EVALUATION OF THE STIFFNESS OF BEAM AND FRAME ELEMENTS USING SYSTEM IDENTIFICATION TECHNIQUES

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ABSTRACT

Standard Methods of structural analysis become inapplicable for computing stiffness matrices for existing structures that have witnessed material deterioration and old age. Helou presented a method to overcome such a difficulty through the creation of an error function followed by a minimization technique. However, that presentation was limited to both determinate and indeterminate trusses. The following work is an extension of the same minimization techniques to include beam and framed structures.

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INTRODUCTION

Traditional methods of structural analysis begin usually by the construction of a mathematical model that best represents the prototype structure. Furthermore, the material and geometric properties of the structure appear in the what is known to be either the stiffness matrix or the flexibility matrix, depending on whether the displacement or the force method is used.

The subsequent analysis , i.e. the determination of the response of the system due to any set of loading condition hinges on the availability of such matrices. However, in situations where such information is absent or cannot be accurately computed, the analysis resorts to approximations which undoubtedly lead to erroneous results or to inaccurate ones at best. This is the case with ageing structures that have been in use for long periods of time or when the construction materials have greatly deteriorated. System identification is a method to improve mathematical modelling. Amethod which overcomes the difficulty resulting from the unknown material and geometric properties through a minimization algorithm is known to some applied science disciplines but not widely known to structural engineers. Helou (1)(2) made his presentation limeted to determinate and indeterminate trusses. The present study is an application of the same principles to beam and framed structures, thus the validity of the derived equations would be established and made general to include any structures.

The following include the derivation of the equations, followed by numerical experiment. Noting that here also a loading and a displacement information is also needed at every degree of freedom. This is a shortcounting that demands further research.

REVIEW OF THE MATHMATICAL PROBLEM AND ITS SOLUTION

In standard methods of structural analysis the stiffness matrix which is governed by the structure's topology and material is usually given. However, when this is not the case i.e when the force and the displacement vectors are known and [k] is to be computed an error vector defined by the following equation may be introduced.

$$\{E\} = \{F\} - [k]\{x\} \qquad \text{....(1)}$$

in which

[E] is the error vector

[F] is the loading vector

[K] is the stiffness matrix

and [X] is the associated displacement vector

The minimization procedure starts by writing a typical element in equation 1.

When the error vector is minmized to zero the correct solution will be obtained. A typical element of the error vector is withen as

$$E_i = F_i - \sum_{i=1}^{n} K_{ij} X_j \qquad \text{.....(2)}$$

in which n is the number of degrees of freedom.

The error function to be minimized is obtained by squaring the right hand side of equation 2 and summing it over the number of loading conditions.

$$EF = \sum_{j=1}^{m}\sum_{i=1}^{n}\left[F_i^j - \sum_{l=1}^{n} K_{il}x_l^j\right]^2$$

in which to m is the number of loading cases.

Once the error function is constructed the minimization is to be done with respect to each unknown element of the stiffness matrix. This is accomplished by forming the first derivative of EF with respect to the unknown elements of [k] and setting this equal to zero. i.e

$$\frac{\partial EF}{\partial K_i} = 0.0 \qquad (6)$$

This operation results in a set of linear simultaneous equations equal in number to the number of unknown element stiffnesses. solving the set of equation for the vector {k} of the unknown stiffnesses yields

$$\{k\} = \left(\sum_{j=1}^{m} [J^j]^T[J][K]\right)^{-1} \cdot \sum_{j=1}^{m} [J^j]^T\{F\}^j \qquad \ldots.(7)$$

in which [J] is referred to as Jacobean matrix defined as follows.

$$[J] = \begin{bmatrix} \frac{\partial EF_1}{\partial K_1} & \cdots & \frac{\partial EF_1}{\partial K_m} \\ \vdots & \cdots & \vdots \\ \frac{\partial EF_n}{\partial K_1} & \cdots & \frac{\partial EF_n}{\partial K_m} \end{bmatrix} \qquad (8)$$

joint 2 is

$$[K] = \begin{bmatrix} 12\frac{k1}{L_1^3}+12\frac{k2}{L_2^3} & SYM \\ -6\frac{k1}{L_1^2}+6\frac{k2}{L_2^2} & 4\frac{k1}{L_1}+4\frac{k2}{L_2} \end{bmatrix}$$

The following are the equivalent joint load action for the loading case shown together with the associated displacements used in the present numerical experiments.

$$\{F\} = \begin{Bmatrix} -2.0 \\ 1.0 \end{Bmatrix} \qquad \{X\} = \begin{Bmatrix} -6.7878\ E\text{-}5 \\ 4.848\ E\text{-}6 \end{Bmatrix} m$$

The error vector is written as

$$\begin{Bmatrix} E1 \\ E2 \end{Bmatrix} = \begin{Bmatrix} F1 \\ F2 \end{Bmatrix} - \begin{bmatrix} 12\frac{k1}{L_1^3}+12\frac{k2}{L_2^3} & SYM \\ -6\frac{k1}{L_1^2}+6\frac{k2}{L_2^2} & 4\frac{k1}{L_1}+4\frac{k2}{L_2} \end{bmatrix} \begin{Bmatrix} X1 \\ X2 \end{Bmatrix}$$

The Jacobean Matrix is

$$[J] = \begin{bmatrix} 12\frac{X1}{L_1^3}-6\frac{X2}{L_1^2} & 12\frac{X1}{L_2^3}+6\frac{X2}{L_2^2} \\ -6\frac{X1}{L_1^2}+4\frac{X2}{L_1} & 6\frac{X1}{L_2^2}+4\frac{X2}{L_2} \end{bmatrix}$$

The details of the above mathematical operations are outlined in Helou (1) & (2).

The following numerical experiments on beam and framed structures illustrate the use of the above equation and establish their validity.

Numerical experiment.

The following two examples illustrate the solution .In this examples a beam and one story frame configuration is chosen for simplicity in which displacements were actually computed using the standard direct stiffness method .

EXAMPLE 1 {BEAM}

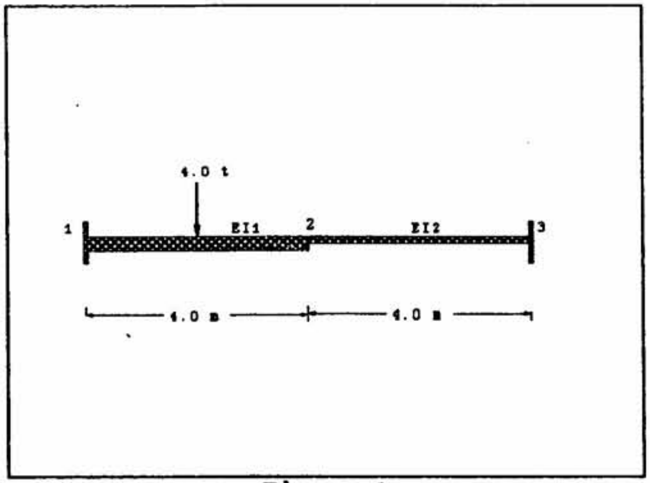

Figure 1

For the beam shown in Figure 1 the modules of elasticity 20,000,000 t/m2 and I_1=.005 m^4 ,I_2=.0025 m^4, k_1=EI_1 ,k_2=EI_2 ,L_1=L_2= 4 m

The global reduced stiffness matrix obtained by standard structural analysis methods for the degree of freedom at

Upon performing the operation described in equation 7 the unknown elements stiffnesses are the readily obtained. They are the same as would be obtained by evaluating {K = EI} for each element.

$$\{K\}=\begin{bmatrix}100000\\50000\end{bmatrix} t/m^2$$

EXAMPLE NO. 2 {FRAME}

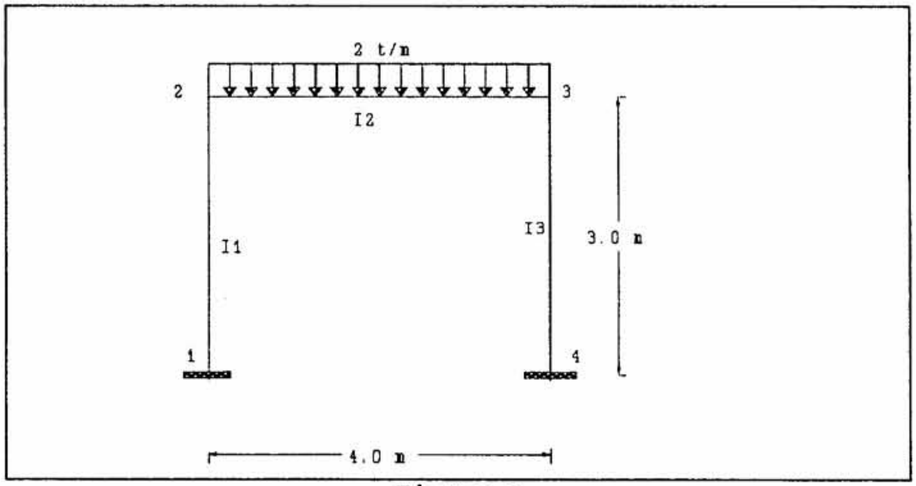

Figure 2

For the frame shown in Figure 1 the modules of elasticity 20,000,000 t/m2 and I_1=.002 m^4 ,I_2=.003 m^4,I_3=.004 m^4 $k_1=EI_1$, $k_2=EI_2$,$k_3=EI_3$,$L_1=L_3$= 3 m ,L_2=4m

The global reduced stiffness matrix obtained by standard structural analysis methods for degree of freedom at joint 2 and 3 neglecting axial deformation is

$$[K]=\begin{bmatrix} 12\dfrac{k1}{L_1^3}+12\dfrac{k3}{L_3^3} & & SYM \\ 6\dfrac{k1}{L_1^2} & 4\dfrac{k1}{L_1}+4\dfrac{k2}{L_2} & \\ 6\dfrac{k3}{L_3^2} & 2\dfrac{k2}{L_2} & 4\dfrac{k2}{L_2}+4\dfrac{k3}{L_3} \end{bmatrix}$$

The following are the equivalent joint load action for the loading case shown together with the associated displacements used in the present numerical experiments.

$$\{F\}=\begin{Bmatrix} 0.0 \\ -2.666 \\ 2.666 \end{Bmatrix} \qquad \{X\}=\begin{Bmatrix} -.10487\ E-4 \\ -.2751\ E-4 \\ .2436\ E-4 \end{Bmatrix}$$

The error vector is written as

$$\begin{Bmatrix} E1 \\ E2 \\ E3 \end{Bmatrix}=\begin{Bmatrix} F1 \\ F2 \\ F3 \end{Bmatrix}-\begin{bmatrix} 12\dfrac{k1}{L_1^3}+12\dfrac{k3}{L_3^3} & & SYM \\ 6\dfrac{k1}{L_1^2} & 4\dfrac{k1}{L_1}+4\dfrac{k2}{L_2} & \\ 6\dfrac{k3}{L_3^2} & 2\dfrac{k2}{L_2} & 4\dfrac{k2}{L_2}+4\dfrac{k}{l} \end{bmatrix}$$

The Jacobean Matrix is

$$[J]-\begin{bmatrix} 12\frac{X1}{L_1^3}+6\frac{X2}{L_1^2} & 0 & 12\frac{X1}{L_3^3}+6\frac{X3}{L_3^2} \\ 6\frac{X1}{L_1^2}+4\frac{X2}{L_1^2} & 4\frac{X1}{L_1}+2\frac{X3}{L_2} & 0 \\ 0 & 2\frac{X2}{L_2^2}+4\frac{X3}{L_2^2} & 4\frac{X3}{L_3}+6\frac{X1}{L_3^2} \end{bmatrix}$$

Upon performing the operation described in equation 7 the unknown elements stiffnesses are the readily obtained. They are the same as would be obtained by evaluating {K = EI} for each element.

$$\{K\}-\begin{bmatrix} 40000 \\ 59814 \\ 79973 \end{bmatrix} t/m^2$$

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