrt{\vartheta_2 Da}$, $\gamma_3 = \frac{\alpha}{\sqrt[3]{\vartheta_1 Da\alpha}}$. $\gamma_4 = \sqrt[3]{\frac{\alpha^2 Da^2 \pi^3}{\vartheta_1}}$, Ai and Bi are the homogeneous Airy's functions, and $N_i(Y^*) = A_i(Y^*)\int_0^{Y^*} B_i(t)dt - B_i(Y^*)\int_0^{Y^*} A_i(t)dt.$

We then obtain the solution satisfying the boundary and matching conditions, and use asymptotic and ascending series representations to evaluate the functions Ai, Bi and Ni. Velocity and shear stress at the interface are then computed.

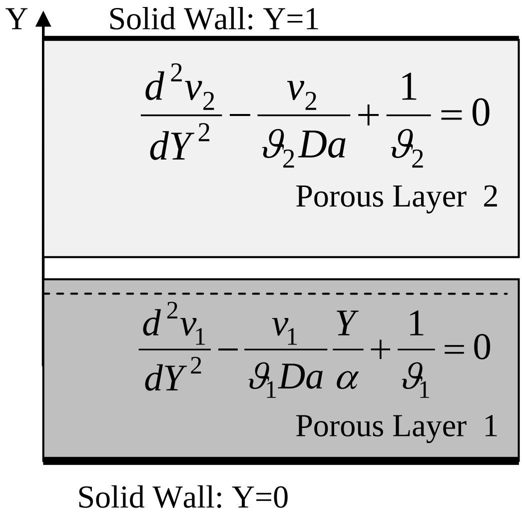

Fig. 1.
Representative Sketch