Simple Analysis of Cracked Concrete Section Under Flexure

by

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Introduction

General design equations are presented for direct calculation of the required tensile reinforcement A_s in rectangular and flanged concrete sections. The effect of compression reinforcement A_s' behaving elastically or yielding is included. If A_s' is required, it may be precisely determined after one trial even if $A_s'=0$ was initially assumed.

Equations to determine the nominal moment strength M_n in terms of the concrete dimensions, b, d, areas of tension and compression steel A_s, A_s', and known material properties f_c', f_y are also presented.

These equations eliminate the trial-and-error numerical solution or design aids needed when analysis equations are used for design and greatly simplify the solution of reinforced concrete sections under bending.

Equations are also provided to determine $(b/b_w)_{max}$, and the minimum tension steel ratio, for singly and doubly reinforced flanged section to work as T-section.

General

In beam design of reinforced concrete members under bending, the factored moment M_u is calculated first, then design is the process by which the cross sectional dimensions and reinforcement are determined. The section effective depth d, area of tensile reinforcement A_s and area of compression reinforcement A_s', are the most common design unknowns.

Investigation of strength is a reverse process in which the nominal moment strength M_n is determined, once a cross section is fully defined; that is, the concrete dimensions, tensile reinforcement area A_s and compression reinforcement area A_s' and materials are known.

Equations developed for studying rectangular and flanged sections have long been known and used [2] in both the design and investigation of strength processes. Design equations of the direct solution of the flexural reinforcement for rectangular sections with tensile reinforcement only are available in References [4,5,11,12].

When strength equations are used for design, a trial- and-error process, or the use of design aids, such as tables and diagrams, is needed [5] because the strength equations are expressed in terms of the design unknowns A_s, A_s', b and d, for rectangular sections. The number of equations of equilibrium available is less than the number of unknowns, consequently implying a non-unique solution .

Equations for the direct calculation of A_s, in terms of concrete section dimensions, material parameters, loads and compression reinforcement behaving elastically or yielding.

The investigation equations discussed solve the general case of a flanged section with compression reinforcement behaving elastically or yielding. The equations presented discuss in a unified treatment the design and investigation processes of reinforced concrete sections in bending in terms of the depth of the concrete equivalent stress block a .

These equations are appropriate for either computer or manual calculations.

The selection of reinforcing bars and their distribution is not a topic of concern in this research.

Assumptions in Ultimate Strength Design

The strength design method is based on the following assumptions in accordance with the ACI code [1,2] :

1- The strength of a member computed by the strength design method requires that two basic conditions be satisfied :

(a) static equilibrium.

(b) compatibility of strains.

2- The distribution of strains is linear across a reinforced concrete section, even near ultimate strength. The strain in both the reinforcements and the concrete is assumed to be directly proportional to the distance from the neutral axis. This assumption is of primary importance in design for determining the strain and corresponding stress in the reinforcements.

3- The maximum concrete strain at which ultimate moments are developed is usually 0.003 for a member of normal proportions and materials.

4- For deformed reinforcement, it is reasonably accurate to assume that the stress in the reinforcements, below the yield strength f_y, is proportional to the strain multiplied by the modulus of elasticity. The increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computation. The stress in the reinforcements is computed as :

when (i) $\epsilon_s < \epsilon_y$ (yield strain)

$$f_s = E_s \epsilon_s$$

(ii) $\epsilon_s \geq \epsilon_y$

$$f_s = f_y$$

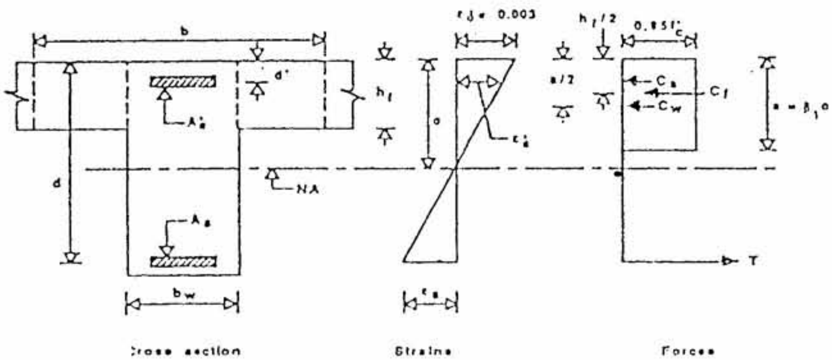
where ϵ_s is the value from the strain diagram at the location of the reinforcement. For design, the modulus of elasticity of steel reinforcement, E_s, may be taken as (29,000 ksi) (200,000 MPa) [10].

5-The tensile strength of concrete in flexure is neglected in strength design. For members with a normal percentage of reinforcement this assumption is in good agreement with test results. For very small percentages of reinforcement, neglecting the tensile strength at ultimate is usually correct.

6- An equivalent rectangular compressive stress distribution is to replace the more exact concrete stress distribution. In the equivalent rectangular stress block, an average stress of 0.85 f_c' is used with a rectangle of depth $a=\beta_1 c$. The value of β_1 is 0.85 for concrete with $f_c' \leq 30$ MPa and reduces by 0.008 for each MPa of f_c' in excess of 30 MPa, but $\beta_1 \geq 0.65$.

Basic Relationships

The following relationships can be established using the provisions of the ACI code [1]. From strain compatibility, an ultimate strain in the concrete of $\epsilon_c = 0.003$ and



Reinforced concrete flanged section[6] .

(1) Assuming $d' < c < d$ and $a > h_f$

$$\epsilon_s' = 0.003(1 - d'/c) \qquad ......1$$

$$\epsilon_s = 0.003(d/c - 1) \qquad ......2$$

From equilibrium $\Sigma H = 0$

$$T = C_f + C_w + C_S \qquad ......3$$

In which

$$T = A_s f_y \text{ when } \epsilon_s \geq \epsilon_y \qquad ......4$$

$$C_w = 0.85 f_c' b_w a \qquad ......5$$

$$C_f = 0.85 f_c'(b - b_w)h_f \qquad ......6$$

where $a = \beta_1 . c$

then $\beta_1 = 1.05 - f_c'/20$ (where f_c' is in ksi)

or $\beta_1 = 1.05 - f_c'/138$ (where f_c' is in MPa) 7

but $0.65 \leq \beta_1 \leq 0.85$

$$C_s = A_s'(f_s' - \gamma 0.85 f_c') \quad \text{......8}$$

where $\gamma = 1$ if $a > d'$ otherwise γ=0.0

and f_s' is the stress in the compressive reinforcement at ultimate strength .

$$f_s' = \epsilon_s' E_s$$

$$= 0.003 E_s (1 - \beta_1 d'/a) \quad \text{......9}$$

where E_s=29,000 ksi (200,000 MPa) [1].

$$f_s' = 87(1 - \beta_1 d'/a) ksi$$

$$f_s' = 600(1 - \beta_1 d'/a) MPa \quad \text{......10}$$

$f_s' = f_y$ if $\epsilon_s' \geq \epsilon_y$ and $a' < a$

where a' is determined by putting $f_s' = f_y$ in Eq 10 .

$$f_y = 87(1 - \beta_1 d'/a')$$

$$a' = \beta_1 d'/(1 - f_y/87) \quad \text{(where } f_y \text{ is in ksi)}$$

$$a' = \beta_1 d'/(1 - f_y/600) \quad \text{(where } f_y \text{ is in MPa)} \quad \text{......11}$$

$$f_s' = f_y \text{ if } a \geq a'$$

from Eq.3 if $a > h_f$ the following is obtained :

$$A_s f_y = 0.85 f_c' [b_w a + (b - b_w) h_f - \gamma . A_s'] + A_s' f_s' \quad \text{......12}$$

Taking $\Sigma M = 0$ with respect to A_s

$$M_u = \phi\{0.85 f_c' [b_w a (d - a/2) + (b - b_w) h_f (d - h_f/2) -$$
$$\gamma A_s' (d - d')] + A_s' f_s' (d - d')\} \quad \text{......13}$$

Alternatively, taking ΣM=0, with respect to C_w

$$M_u = \phi\{A_s f_y (d - a/2) + 0.85 f_c' [h_f (b - b_w)(a - h_f)/2 -$$
$$\gamma A_s' (a/2 - d')] + A_s' f_s' (a/2 - d')\} \quad \text{......14}$$

where ϕ is the strength reduction factor =0.9 .

(2) Assuming $d' < c < d$ and $a \leq h_f$

$$A_s f_y = 0.85 f_c'[ba + -\gamma . A_s'] + A_s' f_s' \qquad ......15$$

Taking $\Sigma M = 0$, with respect to A_s

$$M_u = \phi\{0.85 f_c'[ba(d - a/2) - \gamma A_s'(d - d')] + A_s' f_s'(d - d')\} \qquad ......16$$

Alternatively, taking ΣM=0, with respect to C_w

$$M_u = \phi\{A_s f_y(d - a/2) + A_s'(a/2 - d')(f_s' - 0.85\gamma f_c')\} \qquad ......17$$

Many cases can be obtained from equations 12, 13 and 14 for a section with tension reinforcement only where $A_s' = 0.0$, and $d' = 0.0$

For a singly reinforced rectangular section $A_s' = 0.0$, $d' = 0.0$, $b_w = b$ and $h_f = 0.0$

For a T-section reinforced in tension only and $a \leq h_f$ A_s', $d' = 0.0$, $b_w = b$, $h_f = 0.0$

For a doubly reinforced rectangular section $b_w = b$ and $h_f = 0.0$.

Balanced Strain Condition

A balanced strain condition, as defined in the ACI code [1], exists when $\epsilon_s = \epsilon_y$ and $\epsilon_c = 0.003$, then $c = c_b$

and we have $\epsilon_s = 0.003(d/c - 1) = \epsilon_y$

and $$c_b = d/(1 + \epsilon_y/0.003) \qquad ......18$$

substituting $\epsilon_y = f_y/E_s$ (E_s=29,000 ksi or 200,000 MPa) gives

$$c_b = d/(1 + f_y/87)$$

$$a_b = \beta_1 c_b$$

$$a_b = \beta_1 d/(1 + f_y/87) \qquad (f_y \text{ is in ksi})$$

$$a_b = \beta_1 d/(1 + f_y/600) \qquad (f_y \text{ is in MPa}) \qquad ......19$$

The stress in the compression reinforcement for the balanced condition f_{sb}' is obtained from

$$f_s' = 87(1 - \beta_1 d'/a) \qquad \text{ksi}$$

$$f_s' = 600(1 - \beta_1 d'/a) \qquad \text{MPa}$$

by substituting $a = a_b$, one can get

$$f'_{sb} = 87 - (f_y + 87)(d'/d) \quad \text{ksi}$$

$$f'_{sb} = 600 - (f_y + 600)(d'/d) \quad \text{MPa} \qquad ......20$$

According to the ACI code [1], for sections in flexure without axial load, the tensile reinforcement provided shall not exceed 0.75 of the reinforcement producing balanced strain conditions. Therefore we can get A_{sb} by substituting $a = a_b$ in Eq.12

$$A_{sb} = 0.85(f'_c/f_y)[b_w a_b + (b - b_w)h_f - \gamma A'_s] + A'_s f'_{sb}/f_y$$

$$A_{s(max)} = 0.75 A_{sb}$$

$$A_{s(max)} = 0.6375(f'_c/f_y)[b_w a_b + (b - b_w)h_f - \gamma A'_s] + A'_s f'_{sb}/f_y \qquad ......21$$

and $A_{s(max)}$ can be determined for many cases as follows:

- For a section with tension reinforcement only A'_s=0.0 .

$$A_{s(max)} = 0.6375(f'_c/f_y)[b_w a_b + (b - b_w)h_f] \qquad ......22$$

- For a rectangular section $b_w = b$ and $h_f = 0.0$.

$$A_{s(max)} = 0.6375(f'_c/f_y)(b a_b) \qquad ......23$$

According to ACI code [1], the minimum required positive reinforcement is given by:

$$A_{s(min)} = 200 b_w d/f_y, \; f_y \text{ in psi}$$

$$A_{s(min)} = 1.4 b_w d/f_y, \; f_y \text{ in MPa} \qquad ......24$$

ANALYSIS OF REINFORCED CONCRETE SECTIONS

Analysis is the process by which the nominal moment strength M_n is determined, once a cross section is fully defined ; that is, the concrete dimensions and steel quantities, A_s, A'_s, and materials strength are known .

From Eq. 12 for a flanged section with compression reinforcement and $(a > h_f)$

$$a = [(A_s f_y - A_s' f_s')/(0.85 f_c') - (b - b_w)h_f + \gamma A_s']/b_w \qquad ......25$$

When the Compression Steel has Yielded ($a' < a$)

$$a = [(A_s - A_s')f_y/(0.85 f_c') - (b - b_w)h_f + \gamma A_s']/b_w \qquad ......26$$

we have $\rho_w = A_s/b_w d$ and $\rho_w' = A_s'/b_w d$

Writing $\quad a = Xd \qquad ......27$

$$X = (\rho_w - \rho_w')(f_y/0.85 f_c') - (b/b_w - 1)h_f/d + \gamma\rho_w' \qquad ......28$$

To have the compression reinforcement at yield

$$a = Xd > a' = \beta_1 d'/(1 - f_y/600) \quad \text{(where } f_y \text{ is in MPa)}$$

then $\quad X_{min} = \beta_1(d'/d)/(1 - f_y/600) \quad$ (where f_y is in MPa)

or $\quad X_{min} = \beta_1(d'/d)/(1 - f_y/87) \quad$ (where f_y is in ksi) $\qquad ......29$

to get the tension reinforcement area less than the maximum allowable tension reinforcement area X must satisfy

$$X \leq 0.75 a_b/d$$

then $\quad X_{max} = 0.75\beta_1/(1 + f_y/600) \quad$ (where f_y is in MPa)

or $\quad X_{max} = 0.75\beta_1/(1 + f_y/87) \quad$ (where f_y is in ksi) $\qquad ......30$

When the Compression Steel has not Yielded ($a' > a$)

$$f_s' = 87(1 - \beta_1 d'/a)$$

substitute f_s' in Eq.12 to get:

$$0.85 f_c' b_w a^2 + [0.85 f_c'(b - b_w)h_f + A_s'(87 - 0.85\gamma f_c') -$$
$$A_s f_y]a - 87 A_s' \beta_1 d' = 0.0 \qquad ......31$$

rearrange to get:

$$a^2 + r_1 a - r_0 = 0.0 \qquad ......32$$

$$r_1 = [(b/b_w - 1)h_f/d + (\rho_w'/0.85 f_c')(87 - 0.85\gamma fc') -$$
$$\rho_w f_y/0.85 f_c']d \qquad ......33$$

$$r_0 = 87\rho_w'\beta_1 d' d/(0.85fc') \qquad ......34$$

then $$a = -r_1/2 + \sqrt{(r_1^2/4 + r_0)}. \qquad ......35$$

then once a is known from either Eq.27 or Eq.35 M_u is determined from either Eq.13 or Eq.14.

DESIGN OF REINFORCED CONCRETE SECTIONS

Design of sections with compression reinforcement

Compression reinforcement complicates the design of a reinforced concrete section. Usually, its effects are either ignored in flanged section or approximated in rectangular sections.

Equations are developed for the direct calculation of A_s in terms of section dimensions, material parameters and compression reinforcement behaving elastically or yielding .

Calculation of A_s when the compression steel is elastic $\epsilon_s < \epsilon_y$ and $a < a'$.

In this case the stress in compression reinforcement is given by:

$$f_s' = 87(1 - \beta_1 d'/a) \text{ ksi}$$

$$f_s' = 600(1 - \beta_1 d'/a) \text{ MPa}$$

substitute the value of f_s' in Eq.13 we get

$$a^3 - 2da^2 + [2S + k_0 d^2]a + k_1 d^3 = 0.0 \qquad ......36$$

where $$S = M_u/(0.85\phi f_c' b_w) \qquad ......37$$

$$k_0 = (1 - b/b_w)(2 - h_f/d)h_f/d + 2\gamma\rho_w'(1 - d'/d) - 174\rho_w'(1 - d'/d)/0.85f_c' \qquad ......38$$

and $$k_1 = 174\rho_w'(1 - d'/d)\beta_1(d'/d)/0.85f_c' \qquad ......39$$

solving the cubic equation we get a then A_s is determine by Eq.12, rewritten below

$$A_s = (0.85f_c'[b_w a + (b - b_w)h_f - \gamma A_s'] + A_s' f_s')/f_y$$

Calculation of A_s for flanged sections in which the compression reinforcement has yielded, $\epsilon_s' \geq \epsilon_y$ therefore $f_s' = f_y$

Substitute the value of $f_s' = f_y$ in Eq.14 we get:

$$a = d - \sqrt{(Kd^2 - 2S)} \qquad \text{......40}$$

where $\quad S = M_u/(0.85\phi f_c' b_w)$

and $\quad K = 1 - (1 - b/b_w)h_f/d(2 - h_f/d) - 2\rho_w'(1 - d'/d) +$

$$2\rho_w'(1 - d'/d)f_y/0.85fc' \qquad \text{......41}$$

then the tension reinforcement area is given by

$$A_s = \{0.85f_c'[b_w a + (b - b_w)h_f - \gamma A_s'] + A_s' f_y\}/f_y$$

Calculation of A_s for doubly reinforced rectangular sections or flanged sections when $a \leq h_f$

In this case substituting $b_w = b$, the constant K in Eq.41 is given by

$$K = 1 - 2\rho_w'(1 - d'/d) +$$

$$2\rho_w'(1 - d'/d)f_y/0.85fc' \qquad \text{......42}$$

then $\quad a = d - \sqrt{(Kd^2 - 2S)}$

then the tension reinforcement area is given by

$$A_s = \{0.85f_c'(ba - \gamma A_s')/f_y\} + A_s'$$

Design of Singly Reinforced Sections

Calculation of A_s for a flanged sections, $A_s' = 0.0$ and $a > h_f$

In this, case the constant K in Eq.41, is given by

$$K = 1 - (1 - b/b_w)h_f/d(2 - h_f/d) \qquad \text{......43}$$

then $\quad a = d - \sqrt{(Kd^2 - 2S)}$

then the tension reinforcement area is given by

$$A_s = \{(0.85f_c'[b_w a + (b - b_w)h_f]\}/f_y$$

Calculation of A_s for rectangular sections reinforced in tension only (flanged section $A_s^{'} = 0.0$ and $a \leq h_f$)

In this case K=1.0 since $b_w = b$ and $\rho^{'} = 0$

$$a = d - \sqrt{(d^2 - 2S)}$$

then the tension reinforcement area is given by

$$A_s = (0.85 f_c^{'} ba)/f_y.$$

Minimum Tension Steel Ratio for Singly Reinforced Flanged Sections to Work as T-section

To get the section under consideration as T-section $a > h_f$

$$a = [(A_s f_y / 0.85 f_c^{'}) - (b - b_w) h_f] / b_w > h_f$$

$$A_{sT} > h_f b (0.85 f_c^{'}) / f_y \qquad \text{......44}$$

Dividing equation 44 by $b_w d$ the minimum tension steel ratio is

$$\rho_T > (h_f / d)[b / b_w (0.85 f_c^{'}) / f_y] \qquad \text{......45}$$

Equation 45 gives ρ_T as a function of h_f/d, b/b_w, f_y and $f_c^{'}$.

Minimum Tension Steel Ratio for Doubly Reinforced Flanged Sections to Work as T-section

To get the section under consideration as a T-section $a > h_f$

$$a = [(A_s - A_s^{'}) f_y / 0.85 f_c^{'} - (b - b_w) h_f + \gamma A_s^{'}] / b_w > h_f$$

then

$$A_{sT} = b h_f / \{(1 - \rho^{'}/\rho) f_y / 0.85 f_c^{'} + \rho^{'}/\rho\} \qquad \text{......46}$$

Dividing equation 46 by $b_w d$ to get the minimum tension steel ratio for a doubly reinforced flanged section to work as T-section, $\rho_T = A_s / b_w d$, yields

$$\rho_T = (b/b_w)(h_f/d) / \{(1 - \rho^{'}/\rho) f_y / 0.85 f_c^{'} + \rho^{'}/\rho\} \qquad \text{......47}$$

Equation 47 gives ρ_T as a function of b/b_w, h_f/d, $\rho^{'}/\rho$, $f_c^{'}$ and f_y.

Maximum Flange Width to Web Width b/b_w for Singly Reinforced Flanged section under Consideration as T-section

The maximum area of tension steel from Eq.19 is given by

$$A_{s(max)} = 0.6375(f_c'/f_y)[b_w a_b + (b - b_w)h_f]$$

where $\quad a_b = d\beta_1/(1 + f_y/600) \quad$ where f_y is in MPa

where $\quad a_b = d\beta_1/(1 + f_y/87) \quad$ where f_y is in ksi

Substitute the value of a_b in Eq.19 and dividing by $b_w d$ results in $\rho_{(max)} = A_{s(max)}/b_w d$

$$\rho_{(max)} = 0.6375(f_c'/f_y)[\beta_1/(1 + f_y/600) + h_f/d(b/b_w - 1)] \quad ......48$$

To determine the maximum value of b/b_w to work as T-section equate ρ_T of Eq.45 to $\rho_{(max)}$ of Eq.48 then

$$(b/b_w)_{max} = 3(\beta_1/[h_f/d(1 + f_y/600)] - 1) \quad ......49$$

Equation 49 gives the maximum flange width to web width ratio for a singly reinforced flanged section to work as T-section (neutral axis below flange).

CONCLUSIONS AND RECOMMENDATIONS

The process of designing and analyzing reinforced rectangular and flanged sections was discussed in terms of simplified general equations, which include most common encountered cases. Rectangular and flanged sections with tension reinforcement only are presented as particular cases of the most general case of flanged doubly reinforced sections .

- The analysis equations discussed apply to the general case of doubly reinforced T-section. Analysis equations given in Reference [2] , for flanged and rectangular sections are special cases of the more general solution provided by Eq.13 or 14 after determining the equivalent rectangular stress block depth a, by Eq.27 when the compression steel has yielded or by Eq.35 when the compression reinforcement has not yielded .

- Equations of design as a direct solution of the required tension reinforcement A_s, are presented in terms of the ultimate moment M_u , the material parameters and section dimensions. The area of tension steel is given in terms of the depth of the equivalent rectangular stress block a, which is determined by Eq.36, if the compression reinforcement has not yielded or by Eq.40, if the compression steel has yielded. Eqs.36 and 40 may also be used to determine the required compression reinforcement $A_s^{'}$, by assuming $A_s^{'} = 0.0$ as an initial value and then calculating K from Eqs.41, 42 and 43. If there is a need for compression reinforcement the initial value of $A_s^{'}$ is increased until we get $A_s < A_{s(max)}$.

- The effect of adding compression reinforcement $A_s^{'}$ on the internal lever arm and therefore on the area of tension steel A_s, is considered by the simplified equations discussed. This effect is usually ignored when analysis equations or design aids are used in the design process.

- Simple criteria, in terms of h_f/d gives the maximum flange width to web width ratio $(b/b_w)_{max}$ to get the section work as a T- section; $(b/b_w)_{max}$ is provided by Eq.48 .

- The tension steel ratio ρ_T is determined to get the section work as a T-section for singly and doubly reinforced T-sections; ρ_T is provided by Eqs.45 and 47.

- All equations presented in the thesis are paper programmed, thus simplifying the repetitive nature of the calculation . Examples in the Appendix illustrate the application of the equations to typical reinforced concrete problems using long hand calculations .

- To use the simplified method of design and analysis very shortly, it is recommended to tabulate the constants (K, X, ...etc.) which are presented in the thesis.

APPENDIX

Example 1 (Example1,[1])

For a rectangular section subjected to a factored bending moment $M_u = 100ft - kips$, determine the reinforcement required for the following conditions: $f'_c = 4.0$ ksi, $f_y = 60.0$ ksi, $b = 10$ in, $d = 17.5$ in.

Solution:

the equivalent rectangular stress block depth given by

$a = d - \sqrt{Kd^2 - 2S}$

$S = M_u/(0.85\phi f'_c b)$=1200/(0.85 x 0.9 x 4 x 10)=39.216 in^2

for rectangular section reinforced in tension only $K = 1.0$

a=17.5- $\sqrt{[(17.5)^2 - 2(39.216)]}$=2.4063 in

$A_s = 0.85 f'_c[ab]/f_y = 1.3636\ in^2$

$a_b = \beta_1 d/(1 + f_y/87)$ =8.8036 in

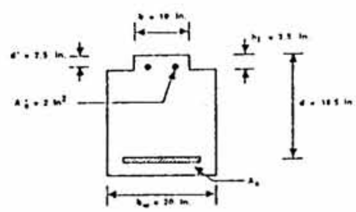
$a < a_b$ then $A_s < A_{s(max)}$

$A_{s(min)}$ = (200/60000)17.5 x 10= 0.5833 in^2

$A_{s(min)} < A_s < A_{s(max)}$ ....ok

Example 2 (Example2,[6])

For the section in the figure below, subjected to a factored moment M_u = 640 ft-kips, determine the tensile reinforcement required if $f'_c = 5.5$ ksi and $f_y = 60.0$ ksi.



Solution:

$a = d - \sqrt{(d^2 - 2S)}$

$S = M_u/(0.85\phi f'_c b_w)$ =100.3922 in^2

$K = 1 - \{(1 - b/b_w)h_f/d(2 - h_f/d) - 2\rho'_w(1 - d'/d)(f_y/(0.85f'_c) - 1)\} = 0.9539$

a=19.5-$\sqrt{[0.9539(19.5)^2 - 2(100.3922)]}$=6.7744 in

$a' = \beta_1 d'/(1 - f_y/87)$=6.4444 in

$a > a'$ compression reinforcement has yielded.

$A_s = 0.85(f_c'/f_y)[b_w a + (b - b_w)h_f - A_s'] + A_s' = 8.976\ in^2$

$a_b = \beta_1 d/(1 + f_y/87)$ =9.233 in

Since $a < a_b$ then $A_s < A_{s(max)}$

$A_{s(min)}$=(200/60000)19.5x20= 1.3 in^2

$A_{s(min)} < A_s < A_{s(max)}$ ....ok

Example 3 (Example4.7,[9])

A T-beam section with b= 30 in, b_w=12 in, d=23 in and h_f =4 in is to have a design flexural strength of 7x10^6 lb-in. Using f_c' =3000 psi and f_y =60,000 psi, calculate the required steel area.

Solution:

$a = d - \sqrt{(d^2 - 2S)}$

$S = M_u/(0.85\phi f_c' b_w) = 254.175\ in^2$

$K = 1 - \{(1 - b/b_w)h_f/d(2 - h_f/d) - 2\rho_w'(1 - d'/d)(f_y/(0.85f_c') - 1)\} = 1.4764$

a=23-$\sqrt{[1.4764(23)^2 - 2(254.175)]}$=6.7744 in

$a_b = \beta_1 d/(1 + f_y/87)$ =11.57 in

Since $a < 0.75a_b$ then $A_s < A_{s(max)}$

$A_{s(min)}$ = (200/60000)23 x 12= 0.92 in^2

$A_{s(min)} < A_s < A_{s(max)}$ ....ok

Example 4 (Example4,[6])

For a flanged section, determine the ultimate moment M_u, for the following conditions: $f_c' = 4.4$ ksi, $f_y = 60.0$ ksi; $h_f = 4$ in.;$b_w = 30$ in.;$d = 16.5$ in.; and $b = 70$ in.

Solution:

Assume compression reinforcement has yielded

$a = Xd$

$X = [(\rho_w - \rho_w')(f_y/0.85f_c) - (b/b_w - 1)h_f/d + \rho_w']$=0.2881

a=0.2881 x 16.5 =4.7539

$a' = \beta_1 d'/(1 - f_y/87)$=6.8472 in

Since $a < a'$ compression reinforcement is elastic

$f_s' = 87(1 - \beta_1 d'/a)$= 49.545 ksi

$a = -r_1/2 + \sqrt{((r_1^2/4) + r_0)}$

$r_1 = \{(b/b_w - 1)h_f/d + (\rho_w'/(0.85f_c'))(87 - 0.85f_c') - \rho f_y/(0.85f_c'\}d$ = -4.318

$r_0 = (87\rho_w'/0.85f_c')\beta_1 d' d$ = 2.9852

a= 4.318/2 + $\sqrt{[(-4.318)^2/4 + 2.9852]}$ = 4.924 in

Since $a > h_f$ the section work as a T-section

$M_u = \phi\{A_s f_y(d - a/2) + 0.85f_c'[h_f(b - b_w)(a - h_f)/2 - A_s'(a/2 - d')] + A_s' f_s'(a/2 - d')\}$

M_u = 14400 in-kips(1200 ft-kips).

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