An-Najah National University

Faculty of Graduate Studies

FINITE VOLUME AND WEIGHTED RESIDUALS METHODS FOR SOLVING MAXWELL'S EQUATIONS

By

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This Thesis was Defended Successfully on 06/03/2025 and approved by

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Dedication

I dedicate this message to my dear and beloved father.

My dear father, I thank you with all my heart for everything you have done for me throughout my life.

I cannot find the words to express my immense love for you.

Father, you are in my eyes the safe place I always turn to.

Thank you for this feeling.

I feel proud to have such a wonderful and supportive father like you.

Words can never truly express my gratitude towards you.

Thank you for all your sacrifices.

I am so grateful to Almighty God for blessing me with a father like you.

Your importance in my life cannot be expressed in words.

My dear father, I want to thank you for making my dreams come true. Thank you for shaping my life. I am proud of you.

Thank you, Father, for giving me love, inspiring me, and filling my life with happiness, peace, and love.

No matter what I do or where life takes me, I will always be grateful to you, Father, because I know my life without you would be worthless.

I truly wish you were here with me during these moments.

Thank you for everything.

Acknowledgments

I sincerely thank God for His abundant grace and for granting me the success and guidance to complete my academic thesis.

To my beloved father, dear mother, and esteemed family: I can never forget your support and what you have done for me. You have my utmost love, and no matter how much gratitude I express, I will never be able to fully repay you for what you deserve.

I would also like to thank my colleague, Mohamed Abu Hatab, for his help and support throughout this thesis.

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As Allah Almighty said in The Quran: "And whoever is grateful is only grateful for the benefit of himself."

قَالَ تَعَالَى: ﴿وَلَقَدْ ءَاتَيْنَا لُقْمَٰنَ ٱلْحِكْمَةَ أَنِ ٱشْكُرْ لِلَّهِ ۚ وَمَن يَشْكُرْ فَإِنَّمَا يَشْكُرُ لِنَفْسِهِۦ ۖ وَمَن كَفَرَ فَإِنَّ ٱللَّهَ غَنِىٌّ حَمِيدٌ﴾

صدق الله العظيم [لقمان: 12]

Declaration

I, the undersigned, declare that I submitted the thesis entitled:

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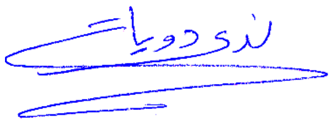
I declare that the work provided in this thesis, unless otherwise referenced, is the researcher’s own work, and has not been submitted elsewhere for any other degree or qualification.

Student's Name: **Nada Taher Dwayyat**

Signature:



Date: **06/03/2025**

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FINITE VOLUME AND WEIGHTED RESIDUALS METHODS FOR SOLVING MAXWELL'S EQUATIONS

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Abstract

In this thesis, the researcher conducted a comparative study between two numerical analysis methods used to solve Maxwell's equations: the finite volume method (FVM) and the weighted residue method (MWR). This is due to the importance of Maxwell's equations in studying electromagnetic fields. Many technological fields rely on them, including electrical engineering, electromagnetic wave patterns, and wireless communications.

The Finite Volume Method (FVM) is a discretization method for approximating a single or a system of partial differential equations (PDEs) expressing the conservation, or balance, of one or more quantities. It is frequently used in solving problems concerning fluid dynamics and electromagnetic simulations and is particularly useful for problems demanding exact flux conservation.

The Method of Weighted Residuals uses specific weighting functions to reduce errors (residuals) in the governing equations after precisely assuming an approximate solution. If the appropriate pilot functions are used, this approach can be highly precise and is adaptable, making it appropriate for complex forms and boundary conditions.

This work scrutinizes the two approaches MATLAB implementations and evaluates how well they solve Maxwell's equations. Precision, computational efficacy, and convenience are the main comparing points. The findings demonstrate that MWR provides greater resilience and can be more precise in complex scenarios, even though FVM is more accurate at saving flux in particular.

All things taken into account, this work contributes to the progress of effective numerical techniques in problem solving strategies, which may be of exceptional advantage in future cutting-edge technologies.

Keywords: Maxwell's Equations, Finite Volume Method, Method of Weighted Residuals.

Chapter One

Maxwell's Equations between Physics and Mathematics

1.1 Introduction

Due to its remarkable impact on a wide spectrum of technologies, such as wireless technology, remote sensors, inter-galactic transmissions, radio astronomy, computer and phone chip design, oil drilling and extraction, and more, Maxwell's equations ranked up high among the most vital in physics at large. Being Valid from subatomic length scales all the way to galaxy length scales; from static to ultraviolet frequencies; and from classical physics to quantum physics, these equations have incredible predictive power for numerous old, new, and futuristic applications.

These equations have a far-ranging impact when solved with modern math and physics concepts using the computational power of parallel computers. This became possible because of the accuracy of Maxwell's equations and the development of computers and programs such as MATLAB.

Therefore, it is important to study and find efficient ways to solve Maxwell's Equations with the hope that they may advance fundamental science towards technology developments and applications for the future, Such as Nano-optics and predicting quantum behavior such as Casimir force, heat transfer, quantum optics, and quantum information.

This chapter continues to present the equations and their abbreviation methods to understand and become familiar with the numerical solution methods used in general and the appropriate numerical methods for solving Maxwell's equations and the systems resulting from their use and to develop accurate computational solutions for scientific applications that arose based on Maxwell's equations.

1.2 Maxwell's equations

An enormous amount of information about the world—the basic rules that govern the behavior of light, the flow of electric current, and how magnetism works—can be boiled down to four beautiful equations, which today are called Maxwell's equations, and you can find them in physics, mathematics, and engineering textbooks.

These equations appeared 150 years ago, when Maxwell presented his theory, which has a unification of the electric and magnetic forces, before the Royal Society of London, and published a full report on it in the following year 1865. It was this work that laid the foundation for the greatest achievements in physics, radio communications, and electrical engineering. [1]

Therefore, there was a big gap between presenting and benefiting from the theory. As the mathematical and theoretical foundations on which Maxwell's equations are based are complex, and this matter made them very neglected after they were presented for the first time.

It took about 25 years for a group of physicists, who were fond of the mysteries of electricity and magnetism, to support Maxwell's equations. They were the ones who gathered the necessary experimental evidence to confirm that light is composed of electromagnetic waves, and they provided these equations in their current form. [2]

The Four Golden Equations: These equations can be written in diverse ways. Today, the relationship between electric force and magnetic force, as well as the wave nature of light and electromagnetic radiation, are summarized, in general terms, in four equations called "Maxwell's equation: [3]

$$\nabla \cdot D = \rho \tag{1}$$

$$\nabla \cdot B = 0 \tag{2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J \tag{4}$$

Here, J stands for electric current density, and E and B stand for electric and magnetic fields, respectively. There are two other fields: the displacement field and its symbol D, and the magnetic field and its symbol H. These two fields are related to E and B by constants that reflect the nature of the medium through which the fields pass (the values of these constants can be summed in to give the speed of light). [4]

One of the main contributions made by Maxwell was the displacement field D, and the last equation describes how electric current and changing electric fields can create and replace magnetic fields. The symbols at the beginning of each equation are differential operators. These calculus symbols contain vectors - mathematical quantities with a direction- and therefore have x, z, and y components. [5]

1.3 Physical interpretation of Maxwell's equations

Maxwell's Equations are four of the most influential equations in science: Gauss's law for electric fields, Gauss's law for magnetic fields, Faraday's law, and the Ampere–Maxwell law. [3]

The first equation is nothing but Gauss's law in its differential form, according to which any point of electric charge in Space must generate an electric field around it, the lines of which radiate from the location of the charge, and this field is static. It does not change with time if the charge is stationary and does not change with time if the charge is changing. [6]

The second equation is nothing but Gauss's law of magnetism in its differential form, which states that there is no existence of magnetic charges, therefore, the field lines must be closed to themselves. [7]

The third equation is nothing but Faraday's law of induction, as Maxwell converted it from its integral form to its differential or point form, and the meaning of this equation is that the changing magnetic field with time is generated around it An electric field whose value and distribution in space is proportional to the rate of change of magnetic field strength with time As well as its direction in space. [8]

The fourth equation is a modified form of Ampère's Law, after Maxwell converted it from its integral form to this displacement current, in its differential form, added to it a new term called the displacement current. This addition is one of the most important contributions of Maxwell in the field of electromagnetism, as it enabled him to predict the existence of electromagnetic waves, and by adding the displacement current to Ampere's equation, the meaning of Maxwell's fourth equation became that the electric current or electric field changing with Time generates a magnetic field around it whose value and distribution in space is commensurate with the value and direction of the current, as well as with the rate of change of the electric field strength with time and its direction in space. [9, 10]

In the year 1865 A.D., Maxwell was able, by merging the third and fourth equations, namely Faraday's law and the modified Ampere's law, to obtain a differential equation of the second degree. When he solved this equation, it became clear to him that electric and magnetic fields must spread in the form of waves in space, and thus he proved and predicted through pure mathematical analysis the existence of so-called electromagnetic waves. [11]

It is possible for us, by examining Maxwell's equations without solving them, to preserve and conclude most of the phenomena of Electromagnetism and especially the fact that electromagnetic waves exist. In the case of the first equation confirms the existence of a field for the existence of static electric charges only. There is only an electric current and no magnetic field since the right-hand side of the fourth equation is zero. [12 ,6]

In the case of a constant electric current (J), the fourth equation confirms the presence of a static magnetic field only and the absence of an electric field, as the right side of the first equation is zero. In the case of changing electric charges only, the first equation confirms the existence of a changing electric field, and this changing electric field will generate a changing magnetic field, as it is clear from the fourth equation, as the second term on its right side is not equal to zero. [13]

This magnetic field generated from the electric field generated by the electric charge, in the beginning, will generate a new electric field around it, as it is clear from the third equation, and so this series continues, as each of the two types of field generates the other according to the third and fourth equations, and thus the entire space will be filled with these interacting electric and magnetic fields which Maxwell called electromagnetic waves. [14, 15]

Such waves can also be obtained from a changing electric current only, as it is clear from the fourth equation, where this current will generate a changing magnetic field, that generates a changing electric field according to the third equation, and so forth.

Maxwell's prediction of the existence of electromagnetic waves was fulfilled by German physicist Heinrich Hertz in 1887 when he was able to generate electromagnetic waves using simple forms of antennas. [16]

Since Maxwell formulated the laws of electromagnetism in his four equations, little has been added to the theoretical science of electromagnetism. In the applied field, these equations have been used extensively by electrical engineers to solve many issues, such as wave propagation in various media, such as transmission lines, waveguides, optical fibers, designing transmitting and receiving antennas, and countless other applications.

1.4 Applications of Maxwell’s equations.

The Maxwell equations describe the interaction of electric and magnetic fields. Important applications are electric machines such as transformers or motors, or electromagnetic waves radiated from antennas or transmitted in optical fibers.

1.4.1 Technical Applications

Maxwell equations are applied in a wide range (limited by quantum effects in the small scale and by relativistic effects in the large scale). For different applications, different terms dominate. If

$$L\omega \ll c = \frac{1}{\sqrt{\varepsilon\mu}} \tag{4}$$

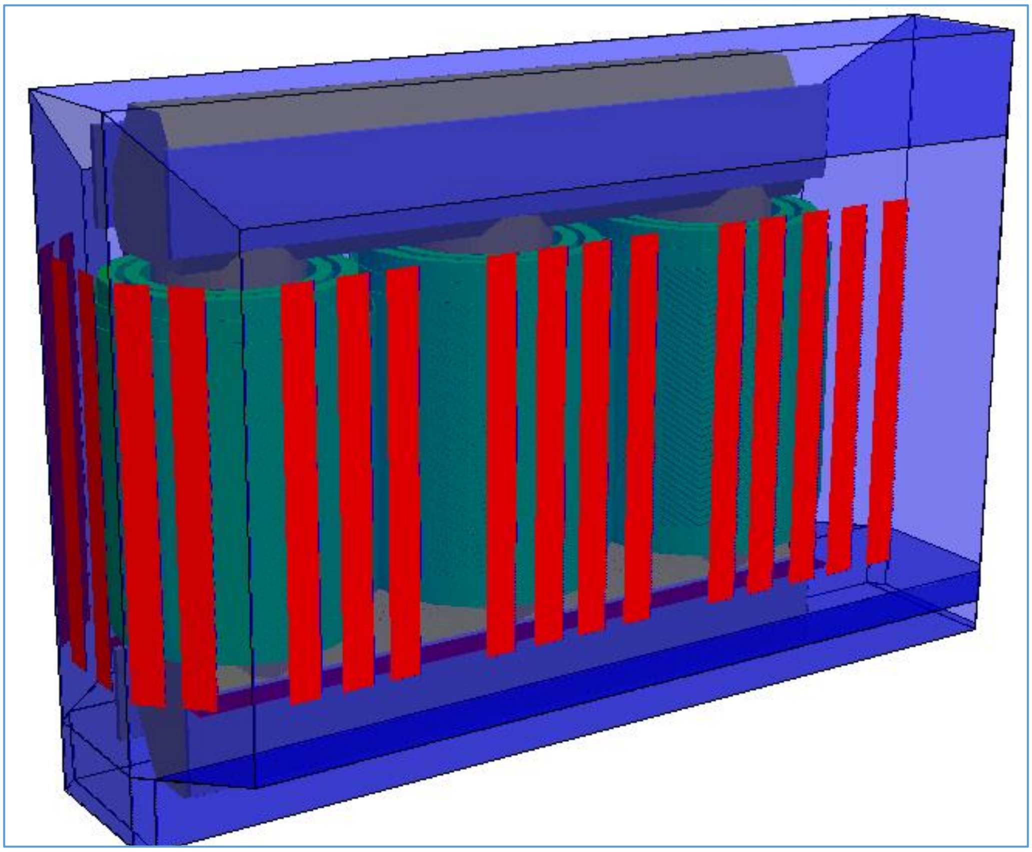
Where L is the length scale, and c is the speed of light, wave effects and thus the second-order time derivative can be neglected. This case is called low-frequency approximation. [17-19]

1.5 Low frequency applications

This is the case with most electric machines, where the frequency is 50Hz. A transformer changes the voltage and current of alternating current. Figure 1.1 shows a three-phase transformer. It has an iron core with high permeability μ. Around the legs of the core are the windings (a primary and a secondary on each leg). The current in the windings is known. It generates a magnetic field mainly conducted by the core. A small amount of the field goes into the air and into the casing. The casing is made of steel and thus highly conducting, which leads to currents and losses in the casing. Thus, one place is highly permeable shields in front of the casing to collect the magnetic flux. The shields are made of layered materials to prevent currents in the shields.

Figure 1.1

Three phase transformers



This problem is a three-dimensional problem, which can only be solved by numerical methods. The induced current density and loss density in the steel casing and interior conducting domains computed by the finite element method is plotted in Figure 1.2 and Figure 1.3.

Figure 1.2

Induced currents

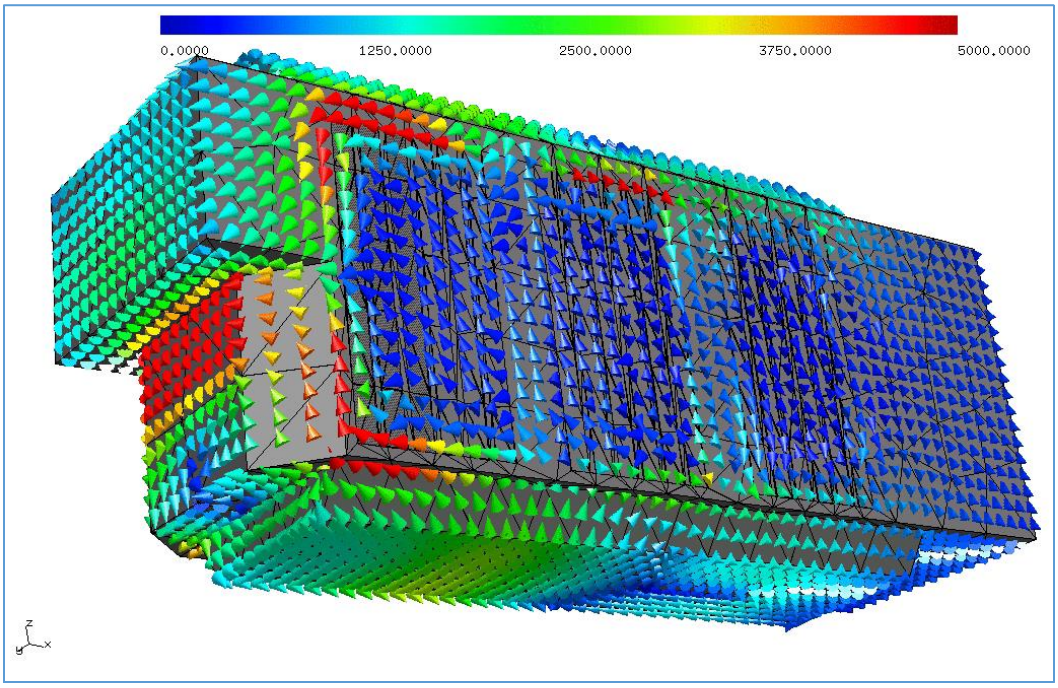
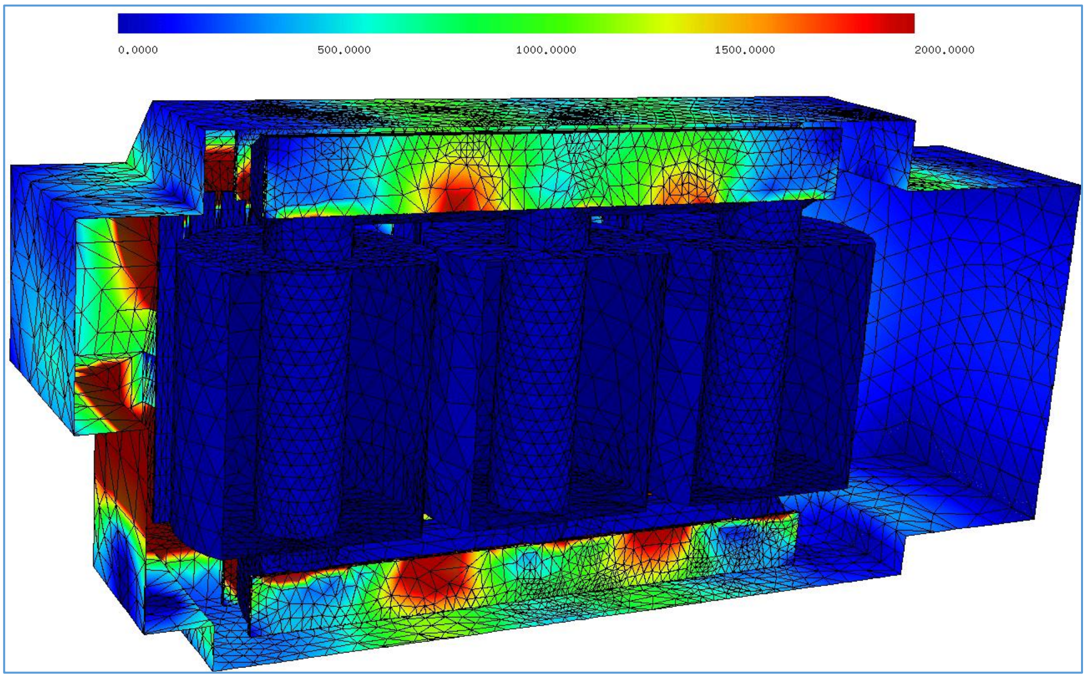


Figure 1.3

Loss density



Other low frequency applications are electric motors and dynamos. Here, the mechanical force (Lorentz force) arising from electric current in the magnetic field is used to transform electromagnetic energy into motion, and vice versa. [20, 21]

1.5.1 High frequency applications

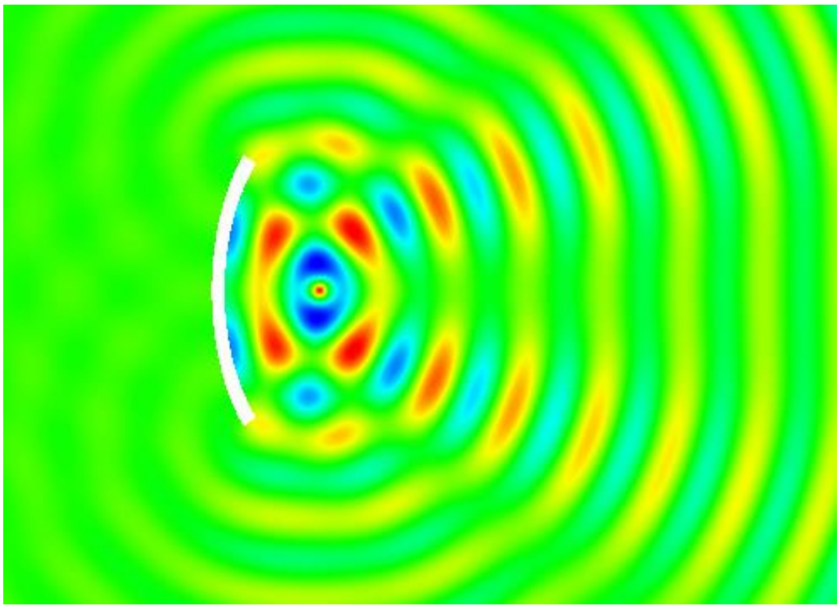
Here, the wave phenomena play the dominating role. Conducting materials ($\sigma > 0$) lead to Ohm's losses. The conductivity term enters with imaginary coefficient into the time harmonic equations.

- **Transmitting Antennas**

Transmitting Antennas are driven by an electric current and radiate electromagnetic waves (ideally) into the whole space. Receiving antennas behaves vice versa. By combining several bars, and by adding reflectors, a certain directional characteristic (depending on the frequency) can be obtained. The radiation of an antenna with a parabolic reflector is drawn in Figure 1.4. The behavior of waves as $x \to \infty$ requires the formulation and numerical treatment of a radiation condition.[22]

Figure 1.4

Parabola antenna.



- **Laser Resonator**

  In a Laser resonator, a standing electromagnetic wave is generated. At a certain, material dependent frequency, the wave is amplified by changing the atomic energy state. The geometry of the resonator chamber must be adjusted such that the laser frequency corresponds to a Maxwell eigenvalue. The case of imperfect mirrors at the boundary of the resonator leads to challenging mathematical problems.

- **Optical Fibers**

  Optical fibers transmit electromagnetic signals (light) over many kilometers. A pulse at the input should be obtained as a pulse at the output. The bandwidth of the fiber is limited by the shortest pulse that can be transmitted. Ideally, the (spatial) wavelength λ of the signal is indirectly proportional to the frequency. Due to the finite thickness of the fiber, this is not true, and the dependency of 1/ λ on the frequency ω can computed and plotted as a dispersion diagram that reflects the transmission behavior of the fiber.

1.6 Mathematical solutions of Maxwell's equations

Maxwell was the first person to calculate the speed of propagation of electromagnetic waves, which was the same as the speed of light, and concluded that EM waves and visible light are similar.

These are the set of partial differential equations that form the foundation of classical electrodynamics, electric circuits, and classical optics along with Lorentz force law. These fields highlight modern communication and electrical technologies.

Maxwell's equations integral form explains how the electric charges and electric currents produce magnetic and electric fields. The equations describe how the electric field can create a magnetic field and vice versa.

1.6.1 Maxwell First Equation

Maxwell's first equation is based on the Gauss law of electrostatic, which states that "when a closed surface integral of electric flux density is always equal to charge enclosed over that surface" [23]

Mathematically Gauss's law can be expressed as, [25 ,24]

Over a closed surface, the product of the electric flux density vector and surface integral is equal to the charge enclosed.

$$\oiint \vec{D}.d\overline{s} = Q_{enclosed} \tag{5}$$

Any closed system will have multiple surfaces but a single volume. Thus, the above surface integral can be converted into a volume integral by taking the divergence of the same vector. Therefore, mathematically it is-

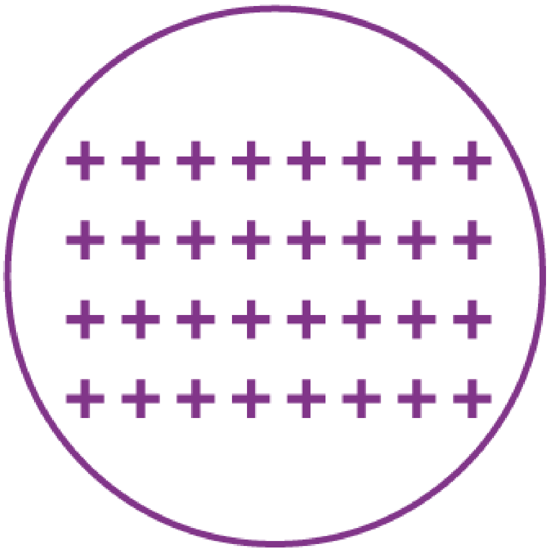
$$\oiint \overline{D}.d\overline{s} = \iiint \nabla.\vec{D}d\vec{v} \tag{6}$$

Thus, combining (1) and (2), we get-

$$\iiint \nabla.\vec{D}d\vec{v} = Q_{enclosed} \tag{7}$$

Figure 1.5

Scalar magnetic flux



Charges in a closed surface will be distributed over its volume. Thus, the volume charge density can be defined as –

$$\rho v = \frac{dQ}{dv}$$

measured using C/m^3

On rearranging, we get-

$$dQ = \rho v dv$$

On integrating the above equation, we get-

$$Q = \iiint \rho v dv \quad (8)$$

The charge enclosed within a closed surface is given by volume charge density over that volume.

Substituting (4) in (3) we get-

$$\iiint \nabla . D dv = \iiint \rho v dv$$

Canceling the volume integral on both sides, we arrive at Maxwell's First Equation.

$$\Rightarrow \nabla . D dv = \rho v$$

1.6.2 Maxwell Second Equation

Maxwell second equation is based on Gauss law on magnetostatics.

Gauss law on magnetostatics states that "closed surface integral of magnetic flux density is always equal to total scalar magnetic flux enclosed within that surface of any shape or size lying in any medium."

Mathematically it is expressed as –

$$\oiint \vec{B} . ds = \phi_{enclosed} \quad (9)$$

Table 1.1

Comparison of Scalar Electric Flux and Scalar Magnetic Flux

Scalar Electric Flux (ψ)	Scalar Magnetic Flux (ϕ)
They are the imaginary lines of force radiating in an outward direction	They are the circular magnetic field generated around a current-carrying conductor.
A charge can be source or sink	No source/sink

Hence, we can conclude that magnetic flux cannot be enclosed within a closed surface of any shape.

$$\oiint \vec{B} . ds = 0 \quad (10)$$

Applying the Gauss divergence theorem to equation (2), we can convert it (surface integral) into volume integral by taking the divergence of the same vector.

$$\Rightarrow \oiint \vec{B}.ds = \iiint \nabla.\vec{B}dv \tag{11}$$

Substituting equation (3) in (2) we get-

$$\iiint \nabla.\vec{B}dv = 0 \tag{12}$$

Here to satisfy the above equation either

$$\iiint dv = 0$$

$$\nabla.\vec{B} = 0$$

The volume of anybody or object can never be zero.

Thus, we arrive at Maxwell's second equation.

$$\nabla.\vec{B} = 0$$

Where,

$$\vec{B} = \mu\overline{H}$$

Is the flux density.

$$\Rightarrow\nabla.\vec{H} = 0$$

[solenoidal vector is obtained when the divergence of a vector is zero. Irrotational vector is obtained when the cross product is zero].

1.6.3 Maxwell Third Equation

Statement: A time-varying magnetic field will always produce an electric field.

Maxwell's 3rd equation is derived from Faraday's laws of Electromagnetic Induction. It states that "Whenever there are n-turns of conducting coil in a closed path placed in a time-varying magnetic field, an alternating electromotive force gets induced in each coil." Lenz's law gives this. Which states," An induced electromotive force always opposes the time-varying magnetic flux."

When two coils with N number of turns, A primary coil and a Secondary coil. The primary coil is connected to an alternating current source, and the secondary coil is connected in a closed loop and is placed at a small distance from the primary coil. When an AC passes through the primary coil, an alternating electromotive force is induced in the secondary coil. See the figure below.

Mathematically it is expressed as –Alternating emf,

$$emf_{alt} = -N\frac{d\phi}{dt} \tag{13}$$

Where,

N is the number of turns in a coil.

ϕ is the scalar magnetic flux.

The negative sign indicates that the induced emf always opposes the time-varying magnetic flux.

Let N=1,

$$\Rightarrow emf_{alt} = -\frac{d\phi}{dt} \tag{14}$$

Here, the scalar magnetic flux can be replaced by –

$$\phi = \iint \vec{B}.d\vec{s} \tag{15}$$

Substitute equation (3) in (2)

$$emf_{alt} = -\frac{d}{dt}\iint \vec{B}.d\vec{s}$$

Which is a partial differential equation given by-

$$emf_{alt} = \iint -\frac{\delta \vec{B}}{\delta t}.d\vec{s} \tag{16}$$

The alternating electromotive force induced in a coil is basically a closed path.

$$\Rightarrow emf_{alt} = \oint \vec{E}.d\vec{l} \tag{17}$$

Substituting equation (5) in (4) we get-

$$\Rightarrow \oint \vec{E}d\vec{l} = \iint -\frac{\delta \vec{B}}{\delta t}.d\vec{s} \tag{18}$$

The closed line integral can be converted into surface integral using Stoke’s theorem. Which states that the “Closed line integral of any vector field is always equal to the surface integral of the curl of the same vector field”

$$\Rightarrow \oint \vec{E}d\vec{l} = \iint \left(\nabla\times \vec{E}\right).d\vec{s} \tag{19}$$

Substituting equation (7) in (6) we get-

$$\Rightarrow \iint \left(\nabla\times \vec{E}\right)d\vec{s} = \iint -\frac{\delta \vec{B}}{\delta t}.d\vec{s} \tag{20}$$

The surface integral can be canceled on both sides. Thus, we arrive at Maxwell’s third equation

$$\nabla\times \vec{E} = -\frac{\delta \vec{B}}{\delta t}$$

Hence, we can conclude that the time-varying magnetic field will always produce an electric field.

Extended Maxwell’s third equation of Maxwell’s third equation for the static magnetic field Which states that the Static electric field vector is an irrotational vector.

Static field implies the time-varying magnetic field is zero,

$$\Rightarrow -\frac{\delta \vec{B}}{\delta t} = 0$$

$$\Rightarrow \nabla \times \vec{E} = 0$$

Hence it is an irrotational vector.

1.7 Maxwell's Fourth Equation

It is based on Ampere's circuit law. To understand Maxwell's fourth equation, it is crucial to understand Ampere's circuit law,

Consider the wire of a current-carrying conductor with the current I. Since there is an electric field, there must be a magnetic field vector around it. Ampere's circuit law states that "The closed line integral of magnetic field vector is always equal to the total amount of scalar electric field enclosed within the path of any shape", which means the current flowing along the wire (which is a scalar quantity) is equal to the magnetic field vector (which is a vector quantity)

Mathematically it can be written as –

$$\oint \vec{H}.d\vec{l} = I_{enclosed} \tag{21}$$

Any closed path of any shape or size will occupy one surface area. Thus, L.H.S of equation (1) can be converted into surface integral using Stoke's theorem, which states that "Closed line integral of any vector field is always equal to the surface integral of the curl of the same vector field"

$$\oint \vec{H}.d\vec{l} = \iint \left(\nabla \times \vec{H}\right).d\vec{s} \tag{22}$$

Substituting equation (2) in (1) we get-

$$\iint \left(\nabla \times \vec{H}\right).d\vec{l} = I_{enclosed} \tag{23}$$

Here,

$$\iint \left(\nabla \times \vec{H}\right).d\vec{l} isavectorquantityandI_{enclosed}isascalarquantity.$$

$$Toconvertthisscalarquantityintothevector, multiplyI_{enclosed}$$
$$bycurrentdensityvector\vec{J}. Thatisdefinedbyscalarcurrentflowing$$
$$perunitsurfacearea.$$

$$\vec{J} = \frac{I}{s}\hat{a}Nmeasuredusing(A/m^2)$$

Therefore,

$$\vec{J} = \frac{Differenceinscalarelectricfield}{differenceinvectorsurfacearea}$$

$$\vec{J} = \frac{dI}{ds}$$

$$dI = \vec{J}.d\vec{s}$$

$$\Rightarrow I = \iint \vec{J}.d\vec{s} \quad (24)$$

Thus, the scalar quantity is converted into vector quantity. Substituting equation (4) into (3) we get-

$$\Rightarrow \iint \left(\nabla\times \vec{H}\right).d\vec{l} = \iint \vec{J}.d\vec{s} \quad (25)$$

In the above equation, R.H.S and L.H.S both contain surface integral. Hence, we can cancel it.

Thus, we arrive at Maxwell's fourth equation-

$$\vec{J} = \nabla\times \vec{H} \quad (26)$$

We can conclude that the current density vector is a curl of the static magnetic field vector.

On applying the time-varying field (differentiating by time) we get-

$$\nabla\times \vec{J} = \frac{\delta\rho v}{\delta t} \quad (27)$$

Apply divergence on both sides of equation (6)-

$$\nabla.(\nabla\times \vec{H}) = \nabla\times \vec{J}$$

The divergence of the curl of any vector will always be zero.

$$\nabla.\left(\nabla\times \vec{H}\right) = 0 \tag{28}$$

Thus, from equations (7) and (8) we can write that-

$$\frac{\delta\rho v}{\delta t} = 0$$

Which contradicts the continuity equation for the time-varying fields.

To overcome this drawback, we add a general vector to the static field equation (6)

$$\left(\nabla\times \vec{H}\right) = \vec{J} + \vec{G} \tag{29}$$

Applying divergence on both sides-

$$\nabla.\left(\nabla\times \vec{H}\right) = \nabla.\left(\vec{J} + \vec{G}\right)$$

The divergence of the curl of any vector will always be zero.

$$0 = \nabla.\bar{J} + \nabla.\vec{G}$$

$$\nabla.\vec{G} = -\nabla.\vec{J} \tag{30}$$

Substituting equation (6) in (10) we get-

$$\nabla.\vec{G} = \frac{\delta\rho v}{\delta t} \tag{31}$$

By Maxwell's first equation,

$$\rho v = \nabla.\vec{D}$$

Substituting the value of ρv in equation (11) we get-

$$\nabla.\vec{G} = \frac{\delta\left(\nabla.\vec{D}\right)}{\delta t} \tag{32}$$

Here,

$$\frac{\delta}{\delta t} is\ \ time\ \ varient\ \ and$$

$$\nabla.\vec{D}$$

Is space variant and both are independent to each other. Thus, on rearranging equation (12) we get-

$$\nabla.\vec{G} = \nabla.\frac{\delta\left(\vec{D}\right)}{\delta t}$$

Thus cancelling the like terms, we get-

$$\vec{G} = \frac{\delta\vec{D}}{\delta t} = \vec{J}_D \tag{33}$$

Substituting them in

$$\left(\nabla\times\vec{H}\right) = \vec{J} + \vec{G}$$

This is an insulating current flowing in the dielectric medium between two conductors.

Hence Maxwell's fourth equation will be

$$\Rightarrow \left(\nabla\times\vec{H}\right) = \vec{J} + \vec{J}_D$$

Or

$$\Rightarrow (\nabla\times\vec{H}) = \vec{J} + \frac{\delta\vec{D}}{\delta t}$$

Where,

$$\vec{J}_D\ is\ \ Displacement\ \ current\ \ density.$$

We know that magnetic flux is equal to the product of electric flux and permittivity.

$$\vec{D} = \varepsilon\vec{E} \tag{34}$$

Substituting equation (14) in (13), we get-

$$\frac{\delta\left(\varepsilon\vec{E}\right)}{\delta t} = \vec{J}_D$$

$$\Rightarrow \vec{J}_D = \varepsilon\frac{\delta\left(\vec{E}\right)}{\delta t}$$

Chapter Two

Finite Volume Method and the Method of Weighted Residuals

2.1 Introduction

The area of mathematical and computer science generally referred to as numerical analysis is concerned with the creation, evaluation, and application of algorithms to resolve mathematical problems and certain equations involving ongoing changes. These contexts can appear in a variety of disciplines, including social sciences, engineering, medicine, and natural sciences as well.

Given its ability to solve problems such as partial differential equations and nonlinear equations that are beyond the reach of traditional methods, numerical analysis is a vital scientific field in its own right. It is used to develop mathematical models that simulate engineering and natural processes, improving technology and performance. It also contributes to opening new horizons for old theories and problems by offering rapid modifications to address new challenges.

The advent of digital computers has made it possible to perform complex algorithmic patterns with complete confidentiality, speed, and efficiency. This has significantly improved the ability of scientists and engineers to use a variety of numerical techniques, such as the Monte Carlo method, the weighted residue method, the finite difference method, the finite element method, and the finite volume method, to address large and complex problems.

2.2 Finite Volume Method

Partial differential equations, especially those based on physical conservation laws and guidelines, can be solved using the finite volume method (FVM). This method works by dividing the problem domain into discrete volumes and using volume integrals to decompose the equations into understandable parts. [26]

Partial differential equations (PDEs), especially those involving physical conservation laws, can be solved numerically using the finite volume method (FVM). Here's a basic summary:

Key Concepts

1. **Volume Integrals to Surface Integrals**
   Using the divergence theorem, the FVM transforms volume integrals into partial differential equations (PDEs) containing divergence terms in surface integrals. This allows the calculation of flows over the surfaces of each small volume.

2. **Conservation**
   One of the key advantages of FVM is its conservation of mass, momentum, and energy. It contributes to the conservation of mass, momentum, and energy by ensuring that the flow entering a volume is equal to the flow exiting the adjacent volume.

3. **Unstructured Meshes**
   FVMs are flexible enough to handle highly complex shapes, due to their ease of application to unstructured meshes.

Applications

- Computational Fluid Dynamics (CFD): Commonly used to simulate heat transfer and fluid flow.
- Diffusion Problems: Used to solve equations related to diffusion, such as thermal conductivity and chemical concentration.

Comparison with Other Methods:

- Finite Difference Method (FDM): Approximates derivatives using values at specific points (nodes).
- Finite Element Method (FEM): Builds local approximations of the solution using local data and combines them to create a global solution.
- FVM: Calculates the average value of the solution over a volume and uses this information to make approximations in each cell.

The Finite Volume Method (FVM) deals with boundary conditions by including them in the flux calculations at the edges of the control volumes. Here are some common approaches:

Types of Boundary Conditions

1. **Dirichlet Boundary Conditions**

   In science and engineering, a Dirichlet boundary condition is also referred to as a constant boundary condition or a boundary condition of the first kind. The value of a function at this boundary is determined by these conditions. This is often done in an FVM by setting the value of the variable at the boundary cell to a specified value.

2. **Neumann Boundary Conditions**

   Neumann boundary condition or boundary condition of the second kind. The Neumann boundary condition specifies the normal derivative at a boundary to be zero or constant. The total value of the derivative (the flux) at the boundary is determined by the current conditions. This is achieved in the FVM by aligning the flux at the boundary surface with the specified value.

3. **Robin (Mixed) Boundary Conditions**

   These conditions define a linear combination of the function's value and its derivative at the limit, combining the Dirichlet and Newman conditions. Induction or ghost cell techniques can be effectively used to address this problem.

Implementation Techniques

1. **Ghost Cells**

   Boundary conditions are applied using "ghost cells," which are, in hypothesis, cells that reside beyond the computational domain. To ensure that the border cell has the appropriate value for a Dirichlet condition, the ghost cell value is previously accounted for and set. The ghost cell value is specified for Neumann conditions to guarantee the proper flux at the boundary.

2. **Extrapolation**

   Within this particular avenue, the boundary value or flux is estimated by fitting a polynomial (linear or higher) to the data adjacent to the boundary. When handling intricate boundary conditions, such as Robin conditions, it is of quite leverage.

3. **Direct Flux Adjustment**

   The flux at the boundary is adjusted to correspond to the designated derivative under Neumann circumstances. This entails including the boundary condition within the flow computation at the boundary.

For absolutely accurate and credible findings, the Finite Volume Method's (FVM) stability with boundary conditions is crucial. Here are some significant points:

Stability Considerations

1. **Consistency and Convergence**

   The discretization of boundary conditions must be consistent with the numerical method and the physical problem to ensure the stability of the FVM. This ensures that the numerical solution gets closer to the exact answer as the mesh resolution increases.

2. **Treatment of Boundary Conditions**

- Dirichlet Conditions: Stability is usually maintained by setting the specified values directly into the boundary cells.
- Neumann Conditions: Stabilization may be more difficult. Flow at the boundary must be carefully calculated to prevent instability.
- Robin Conditions: These mixed conditions need careful handling to stay stable, often using ghost cells or extrapolation methods.

3. **Energy Estimates**

   Techniques like the simultaneous approximation term (SAT) can be used to apply boundary conditions in a way that ensures energy estimates, leading to better stability.

4. **Conditional Stability**

   Some schemes may be conditionally stable, meaning stability is guaranteed only if certain conditions (like time step size) are met.

Practical Implementation

- Ghost Cells: These are used to extend the computational domain and apply boundary conditions without directly affecting the internal cells. This method helps maintain stability by ensuring that boundary conditions are smoothly integrated into the numerical scheme.

- Extrapolation: This involves fitting a polynomial to the values near the boundary and using it to estimate the boundary value or flux. Properly implemented, this method can enhance stability. [26]

2.3 Method of Weighted Residuals

The method of the weighted residuals (MWR), sometimes known as the method of moments (MoM), has traditionally been applied in the frequency domain and has been shown to be effective and efficient, especially in computing open electromagnetic structure problems. Although it has been extended to the time domain in various forms, it is generally employed to solve integral formulations derived from Maxwell's equations. Therefore, it is often considered to be one type of numerical method that is different from other numerical methods, such as finite-difference methods. However, in this paper we will show that the MWR, or MoM, is not just a method per se: it can in fact be a general framework for our approach to unifying or deriving most of the numerical methods developed so far, either in the frequency domain or in the time domain. As a result, all numerical methods can be quite easily understood and can be categorized in one general method, although their conventional derivations may still have their respective advantages. One potential application is that the hybridization of different numerical methods can now be done within a uniform framework. The paper is intended for both beginners and experienced practitioners in numerical electromagnetic modeling.

The Weighted Residuals Method (WRM) is a numerical technique often used in the context of solving differential equations and optimization problems, particularly in the fields of engineering, physics, and applied mathematics. This method focuses on minimizing the residuals (the difference between the observed and calculated values) while placing greater emphasis on more significant or influential data points.[27]

Basic Concept

The core idea of WRM is to approximate the solution of a differential equation by a finite sum of test functions. The method involves the following steps:

1. Assume an Approximate Solution: Start with an assumed form of the solution, often a linear combination of basic functions.
2. Formulate the Residual: Substitute the approximate solution into the differential equation. This results in a residual, which represents the error.

3. Weight the Residual: Multiply the residual by a set of weight functions and integrate over the domain.
4. Minimize the Residual: Change the coefficients of the basic functions to reduce the weighted residual, usually by making the integral of the weighted residual equal to zero.

2.3.1 Types of Weighted Residual Methods

There are several specific methods under the WRM umbrella, each using different weight functions:[28]

1. Galerkin Method: Uses the same functions for both the basis and weight functions.
2. Collocation Method: Chooses weight functions that are Dirac delta functions, focusing on specific points in the domain.
3. Least Squares Method: Minimizes the square of the residual over the entire domain.
4. Subdomain Method: Splits the domain into smaller subdomains and minimizes the residual in each one.

Applications

WRM is widely used in various fields, including:

- Finite Element Method (FEM): The weighted residual method (WRM) is often used in finite element methods (FEM) to derive the weak form of differential equations. It is commonly applied in structural analysis, fluid dynamics, and heat transfer.
- Parameter Estimation: In statistical models, weighted least squares can be used to model risk estimation by giving different weights to data points based on their reliability or variance.
- Variational Methods: In variable analysis, WRM is used to create problems where residuals are minimized using weights, leading to more accurate solutions for physical systems described by PDEs.
- Computational Fluid Dynamics (CFD): Helps solve complex fluid flow problems.
- Electromagnetics: Used in solving Maxwell’s equations for electromagnetic fields.

Choosing Weights

Choosing the appropriate weight function is important. Common methods include:

- Constant Weights: (Fixed weights) A simple method in which all residuals are given the same weight.
- Variance-based Weights: When the variance of data points is known, weights can be adjusted inversely to the variance, giving greater weight to less noisy data.
- Spatially Varying Weights: In spatial problems, weights can change based on the location or behavior of the system.

Advantages and Challenges

Advantages

- Flexibility in addressing different errors or influences in the model.
- Improved accuracy in fitting or optimization by focusing on significant data points.

Challenges

- Choosing appropriate weight functions can be challenging and may require experimental testing.
- Failure to choose weights carefully can lead to overfitting.

In short, the weighted residuals method is a useful technique that allows you to consider different levels of confidence in the data when solving differential equations or optimization problems. Its flexibility makes it suitable for a wide range of applications.

2.4 Compare the two methods

1. Fundamental Approach

Finite Volume Method (FVM)

- Concept: FVM focuses on the conservation laws and integral formulations of the governing equations. It discretizes the problem domain into small control volumes and applies the integral form of conservation laws over these volumes. The fluxes across the boundaries of each volume are calculated to ensure conservation properties are maintained.
- Strengths: Particularly effective for problems where conservation laws are crucial, such as fluid dynamics, electromagnetics, and heat transfer. It inherently maintains conservation of fluxes which makes it reliable for engineering applications.

- Limitations: FVM can be more diffusive, especially when handling high-gradient phenomena because it inherently averages over control volumes.[28]

Method of Weighted Residuals (MWR)

- Concept: MWR works by choosing a trial solution that approximately satisfies the governing equations and then minimizes the residual (the error in these equations) in a weighted integral sense. It is a broader category that includes methods like Galerkin, Collocation, and Least Squares.
- Strengths: Highly flexible in handling complex boundary conditions and irregular geometries. It can achieve higher accuracy if the trial functions are well chosen.
- Limitations: The choice of trial and weighting functions can significantly affect the solution's accuracy and stability. It might require more computational effort to determine the optimal functions, especially in nonlinear problems.

2. Ease of Implementation

- FVM:
- Generally straightforward to implement, especially with commercially available software and libraries that support structured and unstructured grids.
- FVM's reliance on local values and fluxes makes it somewhat easier to program from scratch in environments like MATLAB or Python.
- MWR:
- Implementation complexity can vary significantly based on the choice of trial and weighting functions. Some forms like the Galerkin method may require sophisticated programming to handle the integration and assembly processes.
- Often involves solving systems of algebraic equations which might be dense or require special solvers, especially in three-dimensional or high-order systems.

3. Computational Efficiency

FVM

- Computationally efficient for large-scale problems due to its local approach to flux calculation and the typical sparsity of its discretized system matrices.
- Well-suited for parallel computing as the computation for each volume can often be performed independently.

MWR

- It can be less efficient if the system of equations is large or if calculating the residuals requires a lot of computational power.
- Efficiency depends on the nature of the trial functions; for example, global functions like those used in spectral methods can lead to dense system matrices.

4. Accuracy and Usefulness

FVM

- It provides good accuracy for many problems, especially those with shocks or discontinuities. Its conservative nature makes it dependable for engineering applications.
- May suffer from numerical diffusion in cases with high gradients or when using low-resolution grids.

MWR

- It can offer high accuracy if the trial and weighting functions closely match the solution. This is especially true in areas where the solution behavior is well-known.
- Best suited for problems where higher order continuity is required or where the solution varies smoothly.[29]

Chapter Three

Numerical solutions FVM and MWR

3.1 Introduction

Because of the growing efficiency of computing facilities, constructing efficient computational methods for simulation of partial differential equations, in two and three dimensions, has become a reality[30].

This chapter looks at solving Maxwell's equations using two advanced methods: the Finite Volume Method (FVM) and the Method of Weighted Residuals (MWR), both implemented in MATLAB. These methods are important for finding approximate solutions to Maxwell's equations, which describe how electromagnetic fields behave.

The connection between electric and magnetic fields is described by the equations Ampere's Law with Maxwell's Correction and Faraday's Law. This chapter delineates the numerical solution of these equations, the practical application of boundary conditions, and the visualization and comparison of the results.

3.2 Maxwell's equations in our code

Maxwell's equations: Faraday's law and Ampere's law with Maxwell's adjustment.

- The relationship between magnetic flux density (B) and electric field density (E) is explained by Faraday's law. This law states that the negative value of the rotation of the electric field around a surface is equal to the velocity of change of the magnetic flux through it.
  In terms of mathematics, it may be expressed as:

$$\nabla \times E = -\partial B/\partial t \tag{35}$$

- The relationship between the electric displacement field (D), the electric current density (J), and the magnetic field (H) is described by Ampere's law with Maxwell's correction. According to this theory, the rotation of the magnetic field around a closed loop is equal to the sum of the current density flowing through the loop and the rate of change of the electric displacement field. This can be expressed mathematically as follows:

$$\nabla \times H = J + \partial D/\partial t \quad (36)$$

The code will solve two of Maxwell's equations, specifically the time-domain versions of Faraday's law of electromagnetic induction and Ampere's law with Maxwell's correction:

1. Faraday's Law (a changing magnetic field creates an electric field):
- This equation explains how a changing magnetic field creates an electric field.
- In the code, this is shown by the updated equations for the electric fields Ex and Ey.

2. Ampere's Law with Maxwell's Addition (time-varying electric field induces magnetic field): - This equation describes how a time-varying electric field induces a magnetic field and includes the current displacement term according to Maxwell's addition.
- It is represented in the code by the updated equation for the Hz magnetic field.

3.3 Boundary conditions

Boundary conditions are essential in electromagnetic simulations, including those that solve Maxwell's equations. It determines how electromagnetic fields behave at the boundaries of the simulation field. Without appropriate boundary conditions, the simulation may not accurately represent the physical behaviour of the fields. Here is a brief explanation of the types of boundary states commonly used in electromagnetic simulations:

1. Dirichlet boundary conditions: These conditions describe the values of electromagnetic fields (usually electric fields) at the field boundaries. Dirichlet boundary conditions are necessary when the exact values of the fields at the boundaries are known. Examples include setting the electric field to zero at an ideal conductance boundary or setting a known voltage.
2. Newman boundary conditions: Newman boundary conditions describe the natural derivative of electromagnetic fields at the boundary. These conditions are typically applied when you know the flow of fields across boundaries but not their values. For example, you can determine the surface current density or the rate of change of the electric field at the boundary.

3. Periodic boundary conditions: These conditions are applied in periodic structures to model electromagnetic waves that propagate through recurring structures, such as periodic networks. It makes sure that the fields on one boundary are the same as the fields on the corresponding boundary of the adjacent unit cell.
4. Radiation boundary conditions are a strategy used to cease reflections back into the field in open boundary problems when electromagnetic waves radiate in the opposite direction from the computing field. Perfectly matched layers (PMLs) or other absorbing techniques are rampantly used to create these circumstances, which absorb outgoing waves.

Applying boundary conditions in the code template necessarily entails including the appropriate circumstances determined by the problem's geometry and physics. The kind of boundary conditions required for the simulation will determine how we approach this. In order to meet the requirements, the stiffness matrix (A), mass matrix (M), and load vector (b) must often be recalibrated as needed.

For instance, a Dirichlet boundary condition can be used to set the electric field components (Ex and Ey) at the boundary nodes to zero in the context of a perfect conductor border. Radiation boundary conditions can be used for open boundaries in order to prevent reflections.

Choosing the boundary conditions that best suit the situation is crucial to ensuring accurate physics results, as they have a significant impact on the simulation results.

3.4 Finite Volume Method (FVM)

The finite volume method focuses on maintaining flows within control volumes. Maxwell's equations are integrated over these volumes, and the integrals are transformed into surface integrals using Gauss's theorem.

Steps for FVM Implementation

1. Identify the domain: The first step is to define the region in which Maxwell's equations will be solved. This region can be one, two, or three dimensions.
2. Making the mesh: The next step is to create a grid for the domain. The domain is divided into smaller parts by the cells that form the grid. These cells may be hexagonal, tetrahedral, or triangular.

3. Initialize fields and boundary conditions: The fields and boundary conditions will then be initialized. It is necessary to set the electric and magnetic fields at each applicable position within the field. You can set the fields to zero or some other value and specify the boundary conditions for Maxwell's equations.
4. Solve the equations: After the fields have been initialized, the next phase is to solve Maxwell's equations.
5. We do this by iterating over the cells in the mesh and updating the fields. The update procedure involves calculating the fluxes of the fields across the faces of the cells and then using those fluxes to update the fields inside.
6. Repeat steps 3 and 4 until convergence: Repeat steps 3 and 4 until the fields converge to a solution. It means the fields have stopped changing significantly from iteration to iteration.
7. Plot the results: Once the fields converge, the next step is to plot the results. This can be done using a variety of planning software.

Sample Code

```
clear;

clc;

% Grid setup

Nxx = 100;

Nyy = 100;

Lxx = 1.0;

Ly = 1.0;

Lsp = 3e8;

N = 100;

% Spatial grid

[x, y] = meshgrid(linspace(0, Lxx, Nxx), linspace(0, Ly, Nyy));
```

```
dx = x(1, 2) - x(1, 1);

dy = y(2, 1) - y(1, 1);

dt = 0.9 / Lsp / sqrt(1/dx^2 + 1/dy^2);

% Initialize fields

Ex = zeros(Nxx, Nyy);

Ey = zeros(Nxx, Nyy);

Hz = zeros(Nxx, Nyy);

% Impulse at the center

w = zeros(Nxx, Nyy);

w(50, 50) = 1.0;

% Boundary conditions

Ex(1,:) = 0;

Ex(Nxx,:) = 0;

Ey(:,1) = 0;

Ey(:,Nyy) = 0;

Hz(1,:) = 0;

Hz(Nxx,:) = 0;

Hz(:,1) = 0;

Hz(:,Nyy) = 0;

% Solve the equations

for n = 1:N
```

```
% Calculate the fluxes
Fx = zeros(Nxx, Nyy);
Fy = zeros(Nxx, Nyy);
Hx = zeros(Nxx, Nyy);
Hy = zeros(Nxx, Nyy);
  for i = 1:Nxx
    for j = 1:Nyy
        if i == 1
            Fx(i, j) = -Ey(i, j) / dx;
            Hx(i, j) = Hz(i, j) / dx - Ey(i, j) / dy;
        else
            Fx(i, j) = (Ey(i, j) - Ey(i-1, j)) / dx;
            Hx(i, j) = (Hz(i, j) - Hz(i-1, j)) / dx - (Ey(i, j) - Ey(i-1, j)) / dy;
        end
        if j == 1
            Fy(i, j) = Ex(i, j) / dy;
            Hy(i, j) = -Hz(i, j) / dy + Ex(i, j) / dx;
        else
            Fy(i, j) = (Ex(i, j) - Ex(i, j-1)) / dy;
            Hy(i, j) = (-Hz(i, j) + Hz(i, j-1)) / dy + (Ex(i, j) - Ex(i, j-1)) / dx;
        end
```

```
        end
    end
    % Update fields
    Ex = Ex + dt * (Fy - Hx) + w * dt;
    Ey = Ey + dt * (Hx - Fx) + w * dt;
    Hz = Hz + dt * (Hy - Fx + Fy);
    % Plotting at every 10th time step
    if mod(n, 10) == 0
        imagesc(abs(Ey));
        title(['Time Step: ', num2str(n)]);
        colorbar;
        drawnow;
    end
end
% Final field plots
figure;
surf(x, y, Ex);
title('Electric Field Ex');
xlabel('x');
ylabel('y');
zlabel('Ex');
```

```
figure;

surf(x, y, Ey);

title('Electric Field Ey');

xlabel('x');

ylabel('y');

zlabel('Ey');

figure;

surf(x, y, Hz);

title('Magnetic Field Hz');

xlabel('x');

ylabel('y');

zlabel('Hz');
```

Figure 3.1

Ex Field at final time step. This figure shows the distribution of the electric field component Ex over the domain, visualized using a 3D surface plot. The peaks represent areas of high field intensity

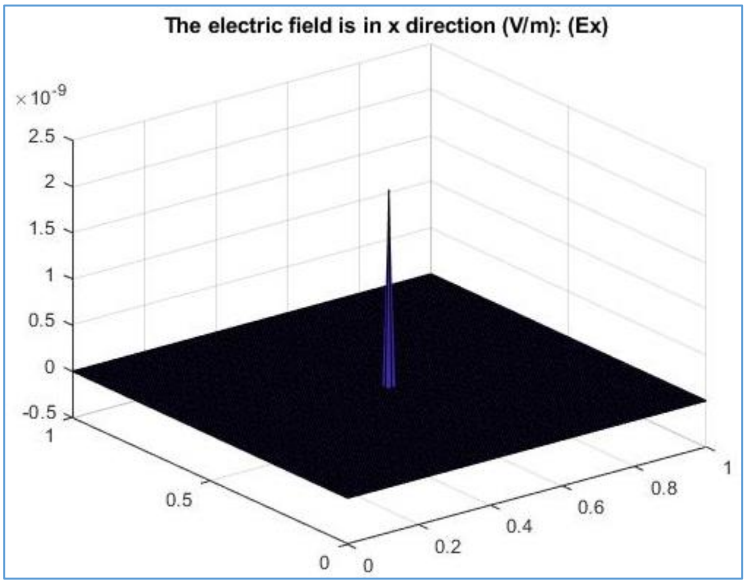


Figure 3.2

Ey Field at final time step. Like Figure 3.1, this figure displays the Ey component, illustrating the variation of the electric field in the y direction

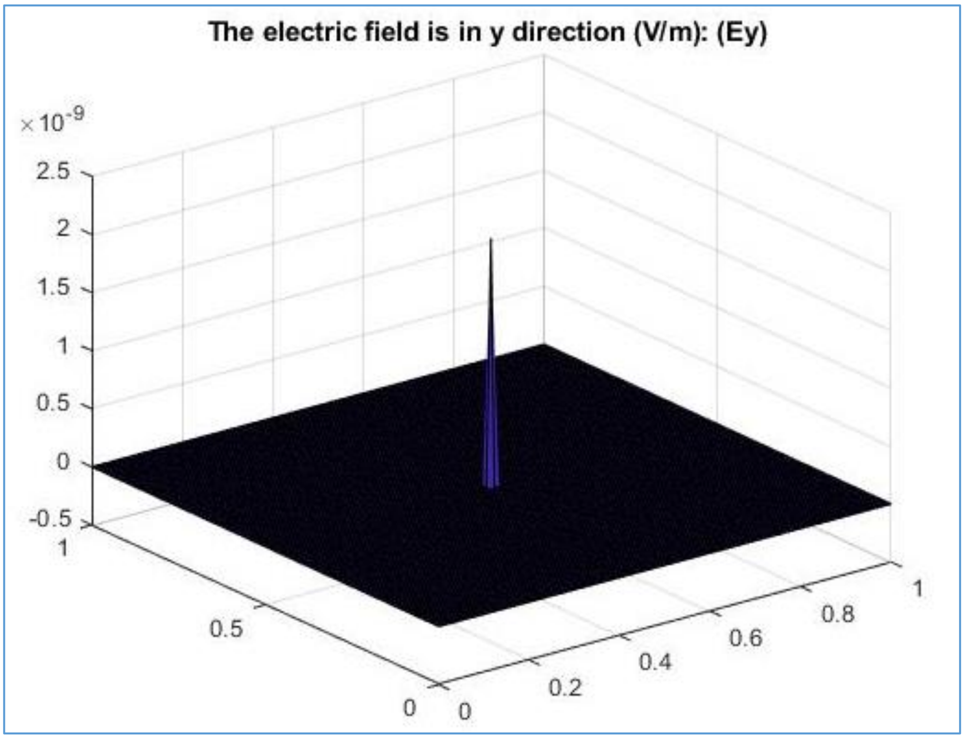
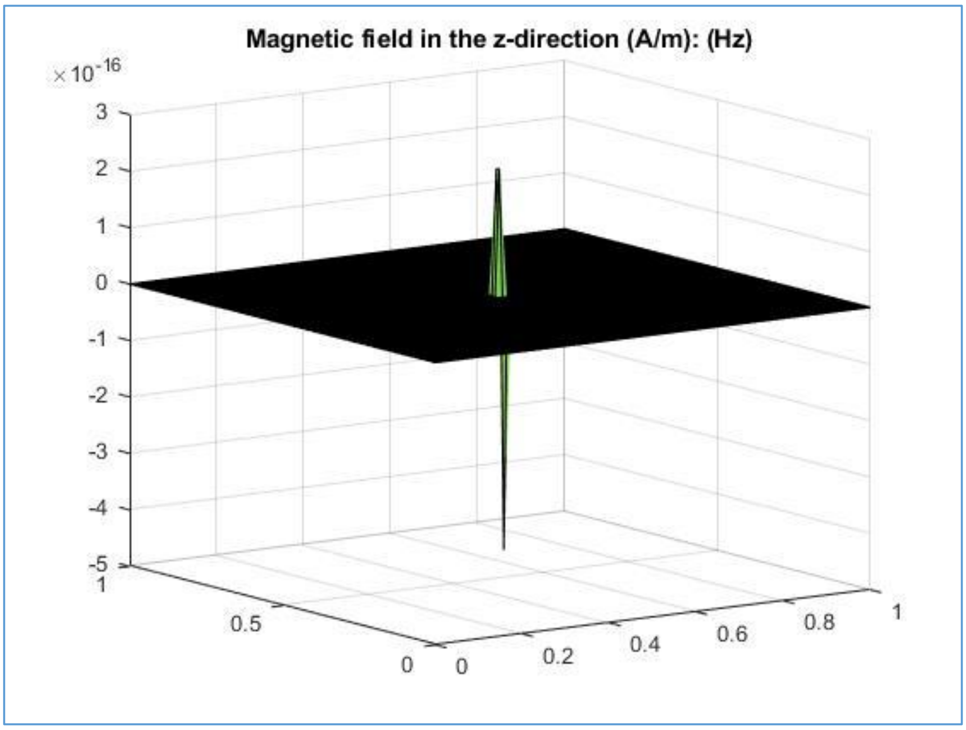


Figure 3.3

Hz Field at final time step. This figure plots the magnetic field component Hz, which shows the induced magnetic field because of the time-varying electric fields



3.5 Method of Weighted Residuals (MWR)

The Method of Weighted Residuals (MWR) minimizes the residual of the governing equations across the domain by applying weights. Trial solutions are selected, and the residuals (errors) are minimized to improve accuracy.

Steps for MWR Implementation

1. Choose a Trial Solution: A trial solution is assumed for the electric and magnetic fields.
2. Set Up Residual Equations: Residuals are formulated based on the trial solution and the governing equations.
3. Minimize the Residuals: The residuals are minimized using appropriate weighting functions to obtain a more accurate solution.

Sample Code

```
clear;

clc;

% Constants

Ls = 3e8;              % Speed of light (m/s)

M0 = 4 * pi * 1e-7;    % Permeability of free space (H/m)

epsi0 = 8.854e-12;     % Permittivity of free space (F/m)

% Grid parameters

LLx = 1.0;             % Domain length in the x-direction (m)

LLy = 1.0;             % Domain length in the y-direction (m)

NNx = 50;              % Number of nodes in x-direction

NNy = 50;              % Number of nodes in y-direction

Nt = 200;              % Number of time steps
```

```
% Mesh creation
[x, y] = meshgrid(linspace(0, LLx, NNx), linspace(0, LLy, NNy));
dx = x(1, 2) - x(1, 1); % Spatial step in x-direction
dy = y(2, 1) - y(1, 1); % Spatial step in y-direction
dt = 0.9 / Ls / sqrt(1/dx^2 + 1/dy^2); % Time step (s)
% Initialize field arrays
Ex = zeros(NNx, NNy); % Electric field in x-direction (V/m)
Ey = zeros(NNx, NNy); % Electric field in y-direction (V/m)
Hz = zeros(NNx, NNy); % Magnetic field in z-direction (A/m)
% Main time-stepping loop
for n = 1:Nt
   % Update electric fields (Ex and Ey)
   for i = 2:NNx - 1
      for j = 2:NNy - 1
         Ex(i, j) = Ex(i, j) + (dt / epsi0) * (Hz(i, j) - Hz(i, j - 1)) / dy;
         Ey(i, j) = Ey(i, j) - (dt / epsi0) * (Hz(i - 1, j) - Hz(i, j)) / dx;
      end
   end
   % Apply boundary conditions using simple copying
   Ex(1, :) = Ex(2, :); Ex(NNx, :) = Ex(NNx - 1, :);
   Ex(:, 1) = Ex(:, 2); Ex(:, NNy) = Ex(:, NNy - 1);
```

```
    Ey(1, :) = Ey(2, :); Ey(NNx, :) = Ey(NNx - 1, :);

    Ey(:, 1) = Ey(:, 2); Ey(:, NNy) = Ey(:, NNy - 1);

    % Update magnetic field (Hz)

    for i = 1:NNx - 1

        for j = 1:NNy - 1

            Hz(i, j) = Hz(i, j) + (dt / M0) * ((Ey(i + 1, j) - Ey(i, j)) / dx - (Ex(i, j + 1) - Ex(i, j)) / dy);

        end

    end

    % Apply source term (Gaussian pulse at the center)

    source = exp(-((n - Nt / 2)^2) / (2 * (Nt / 20)^2));

    Ex(round(NNx/2), round(NNy/2)) = Ex(round(NNx/2), round(NNy/2)) + source;

    % Visualization (every 10 steps)

    if mod(n, 10) == 0

        imagesc(x(1, :), y(:, 1), abs(Hz));

        title(['Time Step: ', num2str(n)]);

        colorbar;

        drawnow;

    end

end
```

3.6 Numerical Solution Preparation and Implementation Using MATLAB

To apply our algorithm,

Using 1D maxwell equations, namely: Faraday and ampere equations given as: Faraday law

$$\frac{\partial E(x,t)}{\partial x} = -\frac{\partial B(x,t)}{\partial t} \quad (37)$$

Ampere law

$$\frac{\partial B(x,t)}{\partial x} = \mu\, J(x,t) + \mu\epsilon\, \frac{\partial E(x,t)}{\partial t} \quad (38)$$

Take the derivative with respect to t in the first equation (Eq.1), and the derivative with respect to x for the second one (Eq.2), yield that:

$$\frac{\partial^2 E(x,t)}{\partial x \partial t} = -\frac{\partial^2 B(x,t)}{\partial t^2} \quad (39)$$

$$\frac{\partial^2 B(x,t)}{\partial x^2} = \mu\, \frac{\partial J(x,t)}{\partial x} + \mu\epsilon\, \frac{\partial^2 E(x,t)}{\partial x \partial t} \quad (40)$$

Substitute Eq. 3 in Eq.4 to get the wave equation:

$$\frac{\partial^2 B(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B(x,t)}{\partial t^2} = \mu\, \frac{\partial J(x,t)}{\partial x} \quad (41)$$

c is the speed of light, μ is magnetic permittivity, and ϵ is electric permittivity of the medium

for a given current density source $J(x,t)$, we can solve the wave equation (Eq. 5) to find the magnetic field as function of time and space, also one may find the electric field by integrate the time derivative of the magnetic field with respect to x, mathematically:

$$E(x,t) = -\int \frac{\partial B(x,t)}{\partial t}\, dx \quad (42)$$

Finite Volume method

To solve Eq. 5 for a given source, for simplicity, c=1, and $\mu = 1$, using FVM

$$\frac{\partial^2 B(x,t)}{\partial x^2} - \frac{\partial^2 B(x,t)}{\partial t^2} = \frac{\partial J(x,t)}{\partial x}$$

Step 1: Discretize the Domain

1. Divide the spatial domain $x \in [0, L]$ into N control volumes (cells) of width Δx.
2. Divide the time domain $t \in [0, T]$ into M time steps of size Δt.

Step 2: Discretize the PDE

1. Use central differencing for the spatial derivatives:

$$\frac{\partial^2 B}{\partial x^2} \approx \frac{B_{i+1} - 2B_i + B_{i-1}}{(\Delta x)^2}$$

where B_i is the value of B at the i-th cell.

2. **Use** central differencing for the time derivatives:

$$\frac{\partial^2 B}{\partial t^2} \approx \frac{B_i^{n+1} - 2B_i^n + B_i^{n-1}}{(\Delta t)^2}$$

where B_i^n is the value of B at the ith cell and nth time step.

3. Discretize the source term:

$$\frac{\partial J}{\partial x} = \mu \frac{J_{i+\frac{1}{2}}^n - J_{i-\frac{1}{2}}^n}{\Delta x}.$$

Step 3: Write the Finite Volume Equation

Substitute the discretized derivatives into the PDE:

$$\frac{B_{i+1}^n - 2B_i^n + B_{i-1}^n}{(\Delta x)^2} - \frac{1}{c^2}\frac{B_i^{n+1} - 2B_i^n + B_i^{n-1}}{(\Delta t)^2} = \mu \frac{J_{i+\frac{1}{2}}^n - J_{i-\frac{1}{2}}^n}{\Delta x}.$$

Rearrange to solve for B_i^{n+1}:

$$B_i^{n+1} = 2B_i^n - B_i^{n-1} + \frac{c^2(\Delta t)^2}{(\Delta x)^2}(B_{i+1}^n - 2B_i^n + B_{i-1}^n) + \mu c^2(\Delta t)^2 \frac{J_{i+\frac{1}{2}}^n - J_{i-\frac{1}{2}}^n}{\Delta x}$$

Step 4: Apply Boundary Conditions

- Use Dirichlet boundary conditions:

  $B_0^n = 0 \quad , B_N^n = 0$

Step 5: Apply Initial Conditions

- Set the initial conditions:

  $B_i^0 = 0 \quad , \frac{\partial B}{\partial t}(x, 0) = 0$

For the second initial condition, use:

$$B_i^1 = B_i^0$$

Step 6: Time Marching

- Use the discretized equation to update B_i^{n+1} at each time step.

MATLAB Implementation

```
% Parameters
L = 1;         % Length of the spatial domain
T = 5;          % Total time (adjusted to match the period of sin(πt/5))
N = 100;       % Number of spatial cells
M = 1000;      % Number of time steps
dx = L / N;    % Spatial step size
dt = T / M;    % Time step size
c = 1;         % Wave speed
mu = 1;        % Permeability
% Initialize arrays
B = zeros(N+1, M+1);  % B(x,t) array
x = linspace(0, L, N+1);  % Spatial grid
t = linspace(0, T, M+1);  % Temporal grid
% Source term J(x,t)
J = zeros(N+1, M+1);  % Initialize J(x,t)
for i = 1:N+1
    for n = 1:M+1
        J(i, n) = ((pi^2/50 - 6) * x(i)^2 + (5 - pi^2/50) * x(i)^4 + (pi^2 * x(i)^6)/150) * sin(pi
* t(n)/5);
    end
end
% Initial conditions
B(:, 1) = 0;          % B(x,0) = 0
B(:, 2) = 0;          % B(x,1) = B(x,0) = 0 (since ∂B/∂t(x,0) = 0)
% Time marching loop
for n = 2:M
```

```
    for i = 2:N
        % Discretized equation
        B(i, n+1) = 2*B(i, n) - B(i, n-1) ...
            + (c^2 * dt^2 / dx^2) * (B(i+1, n) - 2*B(i, n) + B(i-1, n)) ...
            + mu * c^2 * dt^2 * (J(i+1, n) - J(i-1, n)) / dx;
    end
      % Boundary conditions
    B(1, n+1) = 0;    % B(0, t) = 0
    B(N+1, n+1) = 0;  % B(L, t) = 0
end
% Plotting the results
figure;
surf(x, t, B');
xlabel('x');
ylabel('t');
zlabel('B(x,t)');
```

Method of Weighted Residual

Steps to Solve Using Weighted Residual Method

Step 1: Define the Basis Functions

Choose a set of basis functions $\phi_{n,m}(x,t)$ that satisfy the boundary conditions. In this case, the basis functions are:

$$\phi_{n,m}(x,t) = (x - L)x^n t^m$$

where *L*=1 (length of the domain), and *n*, *m* are integers from 1 to *n* max.

Step 2: Approximate the Solution

Express the approximate solution $\psi(x,t)$ as a linear combination of the basis functions:

$$\psi(x,t) = \sum_{n=1}^{nmax} \sum_{m=1}^{nmax} a_{n,m}\phi_{n,m}(x,t)$$

where $a_{n,m}$ are unknown coefficients to be determined.

Step 3: Compute the Residual

Substitute the approximate solution $\psi(x,t)$ into the PDE to compute the residual $R(x,t)$:

$$R(x,t) = \frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\psi(x,t)}{\partial t^2} - \mu\,\frac{\partial J(x,t)}{\partial x} \tag{43}$$

Step 4: Minimize the Residual

Use the Galerkin method (a type of weighted residual method) to minimize the residual. This involves setting the weighted integral of the residual to zero:

$$\int_0^L \int_0^T R\,(x,t)\phi_{i,j}(x,t)dx\,dt \qquad for\ i,j\ = 1,2,\dots\dots\dots,n_{max}$$

This results in a system of linear equations for the coefficients $a_{n,m}$.

Step 5: Assemble the System of Equations

For each basis function $\phi_{n,m}(x,t)$, compute the integral:

$$\sum_{n=1}^{nmax} \sum_{m=1}^{nmax} \int_0^L \int_0^T \phi_{n,m}(x,t)\left(\frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\psi(x,t)}{\partial t^2} - \mu\,\frac{\partial J(x,t)}{\partial x}\right)dxdt = 0$$

This gives a system of *n*max×*n*max equations for the coefficients $a_{n,m}$

Step 6: Solve the System of Equations

Solve the system of equations for the coefficients $a_{n,m}$ This can be done using numerical methods or symbolic computation.

Step 7: Construct the Solution

Once the coefficients $a_{n,m}$ are determined, substitute them back into the approximate solution:

$$B(x,t) \approx \psi(x,t) = \sum_{n=1}^{nmax} \sum_{m=1}^{nmax} a_{n,m}\phi_{n,m}(x,t)$$

MATLAB Implementation

```
% Clear workspace
clear;
clc;
% Define symbolic variables
syms x t;
syms a [5 5]; % Coefficients a(n,m) for n,m = 1 to 5
% Parameters
L = 1;        % Length of the domain
T = 5;        % Total time
nmax = 5;     % Maximum order of basis functions
% Define basis functions
BaseB = @(x, t, n, m) (x - L) * x^n * t^m;
% Define the source term J(x,t)
J = ((pi^2/50 - 6) * x^2 + (5 - pi^2/50) * x^4 + (pi^2 * x^6)/150) * sin(pi * t / 5);
% Compute the derivative of J(x,t) with respect to x
dJ_dx = diff(J, x);
% Initialize the system of equations
eqns = [];
vars = [];
% Assemble the system of equations using Galerkin method
for i = 1:nmax
    for j = 1:nmax
        % Weighted residual equation
        eqn = 0;
        for n = 1:nmax
```

```
            for m = 1:nmax
                % Integrate the weighted residual terms
                integrand = BaseB(x, t, i, j) * (diff(BaseB(x, t, n, m), x, 2) - diff(BaseB(x, t, n, m), t, 2));
                eqn = eqn + a(n, m) * int(int(integrand, x, 0, L), t, 0, T);
            end
        end
        % Add the source term contribution
        source_integral = int(int(BaseB(x, t, i, j) * dJ_dx, x, 0, L), t, 0, T);
        eqn = eqn - source_integral;
        % Add the equation to the system
        eqns = [eqns; eqn == 0];
        vars = [vars; a(i, j)];
    end
end
% Solve the system of equations for the coefficients a(n,m)
sol = solve(eqns, vars);
% Extract the coefficients
coefficients = struct2cell(sol);
a_values = double([coefficients{:}]);
% Reshape the coefficients into a matrix
a_matrix = reshape(a_values, [nmax, nmax]);
% Construct the approximate solution B(x,t)
B_approx = 0;
for n = 1:nmax
```

```
    for m = 1:nmax
        B_approx = B_approx + a_matrix(n, m) * BaseB(x, t, n, m);
    end
end
% Display the approximate solution
disp('Approximate Solution B(x,t):');
disp(vpa(B_approx, 4)); % Display with 4 decimal places
% Evaluate and plot the solution
[X, T] = meshgrid(linspace(0, L, 50), linspace(0, T, 50));
B_eval = double(subs(B_approx, {x, t}, {X, T}));
figure;
surf(X, T, B_eval);
xlabel('x');
ylabel('t');
zlabel('B(x,t)');
```

Example 1:

For current density given by:

$$J(x,t) = \left(\left(\frac{\pi^2}{50} - 6\right)x^2 + \left(5 - \frac{\pi^2}{50}\right)x^4 + \frac{\pi^2 x^6}{150}\right)\mathrm{Sin}\left(\frac{\pi t}{5}\right) \quad (44)$$

It's easy to verify that the exact solution of magnetic field is:

$$B(x,t) = \ x(x^2-1)^2 * \mathrm{Sin}\left(\frac{\pi t}{5}\right) \quad (45)$$

The above solution will be used to test our numerical method, namely: finite volume method, and weighted residual method

Figure 3.4

Density plot for the magnetic field B as function of x and t a) using MWR b) FVM c) the exact analytical solution

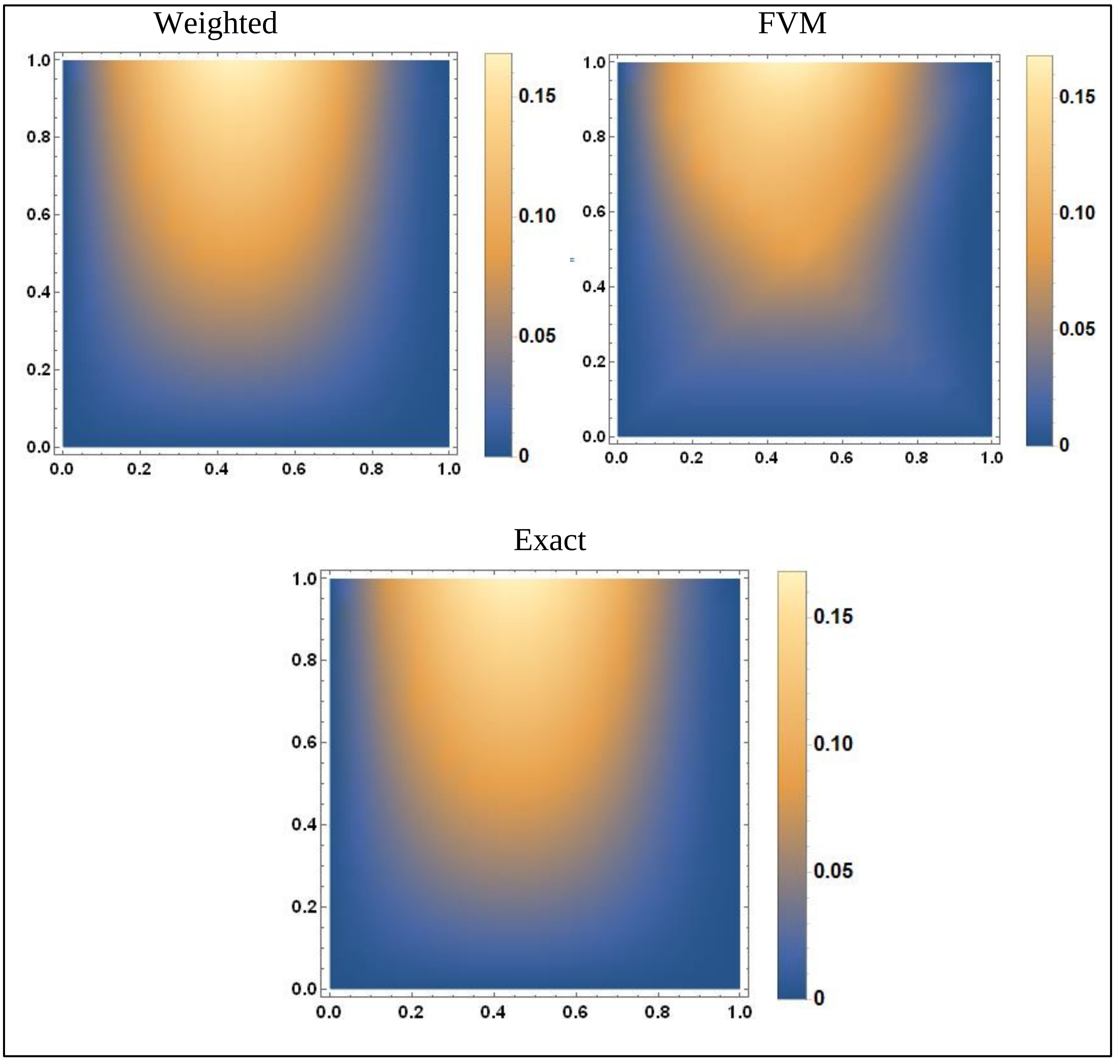


Figure 3.5

B(x) as function of x, for at different time a) t=0.25, b) t=0.5, c) t 0.75, and d) t=1

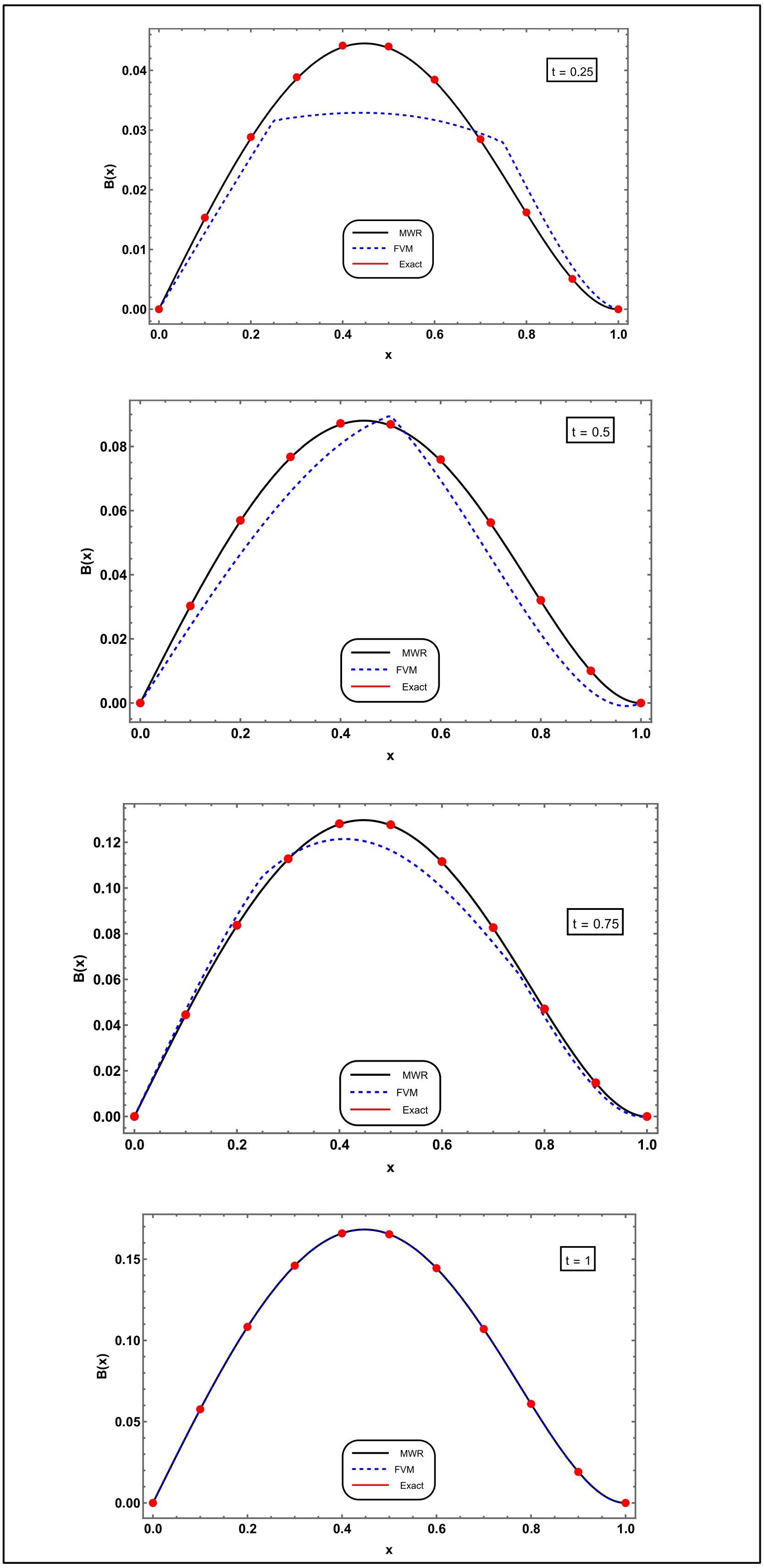


Table 3.1

B(x,t) using MWR , FVM, and exact solution

i \ j	1	2	3	4	5
1	-0.6088415928243034	-0.05114929127772969	0.0228234569752172	0.135237293463735	-0.08556599285169866
2	-0.682019403972548	0.13819199902545662	0.39207617012919094	-1.0444553640422525	0.6081820934786566
3	0.6989045739403678	-0.18388649212441224	-0.6089957744151053	1.706669927246303	-1.0316948900833505
4	0.4961264673868245	0.9011539574572782	-1.9517536126457011	1.4695700594190613	-0.3097477468401946
5	0.12972270131296137	-1.0600566065466	2.829197747621604	-3.007472821032477	1.0971387771765064

Each method outlined above provides a unique approach to solving Maxwell's equations. FVM focuses on conserving quantities locally and is usually easy to implement, but it may need finer discretization for complex shapes. MWR can be more complex to set up because it requires choosing the right trial and weighting functions. However, it is flexible and can produce very accurate results if the functions match the solution well. These examples give a basic understanding of how to implement these methods in MATLAB, but further work is needed to apply them to specific problems and boundary conditions.

Comparison between the two methods

The comparison was made based on several factors, as follows:

3.6.1 Accuracy vs. Speed

FVM is thought to be more precise because it tackles fluxes and boundary conditions in great detail. Since it uses a more stringent and narrow grid and ensures local retention of physical properties, it is the favored approach for issues with very intricate boundaries or sudden changes. However, this raises precision leads to longer computation times.

Because WRM is quicker and easier to implement, it is a better option when computational efficiency is essential. However, it may lose some of its accuracy, especially when working with highly sophisticated field borders or highly complex alterations.

3.6.2 Methodological Trade-offs

The Finite Volume Method (FVM) in Code 1 works effectively for issues that call for precise quantity preserving and the resolution of complex field behaviors. It is particularly useful when there are significant changes to the solution or when maintaining the physical features of the system is crucial.

Code 2's Weighted Residual Method (WRM) is the preferred approach for rapid-onset simulations or specific scenarios where high precision isn't vital to the problem because it is simpler to use and calls for less processing power.

3.6.3 Source Implementation

Code 1 employs a simple impulse source, which may result in numerical issues and produce unpredictable abrupt changes in the field.

Although Code 2's accuracy is dependent on the limitations of the technique itself, it employs a smoother Gaussian pulse, which is more stable and unlikely to cause problems.

3.7 Results

- Although electromagnetic fluxes were efficiently maintained using the Finite Volume Method (FVM), there was some numerical diffusion, especially in regions with abrupt shifts. It was perfect for large-scale structured grid simulations and highly computationally effective.
- In regions where the field shifted smoothly, the Method of Weighted Residuals (MWR) produced highly accurate results. However, because the right trial and weighting functions had to be selected, it was more challenging to integrate. When working with extremely complex shapes that needed a high degree of precision, MWR proved exceptionally helpful.

3.8 Conclusion

This chapter has covered in detail two methods for solving Maxwell's equations: FVM and MWR. Every strategy has clear pros and cons. While FVM is useful for conservation-focused scenarios with simpler shapes, MWR provides more accuracy and resilience for more complex or particularly intricate difficulties. The choice of approach can significantly affect the accuracy and efficiency of the solution, depending on the specific issue at hand and environment.

Each of FVM and MWR has advantages and disadvantages based on the problem at hand and the current situation. While FVM is particularly useful for situations where flux conservation and processing efficiency are of great significance, MWR is preferable for simulations that need high accuracy in complex domains. Actually, the optimal choice between FVM and MWR should depend on the problem's present criteria, including the required accuracy, the available computer resources, and the boundary conditions.

FVM is without unquestionably the best choice when accuracy and comprehensive, conservation-focused simulations are essential, especially for exceptional challenges. WRM is a more practical choice for faster, less resource-intensive simulations when small accuracy trade-offs are acceptable. Which of the two methods is best should depend on the problem's requirements, which should create a logical balance between accuracy and computer economy.

List of Abbreviations

Abbreviation	Meaning
CFD	Computational Fluid Dynamics
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
HFA	High frequency Applications
LFA	Low frequency Applications
MoM	Method of Moments
MWR	Method of Weighted Residuals
ODE	Ordinary Differential Equations
PDE	Partial Differential Equations

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Appendices

Appendix A

Figures

Figure 3.6

Gaussian source at center, Ey pulses

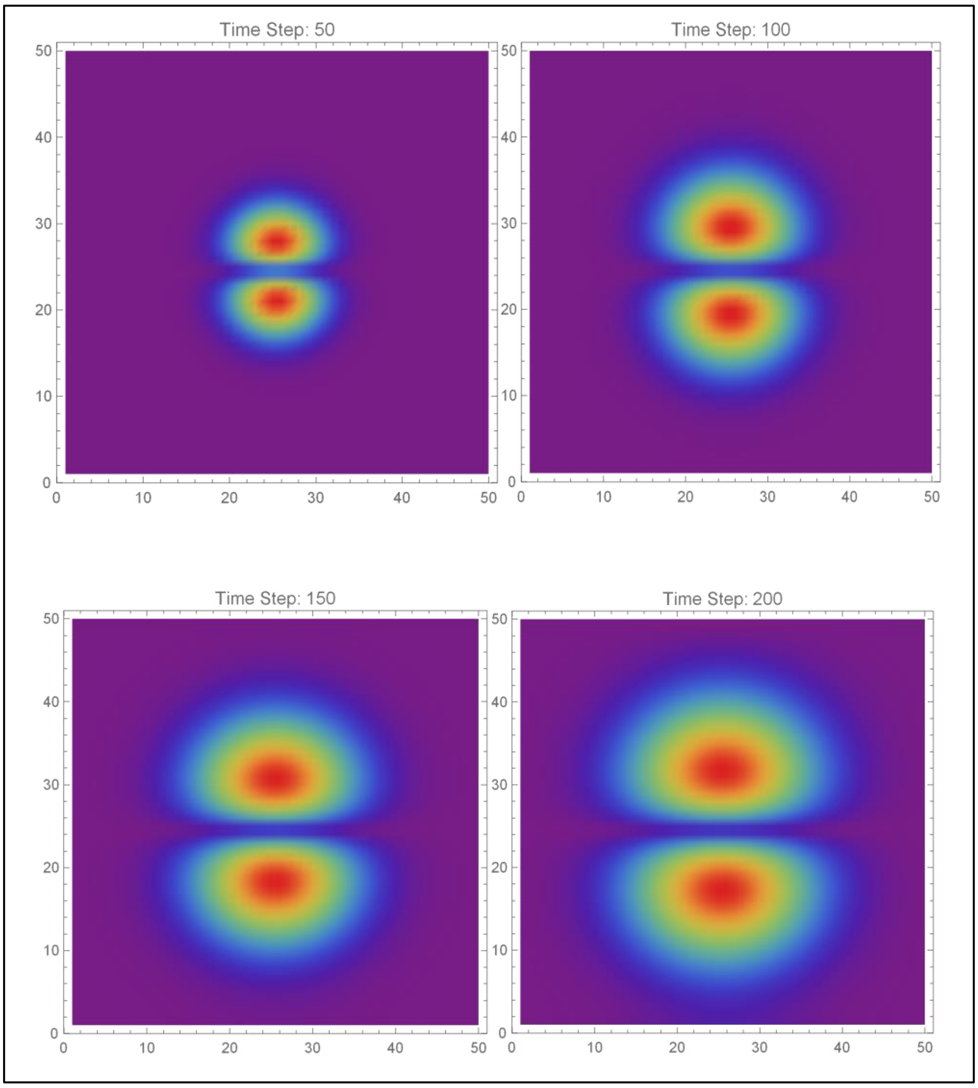
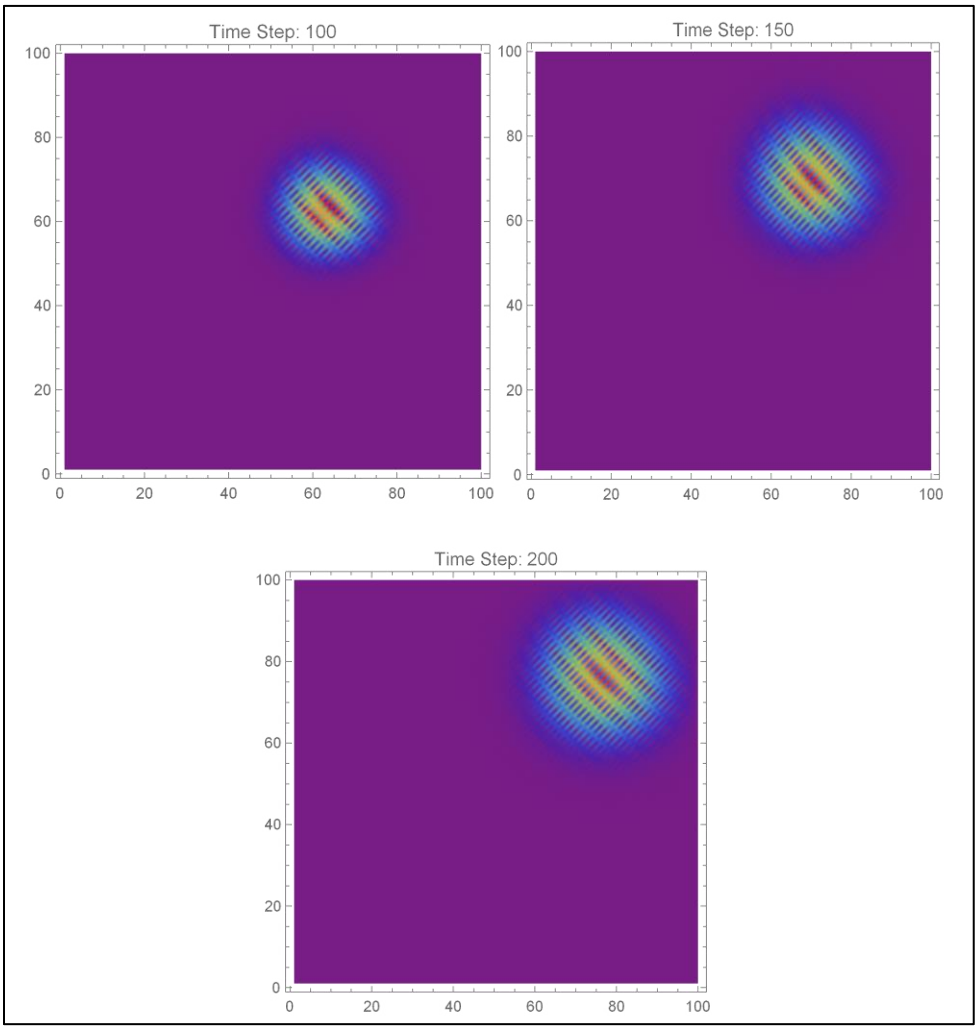


Figure 3.7

Point pulses in center



جـامعـــة النجاح الوطنية

كليـــة الدراســـات العليـــا

طريقة الحجم المحدود وطريقة القيم المتبقية لحل معادلات ماكسويل

إعداد

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إشراف

د. فاطمة عقل

قدمت هذه الرسالة استكمالا لمتطلبات الحصول على درجة الماجستير في الرياضيات المحوسبة، من كلية الدراسات العليا، في جامعة النجاح الوطنية، نابلس– فلسطين.

2025

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الملخص

تقدم هذه الأطروحة دراسة مقارنة لطريقتين عدديتين لحل معادلات ماكسويل هما: طريقة الحجم المحدود (FVM)، وطريقة المتبقيات المرجحة (MWR). تعد معادلات ماكسويل أساسية لفهم المجالات الكهرومغناطيسية وغيرها من الظواهر الفيزيائية. وتعد معادلات ماكسويل حجر الأساس في العديد من التطبيقات التكنولوجية، بما في ذلك الاتصالات اللاسلكية، وانتشار الموجات الكهرومغناطيسية، والهندسة الكهربائية.

تقسم طريقة الحجم المحدود المجال الحسابي إلى أحجام صغيرة وتطبق قوانين الحفاظ على هذه الأحجام، مما يضمن الحفاظ على التدفقات بدقة عبر حدود كل حجم. هذه الطريقة فعالة بشكل خاص للمشاكل التي تتطلب دقة عالية في الحفاظ على التدفق وتستخدم على نطاق واسع في ديناميكيات السوائل الحسابية والمحاكاة الكهرومغناطيسية.

من ناحية أخرى، تتضمن طريقة المتبقيات المرجحة افتراض حل تقريبي ثم تقليل المتبقيات للمعادلات الحاكمة من خلال وظائف الترجيح المناسبة. توفر هذه الطريقة المرونة في التعامل مع الأشكال الهندسية المعقدة وظروف الحدود، ويمكنها تحقيق دقة عالية إذا تم اختيار وظائف التجربة بشكل جيد.

تستكشف هذه الأطروحة تنفيذ كلتا الطريقتين في برنامج MATLAB وتقارن أدائهما في حل معادلات ماكسويل. تتم المقارنة من حيث الدقة والكفاءة الحسابية وسهولة التنفيذ. توضح النتائج أنه في حين توفر FVM دقة أكبر في الحفاظ على التدفق، توفر MWR مرونة أعلى وإمكانية للدقة في السيناريوهات المعقدة.

بشكل عام، تساهم هذه الأطروحة في التطوير المستمر للطرق العددية الفعالة لحل المشكلات، مع التطبيقات المحتملة في التقنيات المتقدمة.

الكلمات المفتاحية: معادلات ماكسويل، طريقة الحجم المحدود (FVM)، طريقة المتبقيات المرجحة (MWR).