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Evaluation of The Stiffness Matrix of An Indeterminate Truss Using Minimization Techniques

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ABSTRACT

For an existing reinforced concrete or steel structure the evaluation of the stiffness matrix may be hampered by certain physical limitations such as material deterioration resulting from prolonged use an adverse or in a corrossive environment. The following is a method that allows the determination of the member stiffness of an indeterminate truss through a minimization technique of a properly constructed Error Function. Thus exact sectional and material properties do not have to be known a priori.

ملخـــص

ان ايجاد مصفوفة القساوة في المنشآت القائمة يعتبر امراً في غاية الصعوبة خاصة اذا أدت الظروف المحيطة في المنشأ الى تغير في خصائص مواد الانشاء. الطريقة المعروضة في هذا السياق تتغلب على هذه الصعوبة وتمكن من ايجاد مصفوفة القساوة لجملون غير محدد استاتيكا بواسطة اسلوب رياضي بحت.

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Introduction

Analysis of an existing old structure is usually difficult to perform due to the fact that material properties change over time. This limitation is frequently encountered in industrial buildings housing a corrossive environment. Particularly relevant are reinforeced concrete buildings in which moisture cause steel rusting and eventual spalling of concrete, specially if this is coupled with lack of effective maintenance. Therefore standard structural analysis methods become inapplicable if accurate results are sought, because such methods hinge upon the availability of the member properties and the geometry of the structure. The following method overcomes such a difficulty through the application of known forces at the nodes and the subsequent measurement of the associated displacements. Ealier presentation of the method(1) were limited to determinate trusses. The following is an extension of the same principles, albeit in more general terms, in order to make the method equally applicable to indeterminate systems.

It should be noted , however , that this solution in its present form is conceptual in nature and requires further development in order to make it suitable for the industrial community.

Problem Statement and Solution :

In structural mechanics the force, displacement equilibrium equation is written in the following form

$$\{F\}=[K]\{X\} \tag{1}$$

In which

$\{F\}$ is the force vector applied at the nodes,

$[K]$ is a global stiffness matrix,

$\{X\}$ is the associated displacement vector at the nodes.

For the exact solution of equation 1 , the following statement is true

$$\{F\}-[K]\{X\}=0 \qquad (2)$$

And when equation 1 is not exact an error vector E may be introduced as follows:

$$\{E\}=\{F\}-[K]\{X\} \qquad (3)$$

A typical element in the error of equation 3 is of the form

$$E_i = F_i - \sum_{i=1}^{n} K_{ij} X_i \qquad (4)$$

where n is the number of degrees of freedom.

The problem is now reduced to minimizing to zero the error vector of equation 4. For this to be achieved an error function has to be constructed. This is done by squaring both sides of equation 4, i.e by forming the inner product of the right hand side of equation 4 with itself and carrying out the summation over the number of loading conditions. The necessity of using more than one load vector will be made clear later on in the text. The error function takes the form

$$EF = \sum_{j=1}^{m} \sum_{i=1}^{n} [F_i^j - \sum_{k=1}^{n} K_{ik} X_k]^2 \qquad (5)$$

in which m is the number of loading cases ,

The solution proceeds by taking the first derivative of the error function , EF, with respect to each unknown element of the stiffness matrix and setting it equal to zero, i.e .

$$\frac{\partial EF}{\partial E_i} = 0 \qquad (6)$$

This operation will result in a set of linear simultaneous equations equal in number to the elements of the structure

$$\sum_{j=1}^{m}[J^j]^J(\{F\}^j - [K]\{X\}^j) = 0 \tag{7}$$

in which [J] is a Jacobean matrix defined as follows

$$[J] = \begin{bmatrix} \dfrac{\partial\ EF_1}{\partial\ k_1} & . & . & \dfrac{\partial\ EF_1}{\partial\ k_m} \\ . & . & . & . \\ . & . & . & . \\ \dfrac{\partial\ EF_n}{\partial\ k_n} & . & . & \dfrac{\partial\ EF_n}{\partial\ k_m} \end{bmatrix} \tag{8}$$

Equation 7 yields

$$\begin{aligned} \sum_{j=1}^{m}[J^j]^T\{F\} &= \sum_{j=1}^{m}[J^j]^T[K]\{X\}^j \\ &= \sum_{j=1}^{m}[J^j]^T[J]\{K\} \end{aligned} \tag{9}$$

where {k} is the vector of element stiffnesses in local coordinates. Furthermore with $[J^j]^T[J]$ invertible the solution for {k} is written formally as

$$\{k\} = (\sum_{j=1}^{m}[J^j]^T[J][k])^{-1} \cdot \sum_{j=1}^{m}[J^j]^T\{F\}^j \tag{10}$$

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From the solution it remaines to be shown that {J}{K}={K}{X}. This will be shown in the course of the illustrative example.

Illustrative Example :

Figure 1-a
Load case No. 1

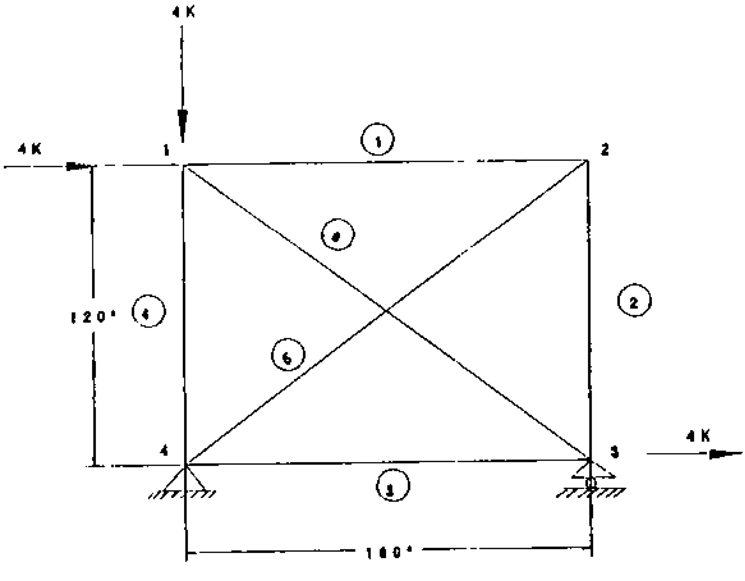
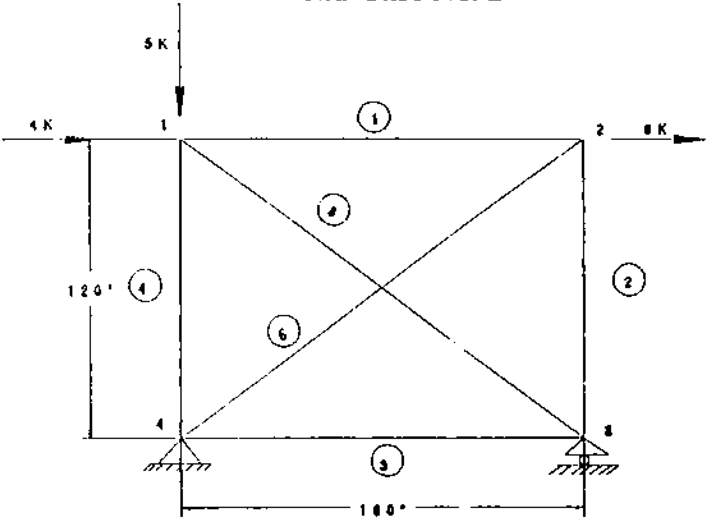


Figure 1-b
Load Case No. 2



For the indeterminate truss shown in Figure 1 -a and b. All elements have an area of 4 in^2 and modulus of elasticity =30000 ksi. The global reduced stiffness

$$\{k\} = \begin{bmatrix} k1+.64k6 & \cdot & \cdot & \cdot & \text{SYM} \\ -.48k6 & k4+.36k6 & \cdot & \cdot & \cdot \\ -k1 & 0 & k1+.64k5 & \cdot & \cdot \\ 0 & 0 & .48k5 & k2+.36k5 & \cdot \\ -.64k6 & .48k6 & 0 & 0 & k3+.64k6 \end{bmatrix}$$

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To assure the existence of a solution two loading cases are used. The following are the loading cases together with the associated displacements used in the present numerical experiments.

$$\{F^1\} = \begin{Bmatrix} 4 \\ -4 \\ 0 \\ 0 \\ 4 \end{Bmatrix} \qquad \{X^1\} = \begin{Bmatrix} .01017 \\ -.002594 \\ .007154 \\ -.001694 \\ .007654 \end{Bmatrix}$$

and

$$\{F^2\} = \begin{Bmatrix} 4 \\ -5 \\ 6 \\ 0 \\ 0 \end{Bmatrix} \qquad \{X^2\} = \begin{Bmatrix} .0663 \\ -.00604 \\ .01733 \\ -.001104 \\ .006037 \end{Bmatrix}$$

The error vector is written as

$$\begin{Bmatrix} E1 \\ E2 \\ E3 \\ E4 \\ E5 \end{Bmatrix} = \begin{Bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \end{Bmatrix} - \begin{bmatrix} k1+.64k6 & . & . & . & SYM \\ -.48k6 & k4+.36k6 & . & . & . \\ -k1 & 0 & k1+.64k5 & . & . \\ 0 & 0 & .48k5 & k2+.36k5 & . \\ -.64k6 & .48k6 & 0 & 0 & k3+.64k6 \end{bmatrix} \begin{Bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \end{Bmatrix}$$

The Jacobean matrix is

$$\begin{bmatrix} x1-x3 & 0 & 0 & 0 & 0 & .64x1-.48x2-.64x5 \\ 0 & 0 & 0 & x2 & 0 & -.48x1+.36x2+.48x5 \\ -x1+x3 & 0 & 0 & 0 & .64x3+.48x4 & 0 \\ 0 & x4 & 0 & 0 & .84x3+.36x4 & 0 \\ 0 & 0 & x5 & & 0 & \end{bmatrix}$$

Upon performing the operation described in equation 10 the unknown elements stiffnesses are readily obtained. They are the same as would be obtained by evaluating EA/L for each element.

$$\{K\} = \begin{bmatrix} 749.36 \\ 1000.63 \\ 749.87 \\ 1000.44 \\ 600.28 \\ 599.48 \end{bmatrix} \text{kip / in}$$

Concluding Remarks :

From the previous presentation and example it is apparent that the proposed method requires a complete set of data i. e. A displacement reading must be available at every degree of freedom of the structure. This is a shortcoming that perhaps can be avoided through further research.

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